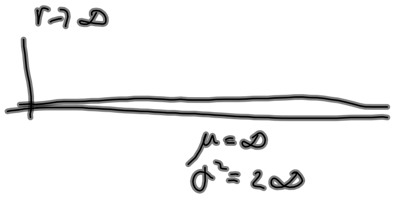
Lecture 29

$\hat{\sigma}_0^2 = \frac{v^T W v}{r}$ 29-1

$\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi_r^2$



$\frac{\hat{\sigma}_0^2}{\sigma_0^2} \sim \frac{1}{r} \chi_r^2$

See plot on notes page.

$y = Ax$, $\Sigma_{yy} = A \Sigma_{xx} A^T$

$y = Ax + Bw$, where x & w are r.v.

$y = [A \ B] \begin{bmatrix} x \\ w \end{bmatrix}$

$\Sigma_{yy} = [A \ B] \, \Sigma_{\binom{x}{w}\binom{x}{w}} \, [A \ B]^T$

$\Sigma_{\binom{x}{w}} = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xw} \\ \Sigma_{wx} & \Sigma_{ww} \end{bmatrix}$, or $\begin{bmatrix} \Sigma_{xx} & 0 \\ 0 & \Sigma_{ww} \end{bmatrix}$

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$\Sigma_{\binom{x}{w}} = \begin{pmatrix} \Sigma_{xx} & 0 \\ 0 & \Sigma_{ww} \end{pmatrix}$ 29-2

$\Sigma_{yy} = [A \ B] \begin{pmatrix} \Sigma_{xx} & 0 \\ 0 & \Sigma_{ww} \end{pmatrix} \begin{bmatrix} A^T \\ B^T \end{bmatrix}$

$= \begin{pmatrix} A\Sigma_{xx} & B\Sigma_{ww} \end{pmatrix} \begin{pmatrix} A^T \\ B^T \end{pmatrix}$

$= A \Sigma_{xx} A^T + B \Sigma_{ww} B^T$

$\Sigma_{\binom{x}{w}} = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xw} \\ \Sigma_{wx} & \Sigma_{ww} \end{bmatrix}$

$\Sigma_{yy} = [A \ B] \begin{bmatrix} \Sigma_{xx} & \Sigma_{xw} \\ \Sigma_{wx} & \Sigma_{ww} \end{bmatrix} \begin{pmatrix} A^T \\ B^T \end{pmatrix}$

$\begin{bmatrix} A\Sigma_{xx} + B\Sigma_{wx} & A\Sigma_{xw} + B\Sigma_{ww} \end{bmatrix} \begin{pmatrix} A^T \\ B^T \end{pmatrix}$

$A\Sigma_{xx}A^T + \left[B\Sigma_{wx}A^T + A\Sigma_{xw}B^T \right] + B\Sigma_{ww}B^T$

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Global Test comparing v's with σ's 29-3

$\hat{\sigma}_0^2$ vs. $\sigma_0^2 \Longleftarrow$

$\uparrow \frac{v^TWv}{r}$

not evaluating choice of σ_0^2

3 quantities

σ^2 : population value

σ_0^2 : we have chosen

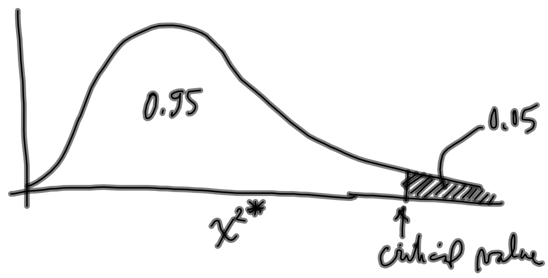
$\hat{\sigma}_0^2$: post adj. est. of

Hypothesis Test

$H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 > \sigma_0^2$

test statistic $\chi^{2*} = \frac{v^TWv}{\sigma_0^2} = \frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi_r^2$



$\alpha = 0.05$

significance level 0.05

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decision rule 29-4

if $\chi^{2*} <$ c.v. accept H_0

otherwise reject H_0, accept alternate H_1

α : level of significance of test

: prob. of Type 1 error

χ^{2*} : $\frac{v^TWv}{\sigma_0^2}$ from your adjustment

choose α, get c.v.

make comparison

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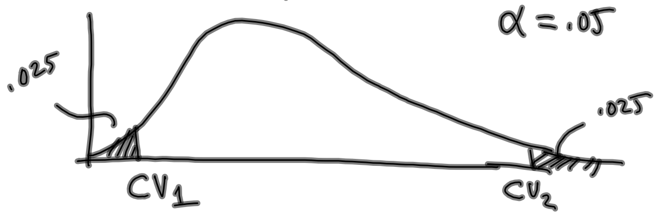
for 2-sided test 29-5

$H_0 = \sigma^2 = \sigma_0^2$

$H_1 = \sigma^2 \neq \sigma_0^2$

$\alpha = .05$



decision rule

if $cv_1 < \chi^{2*} < cv_2$ accept H_0

otherwise reject H_0, accept H_1

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