Lecture 32 Final Exam reschedule ?   32-1

~~tentative on Tues. 15th~~

tentative Sat. 12th

$(\vec{x}-\vec{\mu}_x)^T \Sigma_{xx}^{-1} (\vec{x}-\mu_x)$

assumption Σ_{xx} diagonal

$$\Sigma_{xx}^{-1} \rightarrow \begin{bmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \frac{1}{\sigma_2^2} & \\ 0 & & \ddots & \frac{1}{\sigma_n^2} \end{bmatrix}$$

$$\frac{(x_1-\mu_{x_1})^2}{\sigma_1^2} + \frac{(x_2-\mu_{x_2})^2}{\sigma_2^2} + \cdots + \frac{(x_n-\mu_{x_n})^2}{\sigma_n^2}$$

$$z_1^2 + z_2^2 + \cdots z_n^2 \sim \chi_n^2$$

$$z_1^2 + z_1^2 \sim \chi_2^2$$

1. write probability statement
2. rearrange it

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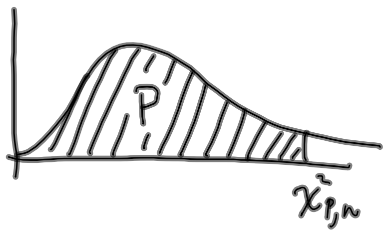
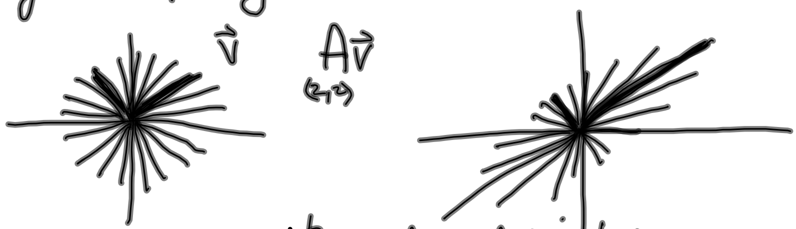
$$(\vec{x}-\vec{\mu}_x)^T \Sigma_{xx}^{-1} (\vec{x}-\vec{\mu}_x) \sim \chi_n^2$$

also true if Σ_{xx} not diagonal

$$\text{prob}\left\{(x-\mu_x)^T \Sigma_{xx}^{-1} (x-\mu_x) < \chi_{p,n}^2\right\} = p$$



eigenvalues / eigenvectors 32-3



2 vectors with unchanged direction
eigenvectors

2 scale factors eigenvalues

A symmetric, λ's are real
e.vectors are orthogonal

We work with Σ_{xx} : symmetric

$Av = \lambda v$ $\qquad (\underset{2,2}{A} - \underset{2,2}{I}\lambda)v = 0$

$Av - \lambda v = 0$ $\qquad$ if nontrivial solution then

$Av - I\lambda v = 0$ $\qquad$ matrix must be singular

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$\det(A - \lambda I) = 0$ 32-4

characteristic equation, solution is λ

$A v_1 = \lambda_1 v_1$ $\qquad \underset{2\times 1}{v_i} : \begin{bmatrix} \vdots \end{bmatrix}$
$A v_2 = \lambda_2 v_2$

$$\underset{2,2}{A}\begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$\boxed{AV = V\Lambda}$ $\qquad \boxed{\begin{matrix} [V, D] = eig(A) \\ AV = VD \end{matrix}}$

$AV = V\Lambda$

$A = V\Lambda V^{-1}$ $\qquad$ A sym : v are orthogonal
interpret V as rotation matrix

eigenvalue decomposition

$\Sigma = V\Lambda V^{-1}$ $\qquad V = R^T$

$\Sigma = R^T D R$ $\qquad \Lambda = D$

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$$\Sigma = R^T D R$$

$$R \Sigma R^T = D, \quad \boxed{D^{-1} = R \Sigma^{-1} R^T}$$

$$\text{Prob}\Big[\underbrace{(X-\mu_x)^T R^T}_{}\underbrace{R \; \Sigma_{xx}^{-1} R^T}_{}\underbrace{R(X-\mu_x)}_{} < \chi^2_{p,2}\Big] = P$$

$$\vec{w} = R(x-\mu_x)$$
$$\vec{w}^T = (x-\mu_x)^T R^T$$

$$D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$$

$$D^{-1} \begin{bmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{bmatrix}$$

$$\text{Prob}\left[w^T D^{-1} w < \chi^2_{p,2}\right] = P$$

$$\text{Prob}\left[\begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} < \chi^2_{p,2}\right] = P$$

$$\text{Prob}\left[\frac{w_1^2}{d_1} + \frac{w_2^2}{d_2} < \chi^2_{p,2}\right] = P$$

$$\text{Prob}\left[\frac{w_1^2}{d_1 \cdot \chi^2_{p,2}} + \frac{w_2^2}{d_2 \cdot \chi^2_{p,2}} < 1\right] = P$$

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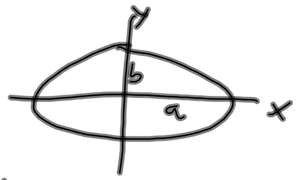
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



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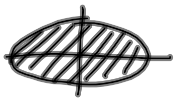
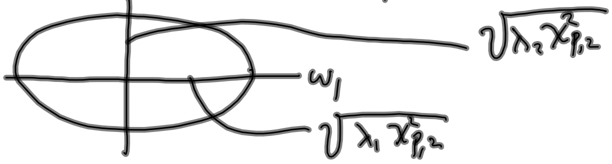
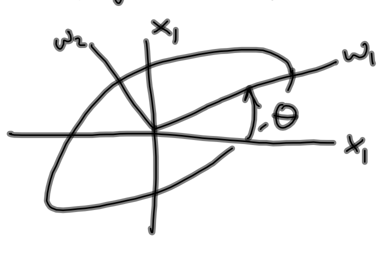




$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = R(x-\mu_x)$$



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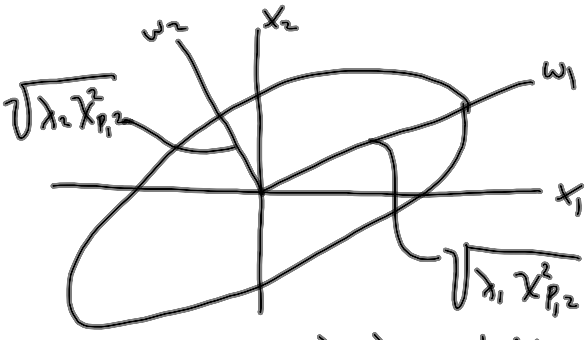

λ_1, λ_2 sorted by magnitude

<u>interpret w.r.t. region</u> (next time!)

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