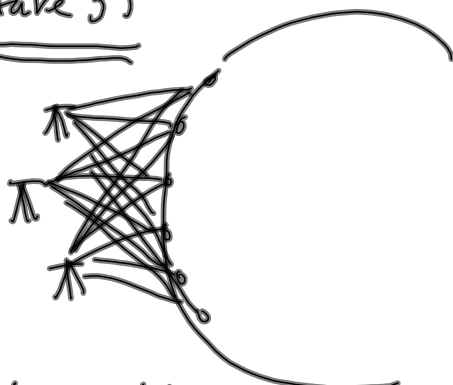


Lecture 39

39-0



constrain points to lie along a circle

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

yields
equation of
circle through
3 pts.

if replace x, y with x_4, y_4 constrains all
4 points to lie along circle

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line through 2 pts.
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \quad 39-1$$

conic section through 5 pts

$$\begin{vmatrix} x^2 & xy & y^2 & x & y & 1 \\ x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_5^2 & x_5 y_5 & y_5^2 & x_5 & y_5 & 1 \end{vmatrix} = 0$$

plane through 3 pts

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

sphere through 4 pts

$$\begin{vmatrix} x^2+y^2+z^2 & x & y & z & 1 \\ x_1^2+y_1^2+z_1^2 & x_1 & y_1 & z_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_4^2+y_4^2+z_4^2 & x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

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2nd approach to constraints is to introduce

39-2

"added parameters" to make it easy to write eqn

new parameters x_c, y_c, R

$$\begin{aligned} R - [(x_1 - x_c)^2 + (y_1 - y_c)^2]^{1/2} &= 0 \\ \vdots & \\ R - [(x_5 - x_c)^2 + (y_5 - y_c)^2]^{1/2} &= 0 \end{aligned}$$

introduce 1 add'l constraint for each added param.

const eqn's s'

added params q

$$C + s' = r + m + q$$

$$\begin{aligned} A \quad v &+ B \quad \Delta &= f \\ s'_{1,m} \quad v_{1,l} &+ s'_{1,f} \quad \Delta_{1,l} &= f_{1,l} \\ D_1 \quad \Delta &+ D_2 \quad \Delta' &= h \\ s'_{1,m} \quad v_{1,l} &+ s'_{1,f} \quad \Delta'_{1,l} &= h_{1,l} \end{aligned}$$

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$$\Phi' = V^T W V - 2K^T (Av + B\delta - f) - 2k_c^T (D_1 \delta + D_2 \delta' - h) \quad 39-3$$

$$v, \delta, \delta', k, k_c \quad \frac{\partial \Phi'}{\partial \delta} = 0$$

$$\begin{bmatrix} -W & A^T & 0 & 0 & 0 \\ A & 0 & B & 0 & 0 \\ 0 & B^T & 0 & D^T & 0 \\ 0 & 0 & D_1 & 0 & D_2 \\ 0 & 0 & 0 & D_2^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \delta \\ k_c \\ \delta' \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ h \\ 0 \end{bmatrix}$$

$$P = D_1 N^{-1} D_1^T$$

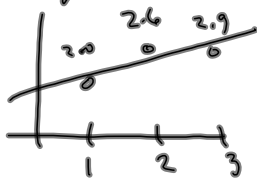
$$R = D_2^T P^{-1} D_2$$

$$Q_{\delta\delta} = N^{-1} \left[I - D_1^T P^{-1} D_1 N^{-1} + D_1^T P^{-1} D_2 R^{-1} D_2^T P^{-1} D_1 N^{-1} \right]$$

$$Q_{\delta\delta'} = R^{-1} \quad (\text{ch. 9 d.l.s.})$$

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Sequential LS, Seq. estimator 39-4



$$l_1 + v_1 = m x_1 + b$$

$$l_2 + v_2 = m x_2 + b$$

$$v - m x - b = -l$$

$$v + \begin{bmatrix} -x & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = -l$$

batch mode

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -2.0 \\ -2.6 \\ -2.9 \end{bmatrix}$$

$$V + B \Delta = f$$

$W = I$
W must be diagonal for seq. tech. to work

$$(B^T W B) \Delta = B^T W f, \quad N \Delta = t$$

$$B^T B \Delta = B^T f \quad W = I$$

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$$\begin{pmatrix} u_1 \\ u_2 \\ u_2 \end{pmatrix} + \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} \begin{pmatrix} u \\ s \end{pmatrix} = \begin{pmatrix} -2.0 \\ -2.6 \\ -2.9 \end{pmatrix} \quad 39-5$$

$$\begin{bmatrix} -1 & -2 & -3 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} = \begin{pmatrix} 1+4+9 & -4-2+3 \\ 1+2+9 & -1-1+1 \end{pmatrix}$$

$$\mathbf{B}^T \quad \mathbf{B} \quad \begin{pmatrix} 14 & 6 \\ 6 & 3 \end{pmatrix} = \mathbf{N}$$

$$\begin{bmatrix} -1 & -2 & -3 \\ -1 & -1 & -1 \end{bmatrix} \begin{pmatrix} -2.0 \\ -2.6 \\ -2.9 \end{pmatrix} = \mathbf{t}$$

$$\mathbf{B}^T \quad \mathbf{f}$$

$$\begin{pmatrix} \underline{1 \times 2.0} + \underline{2 \times 2.6} + \underline{3 \times 2.9} \\ \underline{1 \times 2.0} \quad \underline{1 \times 2.6} \quad \underline{1 \times 2.9} \end{pmatrix}$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} \begin{bmatrix} -2 & -1 \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix} \begin{bmatrix} -3 & -1 \end{bmatrix} = \mathbf{N}$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} (-2.0) + \begin{bmatrix} -2 \\ -1 \end{bmatrix} (-2.6) + \begin{bmatrix} -3 \\ -1 \end{bmatrix} (-2.9) = \mathbf{t}$$

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Sequential formation of normal equations 39-6

- if data coming one obs / time
- if more resource issues
- if eliminate obs

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