Data Adj. Formula Sheet

Indirect Observations

$v + B\Delta = f$

$f = d - \ell$

if nonlinear,

$f = -F(\ell, x^0)$

$N = B^T W B$

$t = B^T W f$

$\Delta = N^{-1} t$

$Q_{\Delta\Delta} = N^{-1}$

$Q_{vv} = Q - B\bar{N}^{-1}B^T$

$Q_{\hat{\ell}\hat{\ell}} = B\bar{N}^{-1}B^T = Q - Q_{vv}$

Observations Only

$Av = f$

$f = d - A\ell$

If nonlinear,

$f = -F(\ell^0) - A(\ell - \ell^0)$

$Q_\ell = AQA^T$

$W_e = Q_e^{-1}$

$k = W_e f$

$V = QA^T k$

$Q_{vv} = QA^T W_e A Q$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$

Mixed Model

$Av + B\Delta = f$

$f = d - A\ell$

if nonlinear,

$f = -F(\ell^0, x^0) - A(\ell - \ell^0)$

$Q_e = AQA^T$

$W_e = Q_e^{-1}$

$N = B^T W_e B$

$t = B^T W_e f$

$\Delta = N^{-1} t$

$Q_{\Delta\Delta} = N^{-1}$

$Q_{vv} = QA^T W_e A Q - QA^T W_e B \bar{N}^{-1} B^T W_e A Q$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$

Constraints

$Av + B\Delta = f$

$C\Delta = g$

$C = \partial F_c / \partial x$

if nonlinear

$g = -F_c(x^0)$

N: full rank

$\Delta = N^{-1}t + \bar{N}^{-1}C^T k_c$

$k_c = (C\bar{N}^{-1}C^T)^{-1}(g - C\bar{N}^{-1}t)$

N: not full rank

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k_c \end{bmatrix} = \begin{bmatrix} -t \\ g \end{bmatrix}$$

$k = W_e(f - B\Delta)$

$V = QA^T k$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\operatorname{atan}(u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{asin}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{acos}(u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\lambda = \frac{\sigma_x^2 + \sigma_y^2}{2} \pm \left[\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2\right]^{1/2}$$

$$2\theta = \operatorname{atan}\left[\frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}\right]$$

$a = \sqrt{\lambda_1 \chi^2_{2,p}}$ or $\sqrt{\lambda_1 \cdot 2F_{2,\infty,p}}$

$b = \sqrt{\lambda_2 \chi^2_{2,p}}$ or $\sqrt{\lambda_2 \cdot 2F_{2,\infty,p}}$

$\hat{X} \pm z\sigma_x$, z from standard normal distr.

$\hat{X} \pm t_r \hat{\sigma}_x$, t_r from t distr., with r degrees of freedom

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$y = Ax$

$\Sigma_{yy} = A\Sigma_{xx}A^T$, or

$Q_{yy} = AQ_{xx}A^T$

$\Sigma_{xx} = \sigma_0^2 Q_{xx}$, or

$\hat{\sigma}_0^2 Q_{xx}$

$w_i = \frac{\sigma_0^2}{\sigma_i^2}$