

3-1

1(a) :  $Q_{\Delta \Delta}$

$-Wv + A^T k = 0$   
 $Av + Bz = f$   
 $B^T k + C^T k_c = 0$   
 $C \Delta = g$

$\Rightarrow$  G.E.

$v = QA^T k$   
 $k = Wz(f - Bz)$   
 $\Delta = N^T t + N^T C^T k_c$   
 $k_c = (CN^T C^T)^{-1} (g - CN^T t)$

$\Delta = \text{[redacted]} \Rightarrow y = Ax + b$   
 make substitutions until you have only 1 random vect.  
 (only "f")  $g = \text{constant}$

$f = d - A\Delta, \quad Q_{ff} = (-A)Q(-A^T) = Q_e \quad \vdots$

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photogrammetry camera distors  $w_1, \rho_1, k_1, x_1, y_1, z_1$   
 $w_2, \rho_2, k_2, x_2, y_2, z_2$

$w_2 = f_w(w_1, \rho_1, k_1)$   
 $\rho_2 = f_\rho(w_1, \rho_1, k_1)$   
 $k_2 = f_k(w_1, \rho_1, k_1)$

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2 cameras	12 unk	6 constr.
3 "	18 unk	12 constr
4 "	24 unk	18 constr
5 "	30 unk	24 constr

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alt. approach 6 parameters

4 5 6 7 transf.

$\text{spy}(B)$

img 1

img 2

B

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added parameters

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y_4 - y_1}{x_4 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

2D

$y_1 = m x_1 + b$  introduce  $m, b$

$y_2 = m x_2 + b$

$y_3 = m x_3 + b$

$y_4 = m x_4 + b$

constraint pts. to a line 3-3  
write constraint eqns

LS treatment of added parameters

$A v + B \Delta = f$   
 $c_1 \times n_1 \quad c_2 \times n_2 \quad c_3$

$D_1 \Delta + D_2 \Delta' = h$   
 $s_1 \times n_1 \quad s_2 \times n_2 \quad s_3$

$g$  added params  
 $s'$  total # constr. eqns

$c + s' = v + (u + p)$

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augmented obj. function:

$\Phi' = v^T W v - 2k^T (A v + B \Delta - f) - 2k_c^T (D_1 \Delta + D_2 \Delta' - h)$  3-4

$\frac{\partial \Phi'}{\partial v} = 2v^T W - 2k^T A = 0$

$\frac{\partial \Phi'}{\partial \Delta} = -2k^T B - 2k_c^T D_1 = 0$

$\frac{\partial \Phi'}{\partial \Delta'} = -2k_c^T D_2 = 0$

$W v - A^T k = 0$

$-B^T k - D_1^T k_c = 0$

$-D_2^T k_c = 0$

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$-W v + A^T k = 0$

$A v + B \Delta = f$

$B^T k + D_1^T k_c = 0$

$D_1 \Delta + D_2 \Delta' = h$

$D_2^T k_c = 0$

full normal equations

$$\begin{bmatrix} -W & A^T & 0 & 0 & 0 \\ A & 0 & B & 0 & 0 \\ 0 & B^T & 0 & D_1^T & 0 \\ 0 & 0 & D_1 & 0 & D_2 \\ 0 & 0 & 0 & D_2^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \\ k_c \\ \Delta' \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ h \\ 0 \end{bmatrix}$$

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if we did G.E. (blocks) then

$$V = QA^T K$$

$$K = W_2 (F - B \Delta)$$

$$\Delta = \bar{N}^T t + \bar{N}^T D_1^T K_c$$

$$K_c = (D_1 \bar{N}^T D_1^T)^{-1} (h - D_1 \bar{N}^T t - D_2 \Delta')$$

$$\Delta' = [D_2^T (D_1 \bar{N}^T D_1^T)^{-1} D_2]^{-1} [D_2^T (D_1 \bar{N}^T D_1^T)^{-1} h - D_2^T (D_1 \bar{N}^T D_1^T)^{-1} D_1 \bar{N}^T t]$$


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inner constraint special case of minimal constraints

$x_1, y_1, y_4$   
 $x_1, y_1, y_3$   
 $x_3, y_3, y_1$   
 $x_1, y_1, x_3 \quad ?? \quad X$

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without constraints  $N$  is singular

if choose  $C$  whose rows are orthogonal to rows of  $N$

then that  $C =$  inner constraint matrix

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