Inner Constraints — special case of min. constraints 4-1

if you fail to completely specify the needed m.c.
then system of equations will be
RANK DEFICIENT or
DATUM DEFECT

another situation leading to Rank Def. configuration defect

2D netw. angles only — Rank Def = 4
2D netw angles & dist. — Rank Def = 3
3D netw. angles only (H+V) — Rank Def = 7
3D netw angles + dist — " " = 6

if you just add constraints to satisfy Rank Def. then
minimal constraints.
interesting property ⇒ all solutions have same residuals

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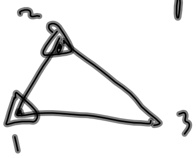
Example of Constraint matrix: 4-2



$X_1 = \cdot\cdot$
$Y_1 = \cdots$
$X_2 = \cdots$
$Y_2 = \cdots$

(a) could impose constr. by substitution, or
(b) by explicit constraints

$X_1 = 500$, $X_1 - 500 = 0$:
$Y_1 = 1000$, $Y_1 - 1000 = 0$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \\ \Delta X_2 \\ \Delta Y_2 \\ \Delta X_3 \\ \Delta Y_3 \end{bmatrix}$$

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if N, square + symmetric, full rank 4-3

then all eigenvalues are NON ZERO, and eigenvectors form a basis for row space of N

if N NOT full rank (order $= u$, rank $= h < u$)

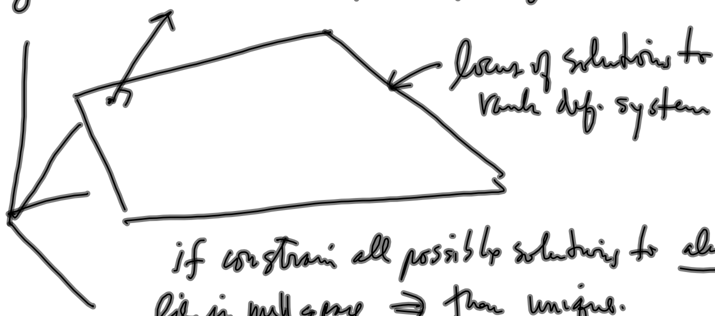
Then defect $d = u - h$, and has

h nonzero eigenvalues

d zero eigenvalues, and

h eigenvectors form basis of row space of N

d eigenvectors form basis of null space of N.



if constrain all possible solutions to <u>also</u> lie in null space $\Rightarrow$ then unique.

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unique: minimum length 4-4

minimum variance

to achieve this solution = may use the eigenvectors of the zero eigenvalues as rows of constraint matrix

(1) resolves deficiency, permits unique sol

(2) of all solutions min magn.

min variance

known in geodetic community as

INNER CONSTRAINT solution

FREE NET solution

for Horiz 2D netw. w/o dist obs

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & 0 & 1 & \cdots \\ Y_1^0 & -X_1^0 & Y_2^0 & -X_2^0 & Y_3^0 & -X_3^0 & \cdots \\ X_1^0 & Y_1^0 & X_2^0 & Y_2^0 & X_3^0 & Y_3^0 & \cdots \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta y_1 \\ \delta x_2 \\ \delta y_2 \\ \delta x_3 \\ \delta y_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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for Horiz 2D w/dist obs 4-5

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & \cdots \\ Y_1^0 & -X_1^0 & Y_2^0 & -X_2^0 & \cdots \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

missing 4th row

for 3D network w/o distance info.

$$C \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 1 & \cdots \\ 0 & Z_1^0 & -Y_1^0 & 0 & Z_2^0 & -Y_2^0 & \cdots \\ -Z_1^0 & 0 & X_1^0 & -Z_2 & 0 & X_2^0 & \cdots \\ Y_1^0 & -X_1^0 & 0 & Y_2^0 & -X_2^0 & 0 & \cdots \\ X_1^0 & Y_1^0 & Z_1^0 & X_2^0 & Y_2^0 & Z_2^0 & \cdots \end{bmatrix} \begin{bmatrix} \delta X_1 \\ \delta Y_1 \\ \delta Z_1 \\ \delta X_2 \\ \delta Y_2 \\ \delta Z_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

for 3D case w/ distance inf in observations
remove last row from C

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Show this is inner matrix by showing that 4-6
rows of C are orthogonal to rows of condition eqn.

$$b = \left[\frac{\partial F}{\partial X_1} \; \frac{\partial F}{\partial Y_1} \; \frac{\partial F}{\partial X_2} \; \frac{\partial F}{\partial Y_2} \; \frac{\partial F}{\partial X_3} \; \frac{\partial F}{\partial Y_3} \right]$$

$$C b^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad b C^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$C N = \underbrace{C B^T}_{} W B = \begin{bmatrix} 0 & 0 & 0 & 0 & \rightarrow \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \\ \downarrow & & & & \end{bmatrix}$$

$\Rightarrow$ C orthog to rows & cols of N (like EV's of zero EVs)
Can serve as inner matrix.

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Consider a 4-par transf between current coords X_0 4-7

and refined coords. X_a in following iteration

$$\underline{X_a} = t + (1+k) R \underline{X_0}$$

shift, scale, rotation

$$\begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + (1+k)\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

assume close identity transformation ⇒ params t_x, t_y, k, α small

$$\begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_0 + \alpha y_0 \\ -\alpha x_0 + y_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \alpha\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix}$$

$$\begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + (1+k)\left[\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \alpha\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix}\right]$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \alpha\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix} + k\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \underline{\underline{k\alpha}}\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix}$$

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$$\begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \alpha\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix} + k\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$ 4-8

represents 1 step in iteration process

$$\begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \alpha\begin{pmatrix} y_0 \\ -x_0 \end{pmatrix} + k\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{matrix} t_x + \alpha y_0 + k x_0 \\ t_y - \alpha x_0 + k y_0 \end{matrix} = \begin{bmatrix} 1 & 0 & y_0 & x_0 \\ 0 & 1 & -x_0 & y_0 \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ \alpha \\ k \end{bmatrix}$$

$$\begin{bmatrix} dx_1 \\ dy_1 \\ dx_2 \\ dy_2 \\ \vdots \\ dx_n \\ dy_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & y_1^0 & x_1^0 \\ 0 & 1 & -x_1^0 & y_1^0 \\ 1 & 0 & y_2^0 & x_2^0 \\ 0 & 1 & -x_2^0 & y_2^0 \\ & & \vdots & \\ 1 & 0 & y_n^0 & x_n^0 \\ 0 & 1 & -x_n^0 & y_n^0 \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ \alpha \\ k \end{bmatrix}$$

looks like

$f \approx Bx$

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if consider as LS problem, solve as 4-9

$$x = (B^T B)^{-1} \underbrace{B^T f}$$

if we want no net shift, scale change, rotation between iterations, then $x = \begin{pmatrix} t_x \\ t_y \\ \alpha \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow B^T f = 0$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & \cdots \\ y_1^0 & -x_1^0 & y_2^0 & -x_2^0 & \cdots \\ x_1^0 & y_1^0 & x_2^0 & y_2^0 & \cdots \end{bmatrix} \begin{pmatrix} dx_1 \\ dy_1 \\ dx_2 \\ dy_2 \\ \vdots \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

We recognize this as the INNER-CONSTRAINT matrix

geometric interpretation:

when you advance from iteration i to iteration i+1
no net shift, scale change, or rotation between the two system.

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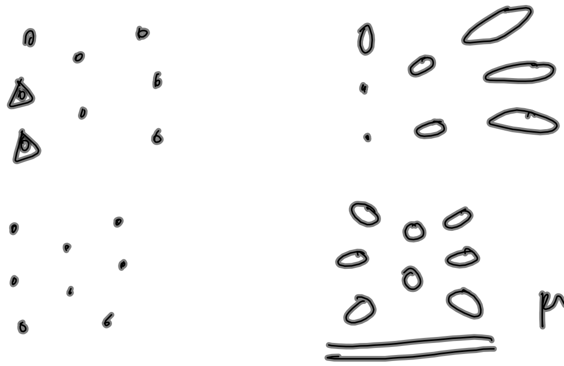
this has big effect on conf. ellipses 4-10



fix 2 pts.

inner constraint

prefer this graphic

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