Lecture 08 Adv. Data Adj
23 June, 2016 Thur.

8-1

next meeting: Tues. 28 June

comment about, $\alpha = .001$ individual residual test

global test α will be different (& closer to "usual" value) since we match β's

Baarda's theory only permits 1 blunder in data

Seems quite restrictive, but in practice no.

just do multiple passes over data, each time detecting and removing 1 blunder.

$$\vec{x} \sim N_n(\vec{\mu}, \Sigma_{xx}) \quad \& \quad \vec{z} = \underset{m \times n}{D}\vec{x}$$

$$\vec{z} \sim N_m(D\vec{\mu}, D\Sigma_{xx}D^T)$$

$$H_0 : w_0 = \frac{v_i}{\sigma_{v_i}} \sim N_1(0, 1)$$

q_{v_i} : $\Sigma_{vv} = \sigma_0^2 Q_{vv}$
assuming you know
σ_i, σ_0

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H_1 : single blunder ∇_i on the i^{th} obs.

$\boxed{\nabla : \text{del, gradient, nabla}}$

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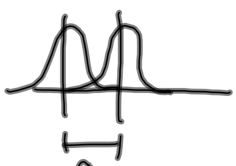
$$v/H_1 : v + \underset{\substack{\uparrow \\ \text{(last time)}}}{Q_{vv}} W[\vec{e}_i \nabla_i], \quad e_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$V/H_1 \sim N_n(\underline{Q_{vv} W e_i \nabla_i}, \underline{\sigma_0^2 Q_{vv}})$$



@ individual residual

$$V_i | H_1 \sim N_1(q_{v_i} w_i \nabla_i, \underline{\sigma_0^2 q_{v_i}})$$

$$H_1 : w_1 = \frac{v_i}{\sigma_{v_i}} = \frac{v_i}{\sigma_0 \sqrt{q_{v_i}}} \sim N_1\Bigg(\underbrace{\frac{q_{v_i} w_i \nabla_i}{\sigma_0 \sqrt{q_{v_i}}}}_{\delta_i}, 1\Bigg)$$

$$H_1 : w_1 = \frac{v_i}{\sigma_{v_i}} \sim N_1(\delta_i, 1)$$

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$H_1 \quad v_i/\sigma_{v_i} \sim N_1(\delta_i, 1)$ 8-3

$$\delta_i = \frac{q_{v_i} w_i \nabla_i}{\sigma_0 \sqrt{q_{v_i}}} = \frac{\nabla_i w_i \sqrt{q_{v_i}}}{\sigma_0} \quad \Big\} \text{ recall } r_i = q_{v_i} w_i$$

$$\sqrt{r_i} = \sqrt{q_{v_i}}\,\frac{\sigma_0}{\sigma_i}$$

$$\sqrt{q_{v_i}} = \frac{\sigma_i}{\sigma_0}\sqrt{r_i}$$

$$\delta_i = \frac{\nabla_i \frac{\cancel{\sigma_0^2}}{\sigma_i^{\cancel{2}}} \frac{\cancel{\sigma_i}}{\cancel{\sigma_0}} \sqrt{r_i}}{\cancel{\sigma_0}}$$

$$\boxed{\delta_i = \frac{\nabla_i \sqrt{r_i}}{\sigma_i}} \qquad \boxed{\nabla_i = \frac{\delta_i \sigma_i}{\sqrt{r_i}}}$$

∇_i : MDE marginally detectable error @ i^{th}
in practice don't know magn. & location of ∇_i so
we get it by working backwards from α_0, β_0

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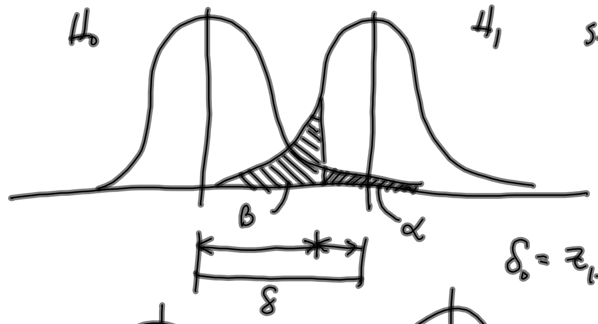
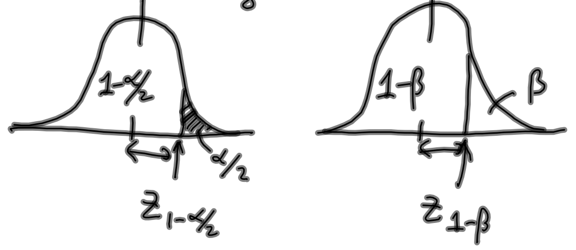


Select α_0, β_0 8-4

$$\delta_0 = z_{1-\alpha/2} + z_{1-\beta}$$



z1ma = icdf('norm', 1-α/2, 0, 1)
z1mB = icdf('norm', 1-β, 0, 1)
δ0 = z1ma + z1mB
δ_0 : rejection threshold for std. residuals

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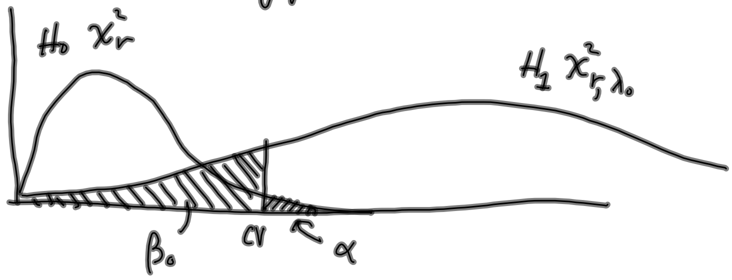
get corresponding CV for global test 8-5

@ same β_0, same reliability

Non centrality parameter for χ^2 : $\delta_0^2 = \lambda_0$



β same for indiv test & global test

$\lambda_0 = \delta_0^2$ NC for χ^2

CV_g = icdf('ncx2', β_0, r, λ_0)

$P_g H\emptyset$ = cdf('chi2', CV_g, r)

$\alpha_g = 1 - P_g H\emptyset$

Preparation

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Execute blunder detection 8-6

1. make global test (must know σ_i's)
2. if accept H_0, done
3. if reject H_0:
   test all std. residuals against δ_0
4. If 1 $\left|\frac{v_i}{\sigma_{v_i}}\right| > \delta_0$ Then eliminate l_i, & do again
5. If multiple $\left|\frac{v_i}{\sigma_{v_i}}\right| > \delta_0$, then eliminate l_i for the largest $\left|\frac{v_i}{\sigma_{v_i}}\right|$, do repeatedly until accept H_0 @ global test

Numerical Example

$\alpha_0 = .001$, $\beta_0 = 0.2$

$\delta_0 = z_{1-\alpha/2} + z_{1-\beta}$

$z_{1-\frac{.001}{2}} + z_{1-.2}$

$z_{1-.0005} + z_{1-.2}$

$z_{.9995} + z_{.8}$

$3.29 + .84 = \boxed{4.13}\ \delta_0$

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$\lambda_0 = \delta_0^2 = (4.13)^2 = 17.06$ 8-7

$\alpha_{\text{global test}} = \underline{.0733}$

<u>Internal Reliability</u>

compute $\nabla_i = \dfrac{\delta_0 \sigma_i}{\sqrt{r_i}}$

look for consistency

equivalent to looking for consistent r_i

$u_i = 1 - r_i$, $u_i + r_i = 1$

absorption number (u_i), red. num. (r_i)

if r_i large (u small) obs. is fully controlled & the residual represents the meas. error.

r_i small (u large) observ. error absorbed into $\left\{\begin{matrix}\text{parameter} \\ \text{other obs residuals}\end{matrix}\right\}$

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<u>External Reliability</u> 8-8

effects on <u>parameters</u> of undetected blunders

$\hat{X} = X^0 + \Delta$

$\Delta = N^{-1} B^T W \left(-(\ell + e_i \nabla_i)\right)$

The effect of MDE in the i^{th} obs

$\Delta_i = N^{-1} B^T W \left[-e_i \nabla_i\right]$

get vector of param shifts for each obs.

but – dependent on choice of minimal constr.

$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} = \sigma_0^2 N^{-1}$

$\Sigma_{\Delta\Delta}^{-1} = \dfrac{N}{\sigma_0^2}$

$\tau_i^2 = \Delta_i^T \Sigma_{\Delta\Delta}^{-1} \Delta_i$ get n of these

look for <u>consistency</u> , networks to be homogeneous

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