Lecture 10 Tuesday 5 July 10-1
CEG37 Adv. Data Adj.

Next Meeting : Thursday 7 July
Today L1 norm estimation
next time finish L1 norm, start Seq. LS

Solutions of minimization problems

$V + B\Delta = f \qquad \Delta = \begin{bmatrix} m \\ b \end{bmatrix}$

$V = f - B\Delta$

$\Phi = V^T W V$ minimize

substituting for V

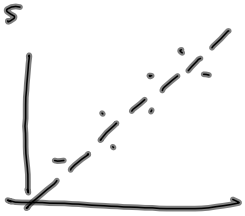
$\Phi = (f - B\Delta)^T W (f - B\Delta)$ minimize

differentiate w.r.t. Δ, set = 0, solve

$(B^T W B)\,\Delta = B^T W f$

Closed form, direct solution for parameters

what if Φ not differentiable ?



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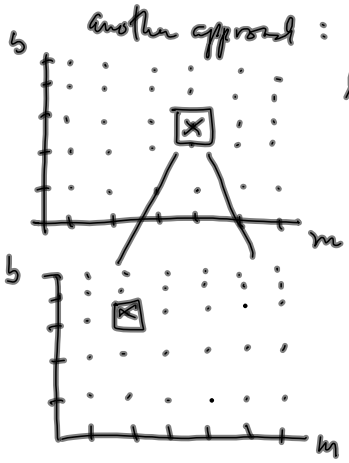
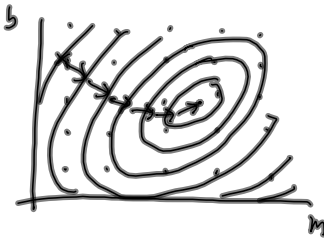
another approach : searching 10-2



look for minimum value of Φ

Could continue subdividing & searching for higher precision



Φ gradient descent method

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look @ 5 obs. of single quantity 10-3

10, 11, 9, 9, 10 $\bar{x} = 9.8$

10, 11, 9, 9, 509 $\bar{x} = 109.6$

9, 9, 10, 11, 509 $\hat{x}_{median} = 10$

median is L_1 norm minimum when single parameter

L_1 norm $\|e\|_1 = \left[\sum_i |e_i|^2\right]^{2/1} = |e_1| + |e_2| + \cdots + |e_n|$

L_2 norm $\|e\|_2 = \left[\sum_i |e_i|^2\right]^{1/2} = \sqrt{e_1^2 + e_2^2 + e_3^2 + \cdots}$

L_3 norm $\|e\|_3 = \left[\sum_i |e_i|^3\right]^{1/3}$

L_n norm $\|e\|_n = \left[\sum_i |e_i|^n\right]^{1/n}$

L_∞ norm $\|e\|_\infty = \left[\sum_i |e_i|^\infty\right]^{1/\infty}$ (:) $= \max_i |e_i|$

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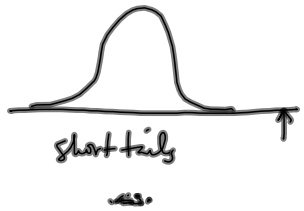
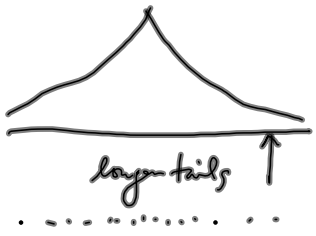
$L_2 \Leftrightarrow$ data is gaussian 10-4

$L_1 \Leftrightarrow$ " " exponential





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L2 est. for sample mean 10-5

$l_1 + v_1 - x = 0$
$l_2 + v_2 - x = 0$
$l_3 + v_3 - x = 0$

Set up some equations, but want all variables to NOT be negative

non-negative

$v_1 = u_1 - w_1$ $\quad x = \alpha_1 - \alpha_2$ auxilliary variables slack variables
$v_2 = u_2 - w_2$
$v_3 = u_3 - w_3$

$\Phi_{L_1} = |v_1| + |v_2| + |v_3|$

$= u_1 + w_1 + u_2 + w_2 + u_3 + w_3 = |u_1 - w_1| + |u_2 - w_2| + |u_3 - w_3|$

$(l_i + v_i - x = 0)$ — obj. fund.

$l_1 + u_1 - w_1 - \alpha_1 + \alpha_2 = 0$
$l_2 + u_2 - w_2 - \alpha_1 + \alpha_2 = 0$
$l_3 + u_3 - w_3 - \alpha_1 + \alpha_2 = 0$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix} = z$$

$$\begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix} = \begin{bmatrix} -l_1 \\ -l_2 \\ -l_3 \end{bmatrix}$$

$\min c^T x = z$
subject to $Ax = b$

LP problem

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$Ax = b$ is underdetermined = inf. # solutions 10-6

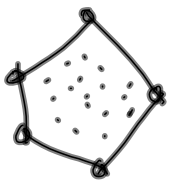
3,8 8,1 3,1

only allow non-neg solutions (feasible set)

$A_{m,n}$: basic solutions — m of unknowns are non-zero, rest are zero

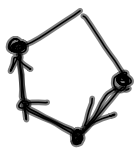
1. only finite number of basic solutions
2. minimum of obj. will be one of basic solutions

basic solution $\Leftrightarrow$ corner



in principle could do exhaustive search of all basic solutions, all corners

simplex algo. $\rightarrow$
interior point algo.



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