Lecture 13 ce697 Adv. Data Adj. 13-1
14 July 2016, Thursday

Next meeting : Tuesday 19^{th}
then decide whether we need to meet on 21^{st}

Review some edits made to notes for Lect. 12

$\dot{x} = f(x)$
$x_{i+1} = \varphi(x_i)$

integration
num. solution of Diff Eq. 1 step
convert to prediction formula

$\underset{\text{u.c.}}{\Phi} = \underset{\text{l.c.}}{\dfrac{\partial \varphi}{\partial x}}$

$\bar{P}_{i+1} = \Phi_i P_i \Phi_i^T + Q$

Reference
KF, LS, and Modeling
–Bruce Gibbs
Wiley, 2011

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Dynamic Models 13-2
Equations of Motion ⇒ Diff. Equations

we need, prediction formula $x_{i+1} = \Phi x_i$
$x_{i+1} = \varphi(x_i)$

linear $\dot{x} = A x$
nonlinear $\dot{x} = f(x)$
⇒ example constant velocity model 1 axis

$$\begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}$$

$\dot{x} = A x$

(euler method)

$$\begin{bmatrix} p \\ v \end{bmatrix}_{k+1} = \begin{bmatrix} p \\ v \end{bmatrix}_k + \begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} \delta t$$

$$\begin{bmatrix} p \\ v \end{bmatrix}_{k+1} = \begin{bmatrix} p \\ v \end{bmatrix}_k + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix} \delta t$$

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$$\begin{bmatrix} p \\ v \end{bmatrix}_{k+1} = \left[I + \begin{bmatrix} 0 & \Delta t \\ 0 & 0 \end{bmatrix} \right] \begin{bmatrix} p \\ v \end{bmatrix}_k$$

$$\begin{bmatrix} p \\ v \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}_k$$

$$x_{k+1} = \Phi_k x_k$$

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example constant accel. model w/ improved euler method

$$\begin{bmatrix} \dot p \\ \dot v \\ \dot a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}, \quad \begin{bmatrix} \ddot p \\ \ddot v \\ \ddot a \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}$$

$$x_{k+1} = x_k + \dot x \Delta t + \ddot x \frac{(\Delta t)^2}{2!} + \cdots$$

$$\begin{bmatrix} p \\ v \\ a \end{bmatrix}_{k+1} = \begin{bmatrix} p \\ v \\ a \end{bmatrix}_k + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix} \Delta t + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix} \Delta t^2/2$$

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$$\begin{bmatrix} p \\ v \\ a \end{bmatrix}_{k+1} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \Delta t^2/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} p \\ v \\ a \end{bmatrix}_k$$

$$\begin{bmatrix} p \\ v \\ a \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}_k$$

$$x_{k+1} = \Phi x_k$$

$$\Phi = e^{A\Delta t} = I + A\Delta t + \frac{(A\Delta t)^2}{2!} + \frac{(A\Delta t)^3}{3!} + \cdots$$

$$e^x = 1 + x + x^2/2! + x^3/3! \cdots$$

matlab <u>expm</u>

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What if model non linear ? $\dot{x} = f(x)$ 13-5

euler step : $x_{k+1} = x_k + \dot{x}\,\Delta t$

$$x_{k+1} = \underbrace{x_k + f(x)\,\Delta t}_{\Phi(x)}$$

improved euler method:

$$x_{k+1} = \underbrace{x_k + \dot{x}\,\Delta t + \ddot{x}\,\frac{\Delta t^2}{2!} + \cdots}_{\Phi(x)}$$

runge-kutta 4th order also widely used

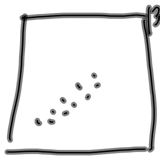
3 axis w/ c.a. model

$$\begin{bmatrix} \dot{p}_x \\ \dot{v}_x \\ \dot{a}_x \\ \dot{p}_y \\ \dot{v}_y \\ \dot{a}_y \\ \dot{p}_z \\ \dot{v}_z \\ \dot{a}_z \end{bmatrix} = \begin{bmatrix} \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} & \phi & \phi \\ \phi & \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} & \phi \\ \phi & \phi & \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} \end{bmatrix} \begin{bmatrix} p_x \\ v_x \\ a_x \\ p_y \\ v_y \\ a_y \\ p_z \\ v_z \\ a_z \end{bmatrix}$$

description of model for first HW prob. 13-6

constant vel. model

state vector = $\begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}$



C.V. model

$$\begin{bmatrix} \dot{p}_x \\ \dot{v}_x \\ \dot{p}_y \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}, \quad \dot{x} = Ax$$

$$\Phi = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}_{k+1} = \Phi \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}_k$$

measurement / obs. model 13-7

$$z_k = H_k x_k + v_k$$

$$\begin{bmatrix} x_{obs.} \\ y_{obs.} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{H} \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}$$

linear KF algo

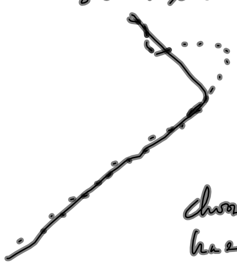
$$\hat{x}_k = \hat{x}_k^- + K_k\left(\underbrace{z_k}_{\text{obs.}} - \underbrace{H\hat{x}_k^-}_{\text{pred obs.}}\right)$$

misclosure

compare to R : $\begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{pmatrix}$

compute σ_x, σ_y

should look out for divergence from model 13-8



one approach to address this
run multiple filters in parallel
choose the lowest order model which
has acceptable misclosure.