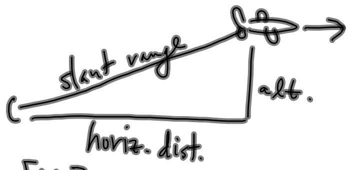
Lecture 14 Adv. Data Adj. 14-1
19 July 2016, Tuesday

next (last!) meeting Thur. 21 July (cover UKF)

today: probs 2 & 3 fr. HW5 due 5 Aug
grades due Tues. 9^{th}
I return HW's 1-4 by end of next week
I am away $30^{th} \to 5^{th}$

2. (HW5) radar tracking
2D (vertical plane)



$$\text{state vector} = \begin{bmatrix} \text{horiz dist.} \\ \text{velocity} \\ \text{altitude} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

x

dyn model: constant velocity, constant altitude

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = 0 + w_2 \qquad \dot{x}_3 = 0 + w_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ w_2 \\ w_3 \end{bmatrix}, \quad \dot{x} = Ax + w$$

linear dynamics

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measurement $r = \sqrt{x_1^2 + x_3^2} + v$ 14-2

$$z = h(x) + v$$

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use EKF $\Phi = I + A\Delta t, \quad x_{i+1} = \Phi x_i$
euler step

$$\text{need } H = \frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{x_1}{\sqrt{x_1^2+x_3^2}} & 0 & \frac{x_3}{\sqrt{x_1^2+x_3^2}} \end{bmatrix}$$

that's all you need to implement the EKF
I provide starting X_0^-, P_0^- + Δt, seq. of range obs.
R for measurement

3. <u>attitude determination</u> (helicopter in hover mode)

state vector $\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$ roll, pitch, yaw — body frame ← inertial system

euler: yaw first, pitch second, roll third

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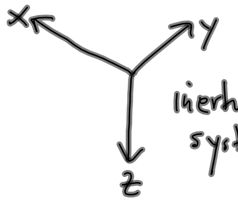

inertial system

$\dot{\phi}, \dot{\theta}, \dot{\psi}$ euler angle rates 14-3

observe directly p, q, r angle rates from gyros

$$M_I^B = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}_{\substack{\text{roll} \\ M_2^B}} \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}}_{\substack{\text{pitch} \\ M_1^2}} \underbrace{\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{\text{yaw} \\ M_I^1}}$$

I-frame

1-frame $\quad r_1 = M_I^1 r_I$

2-frame $\quad r_2 = M_I^2 r_I = M_1^2 M_I^1 r_I$

B-frame $\quad r_B = M_I^B r_I = M_2^B M_1^2 M_I^1 r_I$

<u>axial vector</u>
<u>pseudo vector</u> $\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}, \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

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can write relationship between obs angle rates p, q, r 14-4

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = I_3 \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + M_2^B \underbrace{\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}}_{\text{frame 2}} + M_2^B M_1^2 \underbrace{\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}}_{\text{frame 1}}$$

body frame w/ 3 ortho axes

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

(multiply + rearrange)

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

need $\dot{x} = f(x)$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = [\ \ ]^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

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14-5

$$\begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

can solve 3 linear systems, yield 3 columns of inverse matrix

Cramer's rule

$$\frac{|\ |}{|\ |} \qquad \underset{\parallel}{\begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{vmatrix}} = 1$$

solve for 3 elements in each of 3 systems:

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\dot X = f(x)$$

turn this into a discrete step, use euler

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$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_{i+1} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_i + \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} \Delta t = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} + f(x)\cdot\Delta t$$

$$= \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_i + \begin{bmatrix} p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\cos\phi - r\sin\phi \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \end{bmatrix}\Delta t$$

$x_{i+1} = \phi(x_i)$ prediction step

$$\Phi = \frac{\partial\phi}{\partial x} = I + \frac{\partial[\ ]}{\partial x}\Delta t = I + \frac{\partial f(x)}{\partial x}\Delta t$$

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helicopter in hover mode, accel readings 14-7

$\begin{pmatrix} u \\ v \\ w \end{pmatrix}$ velocities along body axes

$$\underbrace{\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}}_{\text{accel. readings}} = \underbrace{\begin{bmatrix} \dot u \\ \dot v \\ \dot w \end{bmatrix}}_{\text{linear accel}} + \underbrace{\begin{pmatrix} 0 & w & -v \\ -w & 0 & u \\ v & -u & 0 \end{pmatrix}}_{\text{linear vel.}} \underbrace{\begin{pmatrix} p \\ q \\ r \end{pmatrix}}_{\text{angle rates}} + g \begin{bmatrix} \sin\theta \\ -\cos\theta \sin\phi \\ -\cos\theta \cos\phi \end{bmatrix}$$

<u>hover mode</u> $u, v, w = 0$

$\dot u, \dot v, \dot w = 0$

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = g \begin{bmatrix} \sin\theta \\ -\cos\theta \sin\phi \\ -\cos\theta \cos\phi \end{bmatrix} \Rightarrow$$

pitch

$$\theta = \sin^{-1}\left(\frac{f_x}{g}\right)$$

roll

$$\phi = \sin^{-1}\left(\frac{-f_y}{g\cos\theta}\right)$$

These are derived obs. θ, ϕ

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meas. model 14-8

$$z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} + v$$

$$z = Hx + v$$

That should be everything for implementing EKF

$Q, R, \Delta t$, + data stream p, q, r, f_x, f_y, f_z x_0^-, P_0^-

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