CE697 Lect. 15 21 July 2016 15-1

HW5 up by monday (due 5th)

last time: formula accel →
- linear accel.
- angle rates
- gravity
- roll, pitch

derived in Collinson
<u>intro. to avionics</u>

topic for today: Unscented KF, <u>unscented transf.</u>

J. Uhlmann } developed ~1995
S. Julier }

<u>Error Propagation</u>

If linear: $Y = Ax$, $x: \mu_x, \Sigma_{xx}$

$\mu_y = A\mu_x$, $\Sigma_{yy} = A\Sigma_{xx}A^T$

if nonlinear $y = f(x)$ $y \approx f(x^0) + \frac{\partial f}{\partial x}\Delta x$ $\frac{\partial f}{\partial x} = J$

$\mu_y = f(\mu_x)$, $\Sigma_{yy} = J\Sigma_{xx}J^T$

there can arise significant accuracy issues w/ nonlinear case

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How to avoid these E.P. errors? 15-2

2 ways to do it:

1. sampling by monte carlo with MANY samples
   generate samples of x ← $(\underline{\mu_x}, \underline{\Sigma_{xx}})$
   transform by $Y = f(x)$
   compute μ_y sample mean $\bar{y}$
   $\hat{\Sigma}_{yy}$ sample covariance $\frac{1}{n-1}\sum_{i=1}^{n}(y-\bar{y})(y-\bar{y})^T$

   if $w \sim MVN(0, I_n)$
   decompose $\Sigma_{xx} = LL^T$ (Cholesky decomp.)
   $x = Lw + \mu_x$

   however: monte carlo is SLOW

2. instead of 1000's of random pts., choose a few well chosen pts, "sigma points"
   have required mean
   have required covariance

   # sigma points NOT unique
   results close to monte carlo, but much faster

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Sigma pts : χ_i, weights w_i 15-3

$$\bar{x} = \sum_{i=1}^{2n+1} w_i \chi_i$$

$$P_x = \sum_{i=1}^{2n+1} w_i (\chi_i - \bar{x})(\chi_i - \bar{x})^T$$

Sample covariance $\frac{1}{n-1}\Sigma (x_i - \bar{x})(x_i - \bar{x})^T$

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how is this relevant to NLSLS / EKF ? 15-4

x_0^-, P_0^-

$K_k = P_k H^T (H P_k H^T + R)^{-1}$ $\quad$ $H = \frac{\partial h}{\partial x}$

$\hat{x}_k = \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-))$

$P_k = P_k^- - K_k H P_k^-$

$\hat{x}_{k+1}^- = \varphi(\hat{x}_k)$

$P_{k+1}^- = \Phi P_k \Phi^T + Q$ $\quad$ $\Phi = \frac{\partial \varphi}{\partial x}$

EKF

UKF

x_0^-, P_0^-

compute sigma pts + weights χ_i, w_i

prediction + cov $(\hat{x}_k^-, \hat{P}_k^-) = UT(\varphi(\chi_i), w_i, Q)$

predict meas + cov $(z_k, P_z) = UT(h(\chi_i), w_i, R)$

Kalman gain $P_{xz} = \Sigma (f(x_k) - \hat{x})(h(x) - z)^T$

$$\boxed{K_k = P_{xz} \cdot P_z^{-1}} \leftarrow$$

est $\hat{x}_k = \hat{x}_k^- + K_k (z_k - \hat{z}_k)$, $P_k = P_k^- - K_k P_z K_k^T$

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Kalman gain? remember from HW1 cross covariance matrix 15-5

$y = y(x)$, $z = z(t)$

$J_{yx} = \partial y / \partial x \quad J_{zt} = \frac{\partial z}{\partial t}$

$r = \begin{pmatrix} y \\ z \end{pmatrix}$, $s = \begin{pmatrix} x \\ t \end{pmatrix} \qquad J_{rs} = \begin{bmatrix} J_{yx} & 0 \\ 0 & J_{zt} \end{bmatrix}$

$Q_{rr} = J_{rs} Q_{ss} J_{rs}^T$

$$\begin{bmatrix} Q_{yy} & \underline{Q_{yz}} \\ Q_{zy} & Q_{zz} \end{bmatrix} = \begin{bmatrix} J_{yx} Q_{xx} J_{yx}^T & \underline{J_{yx} Q_{xt} J_{zt}^T} \\ J_{zt} Q_{tx} J_{yx}^T & J_{zt} Q_{tt} J_{zt}^T \end{bmatrix}$$

$Q_{yz} = J_{yx} Q_{xt} J_{zt}^T$

$Q_{yz} = J_{yy} Q_{yy} J_{zt}^T$

$Q_{yz} = I$ " " , $= \boxed{Q_{yy} J_{zy}^T = Q_{yz}}$

$\uparrow$

JB derivation of KF eqn's from LS eqn (28) 15-6

$K = P H^T R^{-1}$

$= \underbrace{Q_{xx} J_{zx}^T} R^{-1}$

$K = Q_{xz} R^{-1}$

conventional KF notation

$P_{xz} = Q_{xz}$

$R = P_{zz}, P_z$

$$\boxed{K = P_{xz} \cdot P_z^{-1}}$$

$\hookrightarrow$ UKF algo