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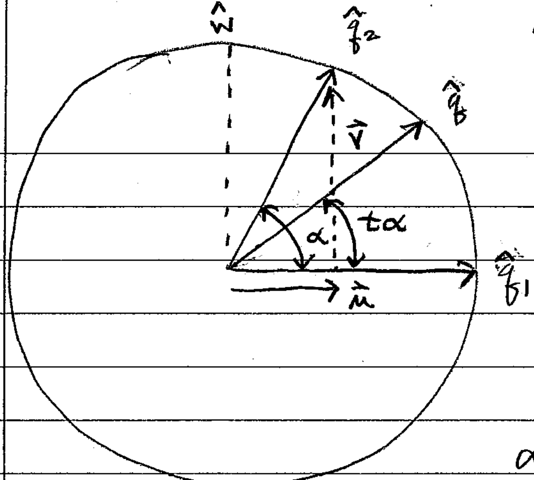
Derive SLERP formula in 2D



unit circle

given $\hat{q}_1, \hat{q}_2$, $t \in [0,1]$

want to interpolate $\vec{q}$ such that

$$t=0 \Rightarrow \vec{q} = \vec{q}_1$$

$$t=1 \Rightarrow \vec{q} = \vec{q}_2$$

$\vec{q}_1 \cdot \vec{q}_2 = \cos\alpha$, $\alpha = \cos^{-1}(\vec{q}_1 \cdot \vec{q}_2)$

$$\alpha = \cos^{-1}(q_{1x}q_{2x} + q_{1y}q_{2y})$$

$$\hat{w} \perp \hat{q}_1$$

$\vec{v}$: comp of of $\hat{q}_2 \parallel \hat{w}$, $\vec{u}$ = comp of $\hat{q}_2 \perp \hat{q}_1$ (or projections)

$\vec{v} = \sin\alpha \cdot \hat{w}$, OR

$\hat{w} = \dfrac{\vec{v}}{\sin\alpha}$, ~~also~~

$\vec{u} = \cos\alpha \cdot \hat{q}_1$

$\hat{q}_2 = \vec{u} + \vec{v}$

$\vec{v} = \hat{q}_2 - \vec{u} = \hat{q}_2 - \cos\alpha \cdot \hat{q}_1$

$\hat{w} = \dfrac{\hat{q}_2 - \cos\alpha \cdot \hat{q}_1}{\sin\alpha}$

$\hat{q} = \cos(t\alpha)\,\hat{q}_1 + \sin(t\alpha)\,\hat{w}$

$\hat{q} = \cos(t\alpha)\,\hat{q}_1 + \sin(t\alpha)\left(\dfrac{\hat{q}_2 - \cos\alpha \cdot \hat{q}_1}{\sin\alpha}\right)$

$\hat{q} = \cos(t\alpha)\,\hat{q}_1 - \dfrac{\sin(t\alpha)\cos\alpha\,\hat{q}_1}{\sin\alpha} + \dfrac{\sin(t\alpha)}{\sin\alpha}\hat{q}_2$

$\hat{q} = \left[\cos(t\alpha) - \dfrac{\sin(t\alpha)\cos\alpha}{\sin\alpha}\right]\hat{q}_1 + \dfrac{\sin(t\alpha)}{\sin\alpha}\hat{q}_2$

$\hat{q} = \left[\dfrac{\cos(t\alpha)\sin\alpha - \sin(t\alpha)\cos\alpha}{\sin\alpha}\right]\hat{q}_1 + \dfrac{\sin(t\alpha)}{\sin\alpha}\hat{q}_2$

use: $\boxed{\sin(a-b) = \sin a \cos b - \cos a \sin b}$

$\hat{q} = \dfrac{\sin(\alpha - t\alpha)}{\sin\alpha}\hat{q}_1 + \dfrac{\sin(t\alpha)}{\sin\alpha}\hat{q}_2$

$$\boxed{\hat{q} = \frac{\sin(\alpha[1-t])}{\sin\alpha}\hat{q}_1 + \frac{\sin(t\alpha)}{\sin\alpha}\hat{q}_2}$$

derivation in 2D, but works in 3D, 4D, ...

quaternions are 4D