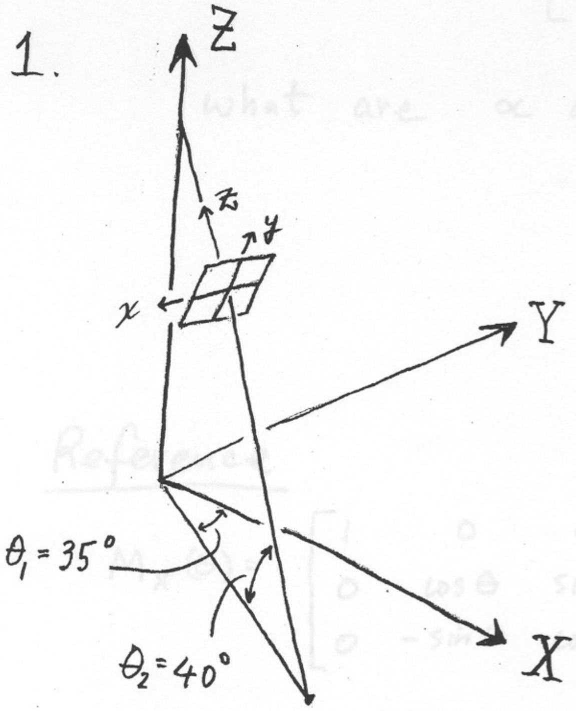
CE 503 Photogrammetry I   Exam I

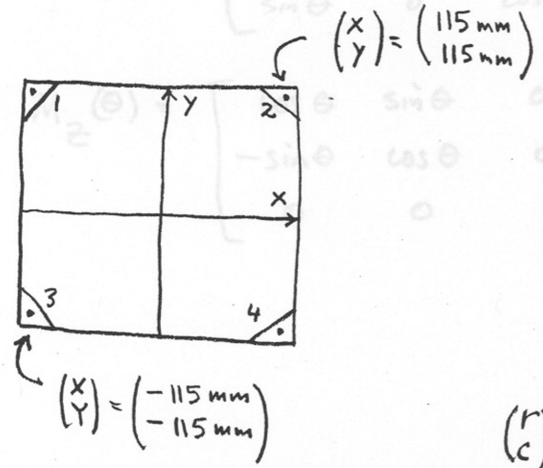
18 Oct 2002   Name ______________

1.

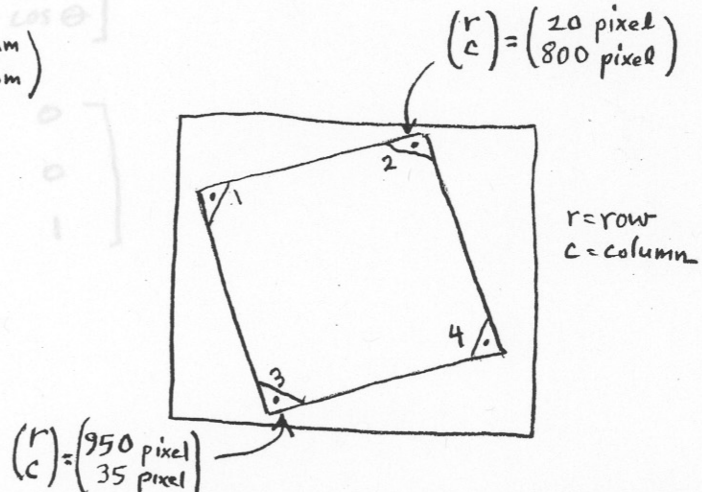


With reference to the figure at the left, show a sequence of rotations that can be applied to the upper case system (X, Y, Z) to bring it into alignment with the image system (x, y, z). Indicate the <u>order</u>, the <u>sign</u>, and the <u>magnitude</u> of each angle. (you do not have to compute the matrix elements.)

2.



calibrated fiducial coordinates



r = row
c = column

measured coordinates in scanned digital image

Show the linear equations necessary to solve for the 4 parameters (a, b, c, d) in the model $\begin{pmatrix} r \\ c \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$, using the 2 measured points. DO NOT SOLVE THE EQUATIONS, only show them.

3. $$M_z(\beta)\, M_x(\alpha) = \begin{bmatrix} .9397 & .3368 & .0594 \\ -.3420 & .9254 & .1632 \\ 0 & -.1736 & .9848 \end{bmatrix}$$

what are α and β ?

Reference

$$M_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$M_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$M_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CE503 Exam I Solution
22 Oct 2002

1. First rotation: clockwise about Z, sign: negative, angle $90° + 35° = 125°$

$\theta_z = -125°$

Second rotation: about X, sign: positive, angle = 50°

$\theta_x = +50°$

$$\Rightarrow \underline{\underline{M = M_x(+50°)\,M_z(-125°)}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .643 & .767 \\ 0 & -.767 & .643 \end{bmatrix} \begin{bmatrix} -.574 & -.819 & 0 \\ .819 & -.574 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -.574 & -.819 & 0 \\ .526 & -.369 & .767 \\ -.627 & .439 & .643 \end{bmatrix}$$

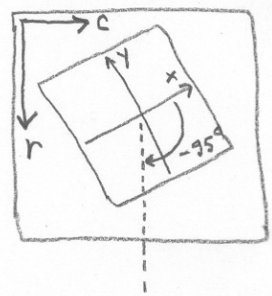
2.
$$\begin{aligned} r_2 &= ax_2 + by_2 + c \\ c_2 &= -bx_2 + ay_2 + d \\ r_3 &= ax_3 + by_3 + c \\ c_3 &= -bx_3 + ay_3 + d \end{aligned}$$

$$\underline{\underline{\begin{bmatrix} r_2 \\ c_2 \\ r_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \\ x_3 & y_3 & 1 & 0 \\ y_3 & -x_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}}, \quad \underline{\underline{\begin{bmatrix} 20 \\ 800 \\ 950 \\ 35 \end{bmatrix} = \begin{bmatrix} 115 & 115 & 1 & 0 \\ 115 & -115 & 0 & 1 \\ -115 & -115 & 1 & 0 \\ -115 & 115 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -.3587 \\ -3.6848 \\ 485 \\ 417.5 \end{bmatrix}$$

$\lambda = \sqrt{a^2+b^2} = 3.7022$

$\theta = \tan^{-1}(b/a) = -95°.5599$



3.
$$M_z(\beta)\,M_x(\alpha) = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & \sin\beta\cos\alpha & \sin\beta\sin\alpha \\ -\sin\beta & \cos\beta\cos\alpha & \cos\beta\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} .9397 & .3368 & .0594 \\ -.3420 & .9254 & .1632 \\ 0 & -.1736 & .9848 \end{bmatrix}$$

$$\Rightarrow \alpha = \tan^{-1}\left(-m_{32}/m_{33}\right) = \tan^{-1}\left(.1736/.9848\right) = \underline{\underline{10°}}$$

$$\beta = \tan^{-1}\left(-m_{21}/m_{11}\right) = \tan^{-1}\left(.3420/.9397\right) = \underline{\underline{20°}}$$