ECI ↔ ECF Transformation

$$\Pi = R_y(-x_p)\, R_x(-y_p)$$

$$\approx \begin{bmatrix} 1 & 0 & +x_p \\ 0 & 1 & -y_p \\ -x_p & +y_p & 1 \end{bmatrix}$$

works because small angles

Small Angle assumptions:

1. $\sin\theta = \theta$
2. $\cos\theta = 1$
3. $\sin\theta_1 \cdot \sin\theta_2 = 0$

$$\Pi = M_y(-x_p)\, M_x(-y_p)$$

$$\begin{bmatrix} \cos(-x_p) & 0 & -\sin(-x_p) \\ 0 & 1 & 0 \\ \sin(-x_p) & 0 & \cos(-x_p) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-y_p) & \sin(-y_p) \\ 0 & -\sin(-y_p) & \cos(-y_p) \end{bmatrix}$$

$$\begin{bmatrix} \cos(-x_p) & \sin(-x_p)\sin(-y_p) & -\sin(-x_p)\underline{\cos(-y_p)} \\ 0 & \cos(-y_p) & \sin(-y_p) \\ \sin(-x_p) & -\cos(-x_p)\sin(-y_p) & \cos(-x_p)\cos(-y_p) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & +x_p \\ 0 & 1 & -y_p \\ -x_p & +y_p & 1 \end{bmatrix} \leftarrow$$

$P = \text{precession}(T)$

3x3 (precession.m)

$[N, ep, delpsi] = \text{nutation}(T)$

3x3 (nutation.m, ian1980n.txt)

GAST ⇒ $\Theta = m_3(\text{GAST})$

3x3 Radians

m1.m, m2.m, m3.m

$\Pi = m2(-x_p) * m1(-y_p)$

3x3

$$\underset{(\text{ITRS})}{X_{ECF}} = \underbrace{\Pi \Theta N P}_{3\times 3} \underset{(\text{ICRS})}{X_{ECI}}$$

$$X_{ECI} = \underbrace{P^T N^T \Theta^T \Pi^T} X_{ECF}$$

What about velocity vector $\dot{}$?

cannot rotate is same way since ECF system is rotating

$X_{ECF} = R \, X_{ECI}$ $\qquad$ $\underline{\underline{R = \Pi \Theta N P}}$

$\frac{d}{dt} X_{ECF} = \dot{X}_{ECF}$

$\dot{X}_{ECF} = \underset{\checkmark}{\underline{R}} \, \underset{\checkmark}{\underline{\dot{X}_{ECI}}} + \underset{?}{\dot{R}} \, \underset{\checkmark}{\underline{X_{ECI}}}$ $\qquad$ product rule ~~for~~ diff.

$\dot{R} = \frac{d}{dt}(\underline{\Pi} \Theta \underline{N} \underline{P})$

Π, N, P constant

$\dot{R} = \Pi \frac{d\Theta}{dt} N P$

$$\frac{d}{dt}\theta, \quad \Theta = \begin{bmatrix} \cos GAST & \sin GAST & 0 \\ -\sin GAST & \cos GAST & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{d}{dt}\theta = \begin{bmatrix} -\sin GAST & \cos GAST & 0 \\ -\cos GAST & -\sin GAST & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{d\,GAST}{dt}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Theta \cdot \underbrace{\frac{d\,GAST}{dt}}_{?}$$

$$GAST = \underbrace{GMST}_{\downarrow} + \Delta\psi \cos\epsilon$$

$$GMST(0h) + \underline{1.002737}\ldots \overset{(t)}{UT1}$$

$$\frac{d}{dt} GAST = 1.002737 \left(\frac{s}{s}\right) \cdot \frac{2\pi}{86400} \left(\frac{Rad}{s}\right)$$

$$\omega_{\oplus} = 7.2921158553 \times 10^{-5} \ R/s$$

rotation rate (sidereal) of earth

$$\dot{X}_{ECF} = R\,\dot{X}_{ECI} + \dot{R}\,X_{ECI}$$

$$\overbrace{\Pi \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Theta N P\, \omega_{\oplus}}^{\Pi \frac{d}{dt}\Theta N P}$$

$$X_{ECI} = R^T X_{ECF}$$

$$\dot{X}_{ECI} = \underset{\checkmark}{\underline{R^T}} \, \underset{\checkmark}{\underline{\dot{X}_{ECF}}} + \dot{R}^T \underset{\checkmark}{\underline{X_{ECF}}}$$

$$P^T N^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Theta^T \Pi^T \omega_{\oplus}$$

<u>Same as</u> $$P^T N^T \Theta^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Pi^T \omega_{\oplus}$$

Note: $$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Theta = \Theta \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ and

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Theta^T = \Theta^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

either pre- or post- multiply by appropriate "1/0" matrix produces the derivative.

We have just shown that

$$\left[\dot{\overline{R^T}}\right] = \left[\dot{R}\right]^T$$

i.e. we can interchange the order of time derivative and transpose.