$$F_x = 0 - x_0 - f\,\frac{u}{w}$$

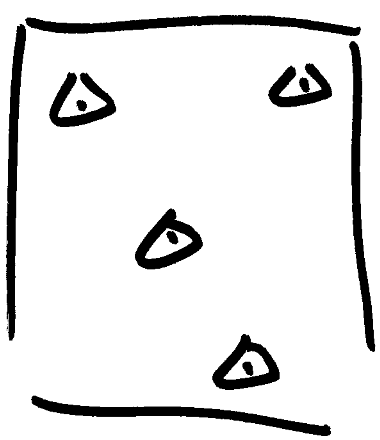
$$F_y = y - y_0 - f\,\underline{v/w}$$

$$dx = \underline{dx_0} + \underline{dx_1}\,t + \underline{dx_2}\,t^2 \qquad t,\ \ell$$

$$dy = dy_0 + dy_1\,t + dy_2\,t^2$$

$$\vdots$$

select : $dx_0, dx_1, dy_0, dy_1\ dz_0\ dz_1$



$$B = \begin{bmatrix} \frac{\partial F_{x_1}}{\partial x_0} & \frac{\partial F_{x_1}}{\partial x_1} & \frac{\partial F_{x_1}}{\partial y_0} & \frac{\partial F_{x_1}}{\partial y_1} & \frac{\partial F_{x_1}}{\partial z_0} & \frac{\partial F_{x_1}}{\partial z_1} \\ \frac{\partial F_{y_1}}{\partial x_0} & \frac{\partial F_{y_1}}{\partial x_1} & \cdots & & & \\ \vdots & & & & & \\ \frac{\partial F_{x_4}}{\partial x_0} & \frac{\partial F_{x_4}}{\partial x_1} & \cdots & & & \\ \frac{\partial F_{y_4}}{\partial x_0} & \frac{\partial F_{y_4}}{\partial x_1} & \cdots & & & \end{bmatrix} \begin{bmatrix} \Delta x_0 \\ \Delta x_1 \\ \Delta y_0 \\ \Delta y_1 \\ \Delta z_0 \\ \Delta z_1 \end{bmatrix}$$

8×6

$$v + B\delta = f \qquad f = -\begin{bmatrix} F_{x_1} \\ F_{y_1} \\ F_{x_2} \\ \vdots \\ F_{x_4} \\ F_{y_4} \end{bmatrix}$$

$$W = I_8$$

$$\Delta = (B^T W B)^{-1} B^T W f$$ Least Squares Est. of parameter corrections

$$\begin{pmatrix} \text{Par} \\ \text{Vect} \end{pmatrix}_{i+1} = \begin{pmatrix} \text{Par} \\ \text{Vect} \end{pmatrix}_i + \Delta$$

monitor size of Δ
when small : <u>converged</u>

$$\Delta = \underbrace{(B^T W B)^{-1}}_{\text{scaled version of covariance matrix of parameter vector}} B^T W f$$

$Q_{\Delta\Delta}$, $\underline{\underline{\Sigma_{\Delta\Delta}}} = \sigma_0^2 Q_{\Delta\Delta}$

a priori or from adjustment

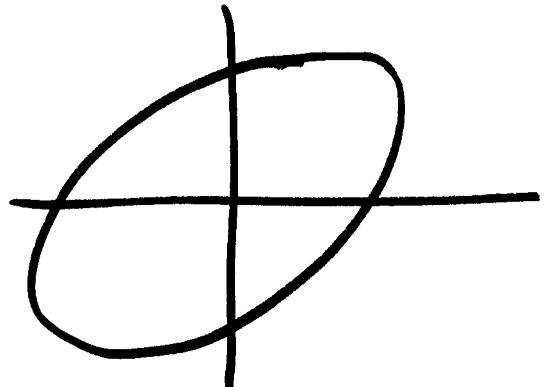
$$W : w_i = \frac{\sigma_0^2}{\sigma_i^2}$$

σ_0^2 : can be any number

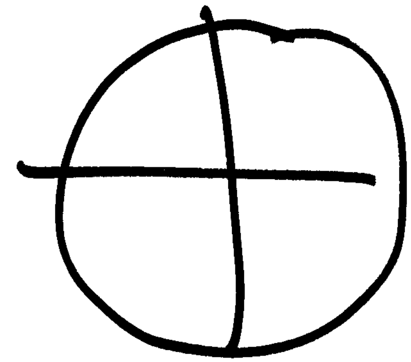
$$\Sigma_{\Delta\Delta} : \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ & \sigma_2^2 & \sigma_{23} & \\ & & \sigma_3^2 & \end{bmatrix}$$

$\Sigma_{\Delta\Delta}$:

90% conf. interval



90% confidence region



90% confidence circle

CE/LE

<u>Error Propagation</u>

polynomial interpolation (for ephemeris data)

2 points: $y = \underline{a_0} + \underline{a_1} x$

3 points $y = a_0 + a_1 x + a_2 x^2$

4 points $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

⋮

n points $y = a_0 + a_1 x + a_2 x^2 + \cdots a_{n-1} x^{n-1}$

$$\begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix}$$

n points ⇒ n equations
n unknowns

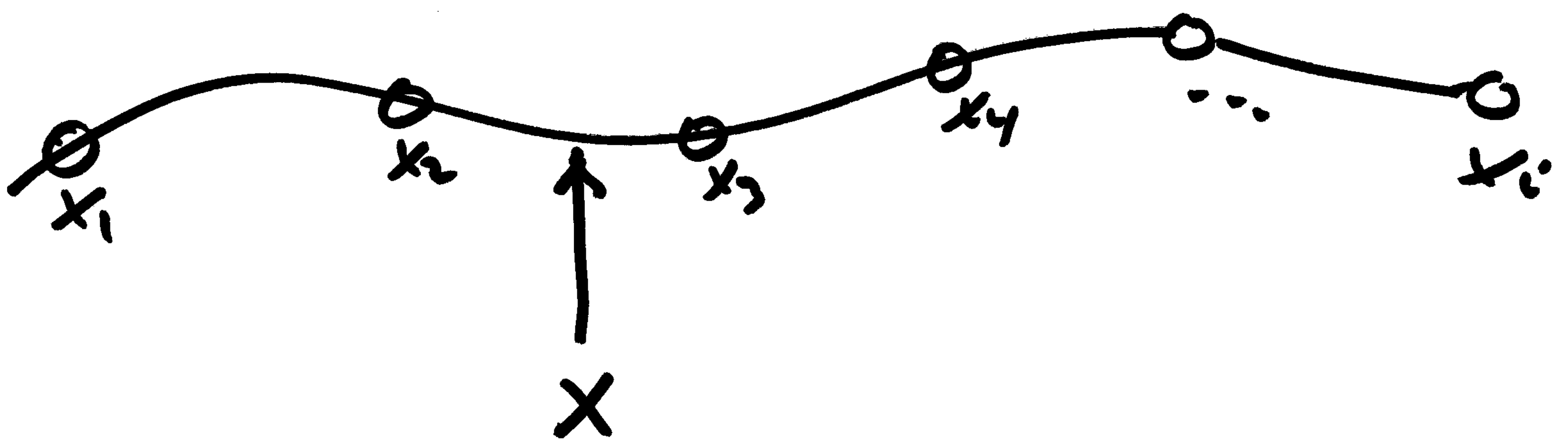
solve linear system

another way: Lagrange Interpolation

$$P(x) = \sum_{i=1}^{n} \frac{(x-x_1)(x-x_2)\cdots(x-x_{i-1})^{*}(x-x_{i+1})\cdots\cdots(x-x_n)}{(x_i-x_1)(x_i-x_2)\cdots(x_i-x_{i-1})_{*}(x_i-x_{i+1})\cdots(x_i-x_n)}\, y_i$$

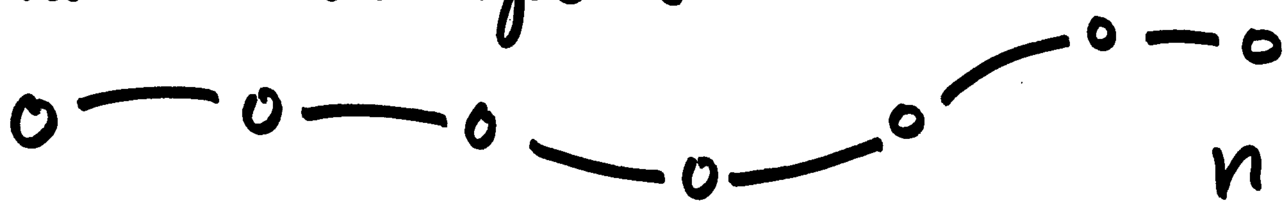
↑ i^{th} factor missing!

n = # points



Q: what order to use?

spline : cubic spline



n points

$n-1$ segments

$$y = \underline{a_0} + \underline{a_1}x + \underline{a_2}x^2 + \underline{a_3}x^3$$

enforce <u>zeroth</u> order continuity

<u>first</u> order "

<u>2nd</u> order "

7 points (n) , 6 segments

unknowns $4 \times 6 = 24$

equations: 1 @ each end — 2

2 @ each interior node — 10

$2(n-2)$

1 @ each int. node — 5

1 @ each int. node — 5

22

missing 2 equations

(1) enforce curv. = 0 @ each end

(2) slope = slope @ last segment

$\boxed{0^{th} \text{ cont.:}}$ enforce continuity

$$a_{i0} + a_{i1}x + a_{i2}x^2 + a_{i3}x^3 = a_{i+1,0} + a_{i+1,1}x + a_{i+1,2}x^2 + a_{i+1,3}x^3$$

$\boxed{1^{st} \text{ cont.}}$

$$y' = \frac{dy}{dx} = a_1 + 2a_2x + 3a_3x^2$$

$$a_{i1} + 2a_{i2}x + 3a_{i3}x^2 = a_{i+1,1} + 2a_{i+1,2}x + 3a_{i+1,3}x^2$$

$\boxed{2^{nd.} \text{ cont.}}$

$$y'' = \frac{d^2y}{dx^2} = 2a_2 + 6a_3x$$

$$2a_{i2} + 6a_{i3}x = 2a_{i+1,2} + 6a_{i+1,3}x$$

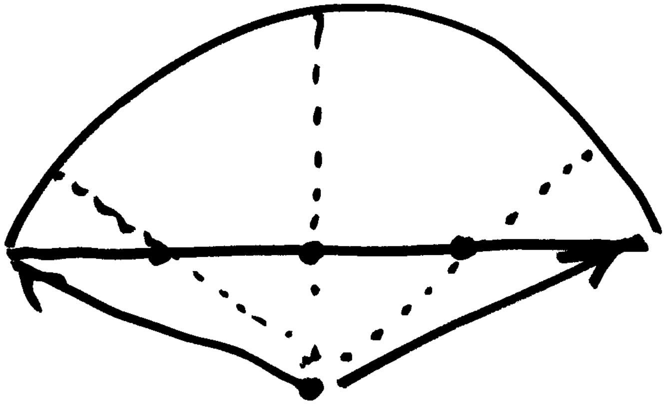
interpolation of attitude

euler angles $\omega_a \varphi_a \kappa_a \rightarrow \omega_b \varphi_b \kappa_b$

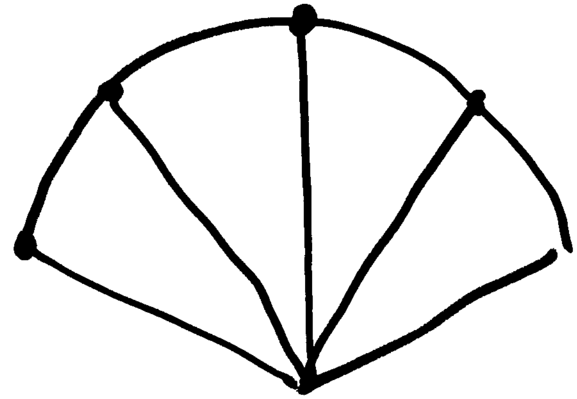
quaternions $q_a \rightarrow q_b$

linear interpolation $\begin{pmatrix} q_s \\ q_i \\ q_j \\ q_k \end{pmatrix}_a \quad \begin{pmatrix} q_s \\ q_i \\ q_j \\ q_k \end{pmatrix}_b$

component wise interpolation



lin. interp components



Spherical linear interpolation

SLERP

better way

$$q_s = (1-t)\, q_{s_a} + t\, q_{s_b}$$

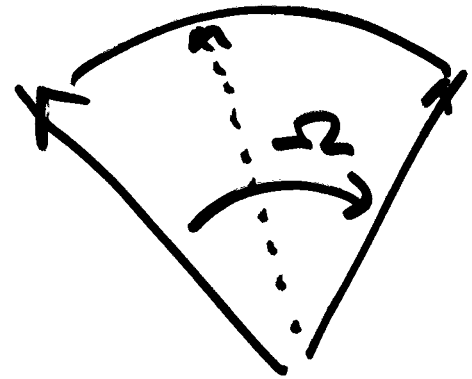
$$q_i = (1-t)\, q_{ia} + t\, q_{ib}$$

$$\vdots$$

normalize to unit length

$q_a \to q_b \quad t\,[0:1]$

$q_a \rightarrow q_b \quad t: (0 \rightarrow 1)$



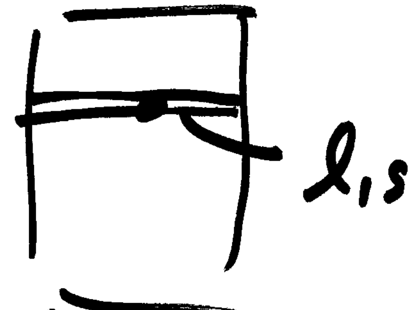
$q_a \cdot q_b = \cos \Omega$

$$\boxed{q_{interp,t} = q_a \frac{\sin((1-t)\Omega)}{\sin \Omega} + q_b \frac{\sin(t\Omega)}{\sin \Omega}}$$

SLERP

constant angular velocity

evaluate cond. equation



l: time

time: interpolate $\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix}$ position

$\phi \lambda h \rightarrow XYZ_{ECF}$

attitude $\begin{pmatrix} q_s \\ q_i \\ q_j \\ q_k \end{pmatrix}$

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ +f \end{pmatrix} = \lambda M_a M_c M \left(\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \left(\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix} - \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \right) \right) = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$