

1. get calibrated fiducial coordinates from the calibration report, assume we have 4 fiducial marks. $(x_{c1}, y_{c1}, x_{c2}, y_{c2}, x_{c3}, y_{c3}, x_{c4}, y_{c4})$
2. configure image viewer application (for example photoshop) to display image coordinates in pixel units, be careful about labeling of line, sample or row, column coordinates.
3. observe the 4 fiducial marks and all data points, then using only the fiducial measurements, solve by least squares for the 6 parameters of the coordinate transformation:

$$\begin{bmatrix} l_1 \\ s_1 \\ l_2 \\ s_2 \\ l_3 \\ s_3 \\ l_4 \\ s_4 \end{bmatrix} + \begin{bmatrix} v_{l1} \\ v_{s1} \\ v_{l2} \\ v_{s2} \\ v_{l3} \\ v_{s3} \\ v_{l4} \\ v_{s4} \end{bmatrix} = \begin{bmatrix} 1 & x_{c1} & y_{c1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{c1} & y_{c1} \\ 1 & x_{c2} & y_{c2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{c2} & y_{c2} \\ 1 & x_{c3} & y_{c3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{c3} & y_{c3} \\ 1 & x_{c4} & y_{c4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{c4} & y_{c4} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} \quad , \quad l + v = Ax$$

$x = (A^T A)^{-1} A^T l$, $v = Ax - l$, units of l and v are pixels
inspect v for quality assurance

4. if v 's are ok (= expected magnitudes) then invert the transformation

$$\begin{aligned} l &= a_0 + a_1 x_c + a_2 y_c \\ s &= b_0 + b_1 x_c + b_2 y_c \end{aligned} \quad , \quad \begin{pmatrix} l \\ s \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

$$l = M \cdot x + t$$

$$x = M^{-1}(l - t)$$

5. use that inverse transformation to transform all data points from raw pixel measurements into the fiducial coordinate system.

6. obtain x_p, y_p (offsets to principal point of symmetry) from the calibration report

7. $\bar{x} = x - x_p$
 $\bar{y} = y - y_p$

8. $r = \sqrt{(\bar{x})^2 + (\bar{y})^2}$

9. obtain k_0, k_1, k_2, k_3 for radial lens distortion, and p_1, p_2, p_3, p_4 for decentering distortion from the camera calibration report. Also find the calibrated focal length, f .

10. compute radial lens distortion correction

$$dx_r = \bar{x} (k_0 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$dy_r = \bar{y} (k_0 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

11. compute decentering lens distortion correction

$$dx_d = (1 + p_3 r^2 + p_4 r^4) (p_1 (r^2 + 2\bar{x}^2) + 2p_2 \bar{x}\bar{y})$$
$$dy_d = (1 + p_3 r^2 + p_4 r^4) (p_2 (r^2 + 2\bar{y}^2) + 2p_1 \bar{x}\bar{y})$$

12. get mean terrain height (above sea level) in km $\rightarrow h$
get flying height (above sea level) in km $\rightarrow H$

compute

$$K = \left[\frac{2410 H}{H^2 - 6H + 250} - \frac{2410 h}{h^2 - 6h + 250} \left(\frac{h}{H} \right) \right] \times 10^{-6}$$

$$dr = K \left(r + \frac{r^3}{f^2} \right)$$

$$dx_{ref} = \bar{x} \cdot dr / r$$

$$dy_{ref} = \bar{y} \cdot dr / r$$

13. compute refined image coordinates

$$x_{refined} = \bar{x} - dx_r - dx_d - dx_{ref}$$

$$y_{refined} = \bar{y} - dy_r - dy_d - dy_{ref}$$