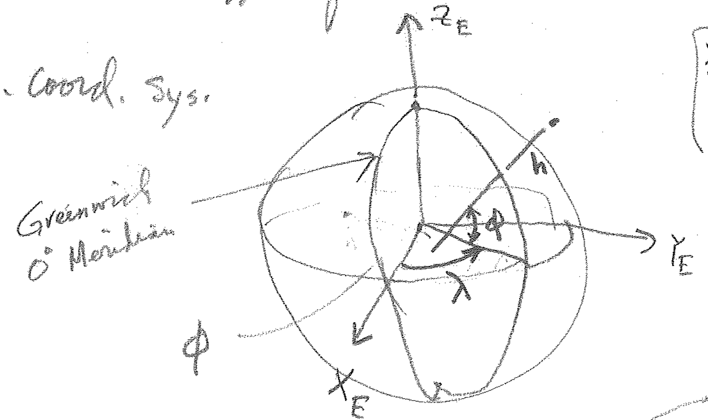
$\boxed{\text{Coord. sys.}}$

1/6

1. product guide
   - L coordinate systems
   - L .geo, .imd, .rpb, .eph, .att

2. look @ all support files

3. Ref. Coord. Sys.



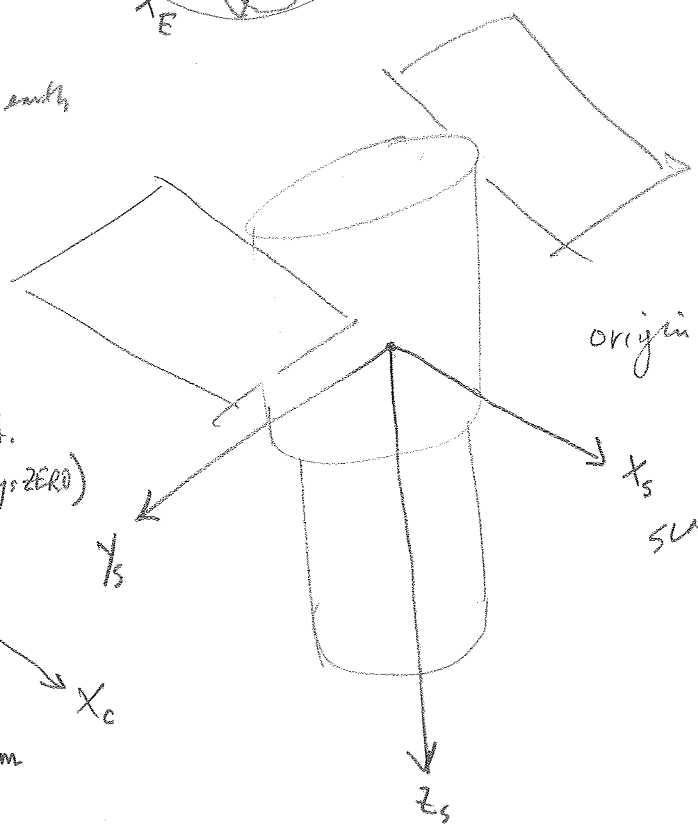
$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_E$: Earth Centered Fixed

$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \leftrightarrow \begin{pmatrix} \phi \\ \lambda \\ h \end{pmatrix}$

origin = center of mass of earth

$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_S$: spacecraft coordinates

C_x, C_y, C_z offset w.r.t.

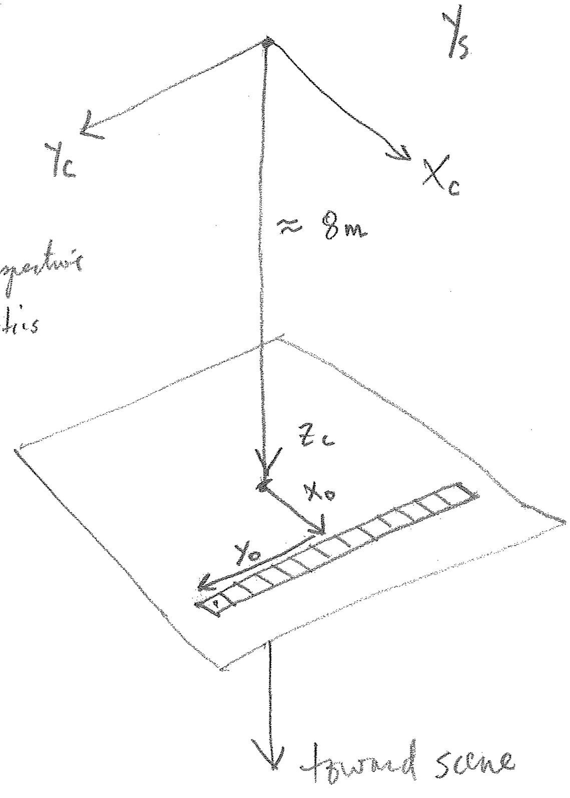


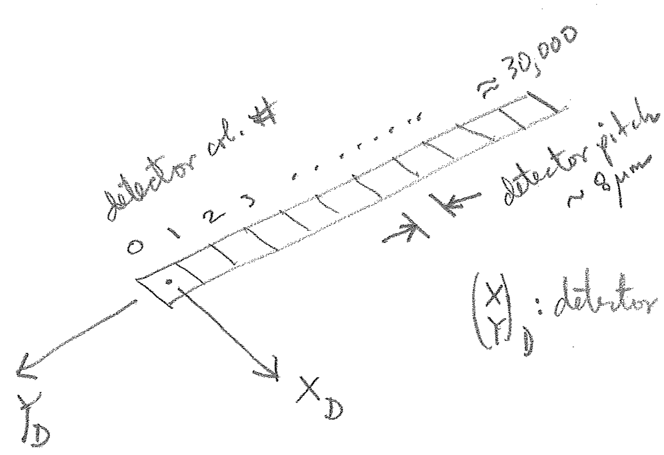
origin = center of mass of s/c

$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_C$: camera coordinates w.r.t. spacecraft (always ZERO)

GEO

origin = perspective center of optics





$\begin{pmatrix} X \\ Y \end{pmatrix}_D$: detector

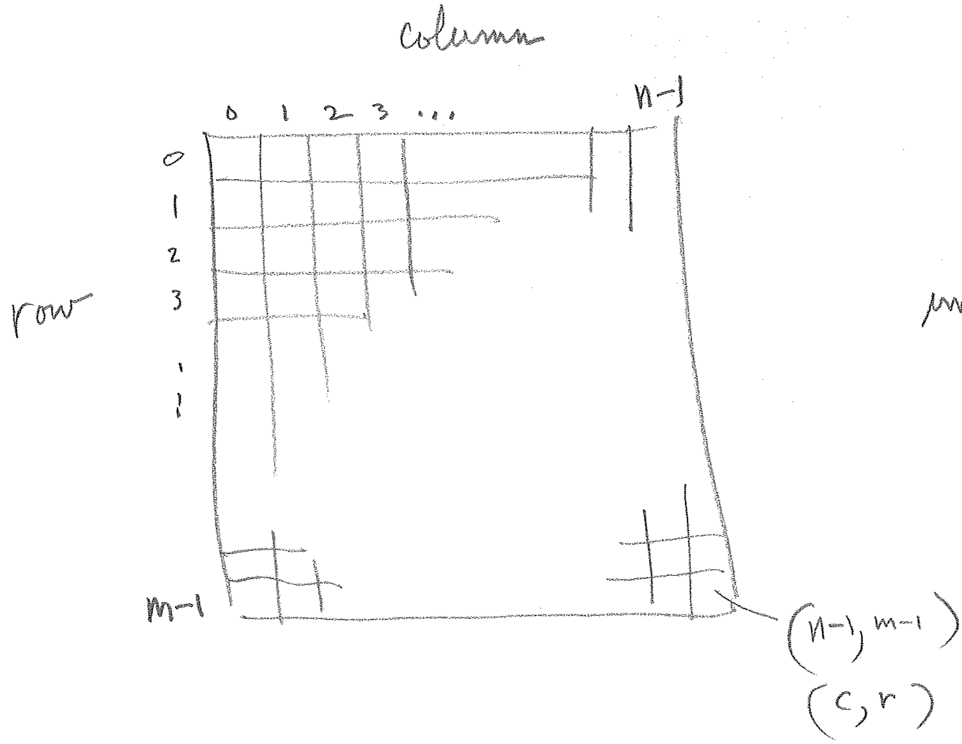
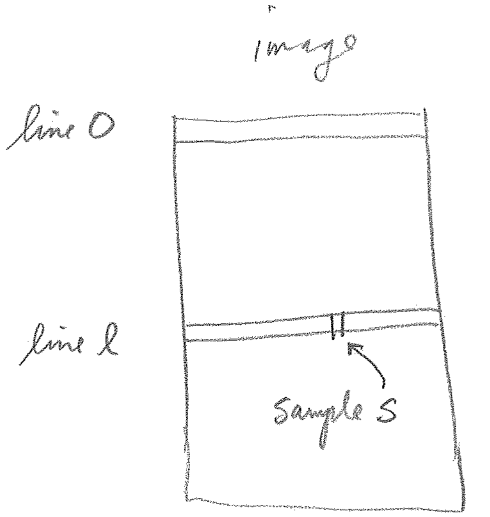
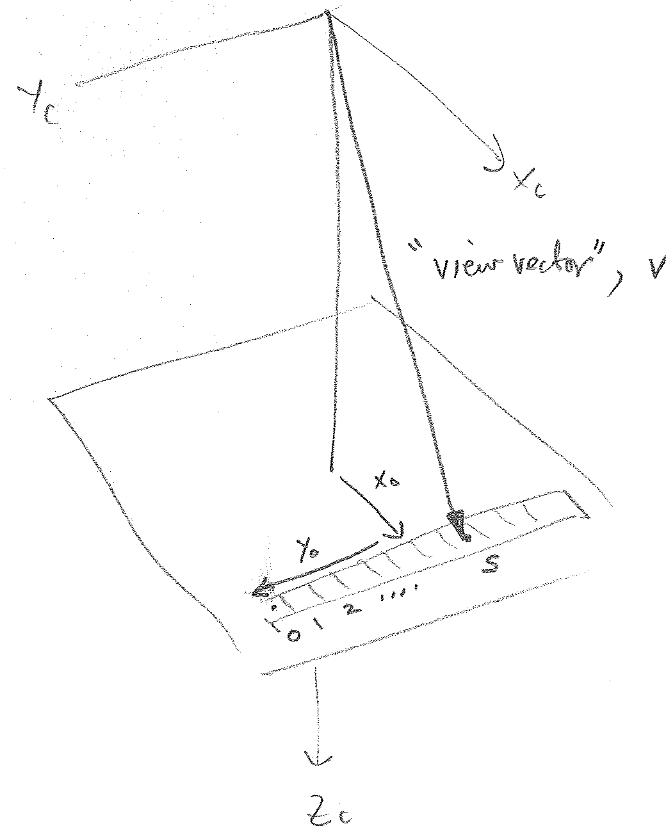


image coords.

$\begin{pmatrix} c \\ r \end{pmatrix}_I$: image coordinates (c, r) left handed !





line l
sample s (l, s)

$$\vec{V}_c = \begin{bmatrix} X_0^{(mm)} \\ Y_0^{(mm)} - S^{(\#)} * (\text{det. pitch})_{(mm)} \\ PD\ (mm) \end{bmatrix}$$

$$\vec{V}_E = M \cdot \vec{V}_c$$

view vector in E.C.F.

IMD

#R, #C

scan direction with motion
against motion

first line time YYYY-MM-DD hh:mm:ss.ssssss Z

Zulu (GMT)

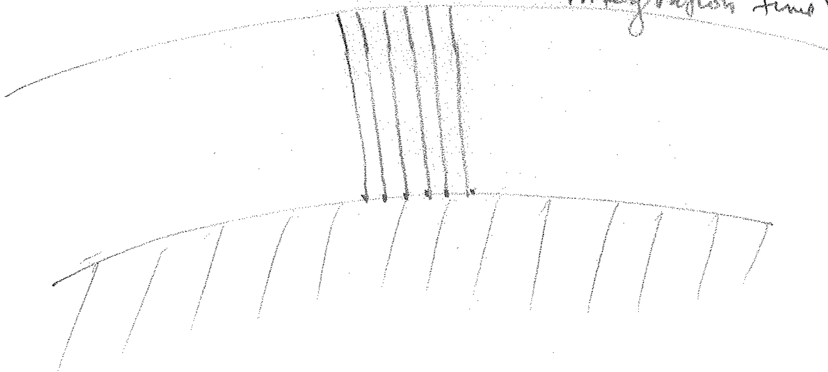
line rate 11895.97 Hz

$$\Delta t_l = 1 / \text{line rate}$$

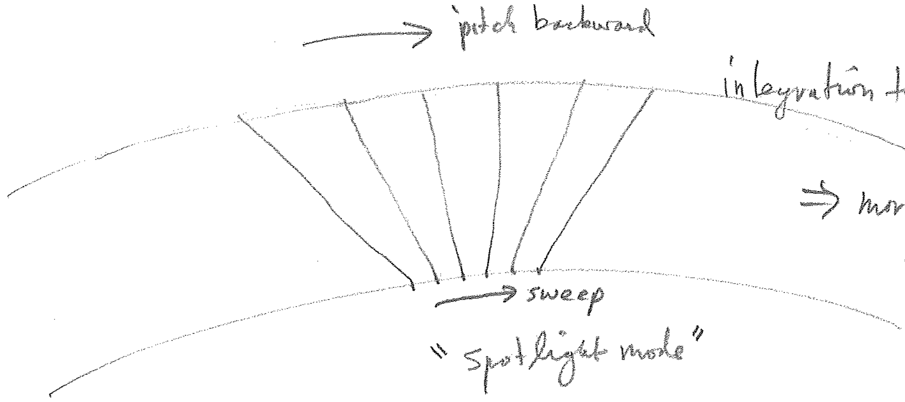
$$\text{time}_l = \text{first line time} + l \times \Delta t_l$$

Synchronous

integration time very short

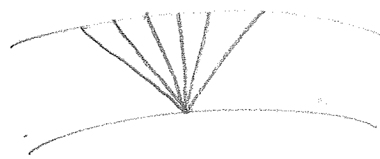


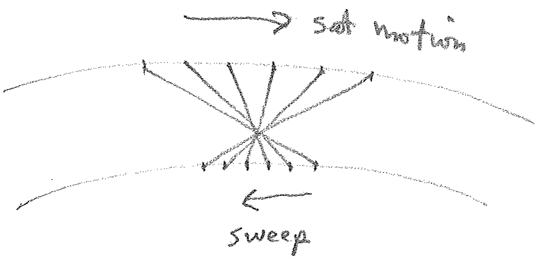
asynchronous



integration time longer

⇒ more energy on the detector





faster

energy on detector

=

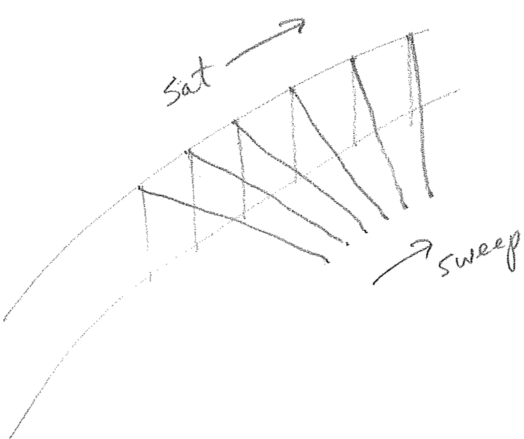
$\propto$ $\boxed{\text{exposure time}}$ ← can adjust !

aperture diameter

area of ground footprint of detector

(det. size, focal length, H)

$1/H^2$



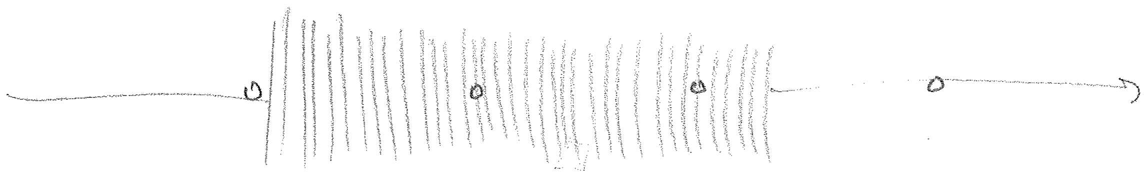
RPB rational polynomial coefficients 5/6

eph $\Delta t_{eph} = 0.02$ sec (50 hz)

$$\frac{\Delta t_{eph}}{\Delta t_{l}} \approx 238 \text{ lines/ephemeris point}$$

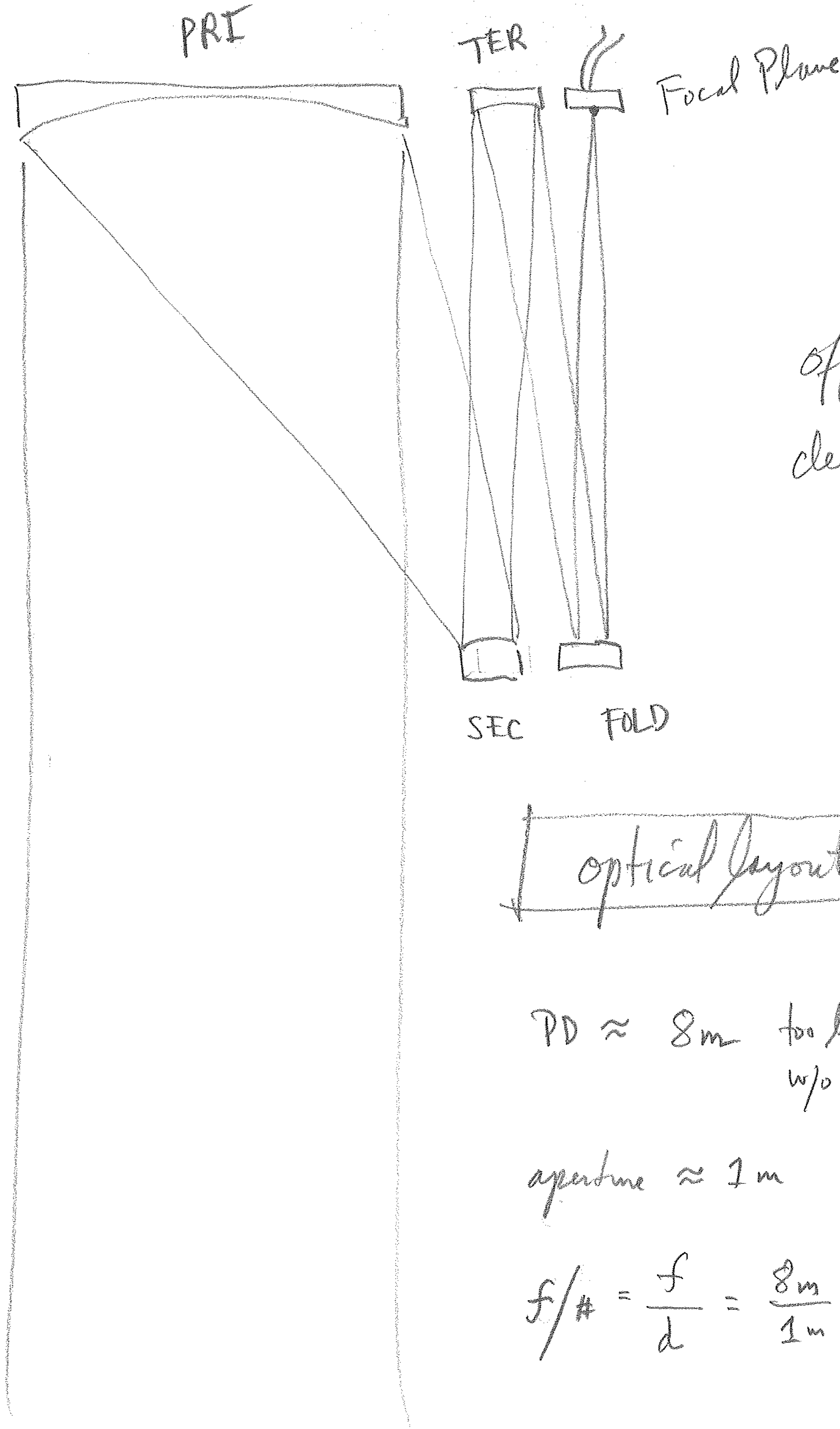
$x \quad y \quad z \quad v_x \quad v_y \quad v_z \quad \sigma_x^2 \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_y^2 \quad \sigma_{yz} \quad \sigma_z^2$

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ & \sigma_y^2 & \sigma_{yz} \\ & & \sigma_z^2 \end{bmatrix}$$ 6 covariances



att $q_1 \; q_2 \; q_3 \; q_5$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{15} \\ & \sigma_2^2 & \sigma_{23} & \sigma_{25} \\ & & \sigma_3^2 & \sigma_{35} \\ & & & \sigma_5^2 \end{bmatrix}$$ + 10 covariances



Off axis
clear aperture

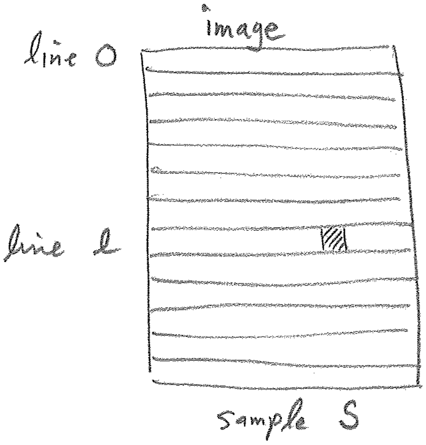
optical layout

PD ≈ 8m too long for design
w/o folding

aperture ≈ 1m

$$f/\# = \frac{f}{d} = \frac{8\,m}{1\,m} = f/8$$

interpolating the ephemeris/attitude data



first line time t_{l_0}

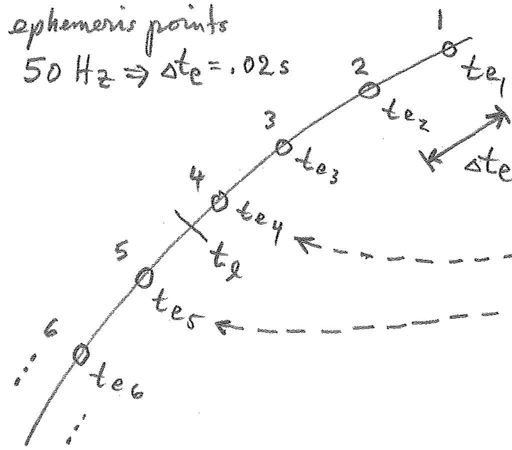
$t_l = t_{l_0} + l \times \Delta t_l$

$$\Delta t_l = \frac{1}{\text{avg. line rate}}$$

for WV1

$$\Delta t_l = \frac{1}{24{,}000} = 4.167e\text{-}5$$

(l, s) measured in <u>pixel</u> <u>units</u>



		example at left
$nintvl =$	$\dfrac{t_l - te_1}{\Delta t_e}$	3.3
$nintvl =$	$fix(nintvl)$	3.0
$index1 =$	$nintvl + 1$	4
$index2 =$	$index1 + 1$	5

$$te_{index1} = te_4 = te_1 + nintvl \times \Delta t_e$$

$$te_{index2} = te_5 = te_{index1} + \Delta t_e$$

now <u>interpolate</u>

$$\left.\begin{aligned} X_l &= X_{index1} + \boxed{\frac{t_l - t_{index1}}{\Delta t_e}} \times (X_{index2} - X_{index1}) \\ Y_l &= Y_{index1} + \boxed{} \times (Y_{index2} - Y_{index1}) \\ Z_l &= Z_{index1} + \boxed{} \times (Z_{index2} - Z_{index1}) \end{aligned}\right\} \text{position}$$

$$\left.\begin{aligned} Vx_l &= Vx_{index1} + \boxed{} \times (Vx_{index2} - Vx_{index1}) \\ Vy_l &= Vy_{index1} + \boxed{} \times (Vy_{index2} - Vy_{index1}) \\ Vz_l &= Vz_{index1} + \boxed{} \times (Vz_{index2} - Vz_{index1}) \end{aligned}\right\} \text{velocity}$$

$$\left.\begin{aligned}
qi_e &= qi_{index1} + \boxed{\qquad\qquad} \times (qi_{index2} - qi_{index1}) \\
qj_e &= qj_{index1} + \boxed{\qquad\qquad} \times (qj_{index2} - qj_{index1}) \\
qk_e &= qk_{index1} + \boxed{\qquad\qquad} \times (qk_{index2} - qk_{index1}) \\
qs_e &= qs_{index1} + \boxed{\qquad\qquad} \times (qs_{index2} - qs_{index1})
\end{aligned}\right\} \text{attitude}$$

<u>unitize</u> result :

$$|q_e| = \left[qi_e^2 + qj_e^2 + qk_e^2 + qs_e^2 \right]^{1/2}$$

$$q_e = q_e / |q_e|$$

parameters for rotation   like unique transf. $\omega, \phi, \kappa \rightleftarrows$ matrix   1/5

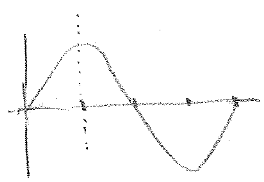
Euler Angles // $M = M_3(\kappa)\, M_2(\phi)\, M_1(\omega)$

Rotations

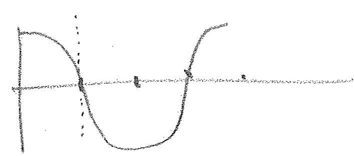
$$M_1(\omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \qquad M_2(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$M_3(\kappa) = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

problem if "middle" angle $= \pm 90°$, $M = M_3(\kappa)\, M_2(90)\, M_1(\omega)$



sin x



cos x

$$\begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} =$$

$$\begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sin\omega & -\cos\omega \\ 0 & \cos\omega & \sin\omega \\ 1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & \sin\omega\cos\kappa + \cos\omega\sin\kappa & -\cos\omega\cos\kappa + \sin\omega\sin\kappa \\ 0 & -\sin\omega\sin\kappa + \cos\omega\cos\kappa & +\cos\omega\sin\kappa + \sin\omega\cos\kappa \\ 1 & 0 & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \cos(w+K) &= \cos w \cos K - \sin w \sin K \\ \sin(w+K) &= \sin w \cos K + \cos w \sin K \end{aligned}}$$

$$\begin{bmatrix} 0 & \sin(w+K) & -\cos(w+K) \\ 0 & \cos(w+K) & \sin(w+K) \\ 1 & 0 & 0 \end{bmatrix}$$

<u>infinite no. combinations of w & K which produce this matrix!</u>

if you try to solve for w & K, infinite # solutions

$\Rightarrow$ <u>singular</u>

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lego model ϕ: $90°$, infinite # w & K produce same position

---

solution to singularity: quaternions like 3D complex #'s
we use only for rotations =

$$\begin{bmatrix} q_i \\ q_j \\ q_K \\ q_s \end{bmatrix} \xleftarrow{\text{D.G. order}} \qquad \begin{bmatrix} q_s \\ q_i \\ q_j \\ q_K \end{bmatrix} \xleftarrow{\text{Mod. Photogr.}}$$

always keep @ unit length

$$q_i^2 + q_j^2 + q_K^2 + q_s^2 = \underline{1}$$

if not, $|q| = \left[q_i^2 + q_j^2 + q_K^2 + q_s^2\right]^{1/2}$

$$q_{unit} = q / |q|$$

$$m = q2m(q_i, q_j, q_k, q_s) \qquad \text{(unit } q\text{)}$$

$$\begin{bmatrix} q_s^2 + q_i^2 - q_j^2 - q_k^2 & 2(q_j q_i - q_s q_k) & 2(q_i q_k + q_s q_j) \\ 2(q_j q_i + q_s q_k) & q_s^2 - q_i^2 + q_j^2 - q_k^2 & 2(q_j q_k - q_s q_i) \\ 2(q_i q_k - q_s q_j) & 2(q_j q_k + q_s q_i) & q_s^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix}$$

---

only need $q \to M$, but here is $M \to q$ (q & $-q$ produce same M)

$$\left.\begin{aligned} 1 + M_{11} + M_{22} + M_{33} &= 4 q_s^2 \\ 1 + M_{11} - M_{22} - M_{33} &= 4 q_i^2 \\ 1 - M_{11} + M_{22} - M_{33} &= 4 q_j^2 \\ 1 - M_{11} - M_{22} + M_{33} &= 4 q_k^2 \end{aligned}\right\}$$

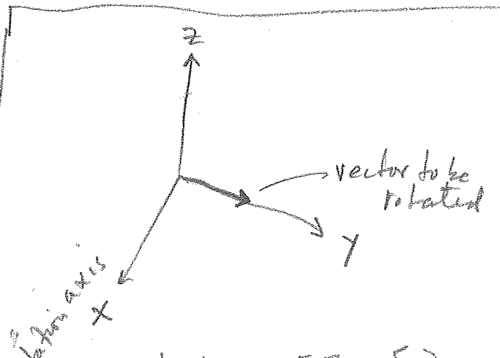
for numerical precision choose largest magnitude

then choose $\underline{3}$ from

$$M_{32} - M_{23} = 4 q_s q_i$$
$$M_{13} - M_{31} = 4 q_s q_j$$
$$M_{21} - M_{12} = 4 q_s q_k$$
$$M_{21} + M_{22} = 4 q_i q_j$$
$$M_{32} + M_{23} = 4 q_j q_k$$
$$M_{13} + M_{31} = 4 q_k q_i$$

axis-angle

(I post derivation of matrix below)



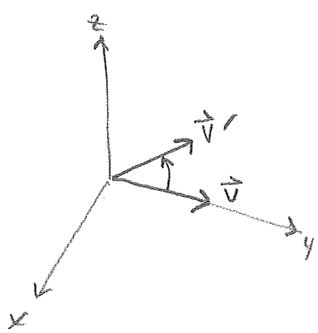
let's take $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

vector $= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\theta = +30^\circ \quad (.5236\,R)$

$m = \text{maxang}(a, b, c, \theta)$

$M * \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ .866 \\ .500 \end{bmatrix}$



So maxang rotates the <u>vector</u> by RH Rule

not coord system!

euler's theorem

any 3D rotation can be expressed as 1 rotation about an axis

axis specified by unit vector

angle θ about that vector

α, β, γ or a, b, c

$$\begin{bmatrix} a^2(1-\cos\theta)+\cos\theta & ab(1-\cos\theta)-c\sin\theta & ac(1-\cos\theta)+b\sin\theta \\ ab(1-\cos\theta)+c\sin\theta & b^2(1-\cos\theta)+\cos\theta & bc(1-\cos\theta)-a\sin\theta \\ ac(1-\cos\theta)-b\sin\theta & bc(1-\cos\theta)+a\sin\theta & c^2(1-\cos\theta)+\cos\theta \end{bmatrix}$$

M

$M \rightarrow a, b, c, \theta$

$$\theta = \operatorname{acos}\left(\frac{\operatorname{tr} M - 1}{2}\right)$$

tr = trace
sum of diag. elems

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{2\sin\theta}\begin{bmatrix} m_{32}-m_{23} \\ m_{13}-m_{31} \\ m_{21}-m_{12} \end{bmatrix}$$

//

q's & a, b, c, θ

$$\boxed{q \to a, b, c, \theta}$$

$$\cos\theta = q_s^2 - \left(q_i^2 + q_j^2 + q_k^2\right)$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{\left[q_i^2 + q_j^2 + q_k^2\right]^{1/2}} \begin{pmatrix} q_i \\ q_j \\ q_k \end{pmatrix}$$

$$\boxed{a, b, c, \theta \to q}$$

$$q_s = \cos\left(\frac{\theta}{2}\right)$$

$$\begin{pmatrix} q_i \\ q_j \\ q_k \end{pmatrix} = \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

useful matlab statement:

```
path(path, 'n:\mfiles\');
```

xyz2geo.m $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \phi, \lambda, h$ (iterative)

$\boxed{\text{coord. conversions} + \text{atmosph. refr.}}$

$a = 6378137$

$f = 1/298.257223563$

$e^2 = 2f - f^2$

$\phi_0 = \mathrm{atan}\left(\frac{z}{\sqrt{x^2+y^2}}\right)$

$\lambda = \mathrm{atan2}\,(y, x)$

$h_0 = 0$

$\phi_{old} = 0$

loop until $|\phi_0 - \phi_{old}| < 1e-10$

$N_0 = a \Big/ \left[1 - e^2 (\sin \phi_0)^2\right]^{1/2}$

$\phi_{old} = \phi_0$

$\phi_0 = \mathrm{atan}\left(\frac{z}{\sqrt{x^2+y^2}} \cdot \frac{1}{\left(1 - e^2 \left(\frac{N_0}{N_0 + h_0}\right)\right)}\right)$

$h_0 = \frac{\sqrt{x^2+y^2}}{\cos\phi_0} - N_0$

lat $= \phi_0$

lon $= \lambda$

$h = h_0$

✓

geo2xyz.m

$\phi \lambda h \rightarrow xyz$

$a = 6378137$

$f = 1/298.257223563$

$e^2 = 2f - f^2$

$$N = \frac{a}{\left[1 - e^2 \sin^2 \phi\right]^{1/2}}$$

$$X = (N+h) \cdot \cos\phi \cdot \cos\lambda$$

$$Y = (N+h) \cdot \cos\phi \cdot \sin\lambda$$

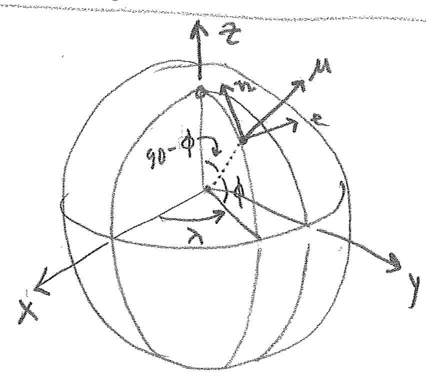
$$Z = (N(1-e^2)+h) \cdot \sin\phi$$

---

2 functions should be inverse of each other!

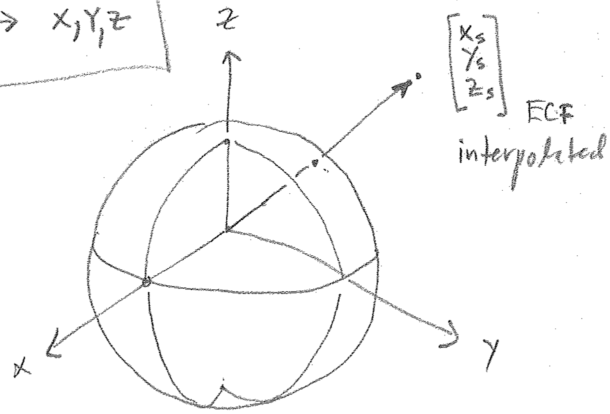
$\begin{matrix} X \\ Y \\ Z \end{matrix}$ or $\begin{matrix} \phi \\ \lambda \\ h \end{matrix}$ $\rightarrow$ local cartesian (topocentric)

local origin (reference) $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_r \rightleftarrows \begin{pmatrix} \phi \\ \lambda \\ h \end{pmatrix}_r$



$$\begin{bmatrix} e \\ n \\ u \end{bmatrix} = M_x(90° - \phi_r)\, M_z(\lambda_r + 90°) \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X_r \\ Y_r \\ Z_r \end{pmatrix} \right]$$

$l, s, h \rightarrow x, y, z$

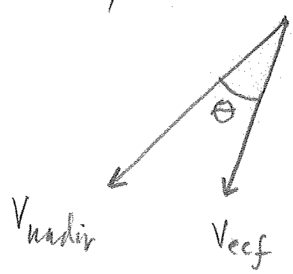


$$V_{nadir} = -\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

$$a_s = |V_{nadir}| \quad ; \text{ meters}$$

$$V_{ecf} = M \cdot V_{sensor}$$

$$\Theta = a\cos(\vec{V}_{nadir} \cdot \vec{V}_{ecf})$$



H_{km} approx sensor height (km)

h_{km}

$$H_{km} = a_s - 6371000 / 1000$$

$$h = h / 1000$$

Sastamoinen MoP p. 293

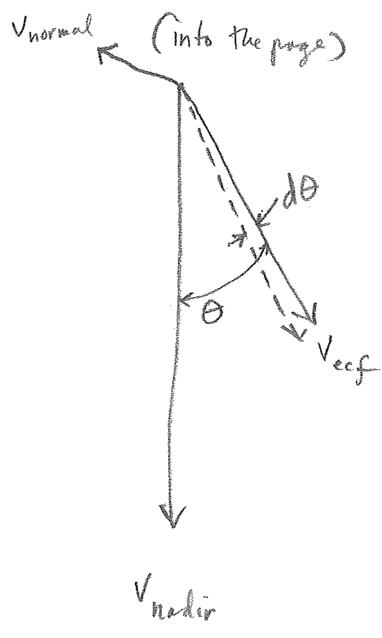
$$t_1 = \frac{2335}{H-h} \cdot (1 - 0.02257 \cdot h)^{5.256}$$

$$t_2 = 0.8540^{(H-11)} \cdot \left(82.2 - \frac{521}{H-h}\right)$$

$$K = (t_1 - t_2) \times 10^{-6}$$

$$d\Theta = K \cdot \tan\Theta$$

(rotate toward nadir)



$$V_{normal} = V_{ecf} \times V_{nadir}$$

(unitize V_{normal})

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = V_{normal}$$

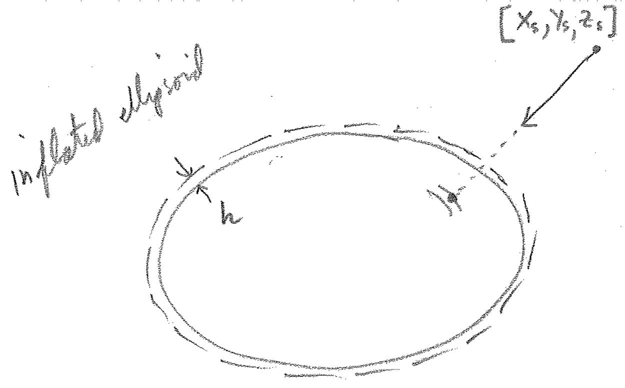
$$M_{AR} = M_{axis\text{-}angle}(\alpha, \beta, \gamma, d\Theta)$$

$$V_{ecf\_corr} = M_{axis\text{-}angle} \cdot V_{ecf}$$

apply velocity aberration correction (next time)

V_{ecf_cor2}

intersect w/ ellipsoid @ h 5/5



$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$ unit vector

$$\frac{X^2+Y^2}{a^2} + \frac{Z^2}{b^2} = 1$$

$=$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} + K \cdot V_{ecs\_corr} \quad \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_s \\ Y_s \\ Z_s \end{pmatrix} + K \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\begin{cases} X = X_s + K V_x \\ Y = Y_s + K V_y \\ Z = Z_s + K V_Z \end{cases}$$

$$\frac{(X_s + KV_x)^2 + (Y_s + KV_y)^2}{a^2} + \frac{(Z_s + KV_z)^2}{b^2} - 1 = 0$$

$$\frac{X_s^2 + K^2V_x^2 + 2X_sKV_x + Y_s^2 + K^2V_y^2 + 2Y_sKV_y}{a^2} + \frac{Z_s^2 + K^2V_z^2 + 2Z_sKV_z}{b^2} - 1 = 0$$

$$\frac{X_s^2}{a^2} + K^2\frac{V_x^2}{a^2} + K \cdot \frac{2X_sV_x}{a^2} + \frac{Y_s^2}{a^2} + K^2\frac{V_y^2}{a^2} + K \cdot \frac{2Y_sV_y}{a^2} + \frac{Z_s^2}{b^2} + K^2\frac{V_z^2}{b^2} + K \cdot \frac{2Z_sV_z}{b^2} - 1 = 0$$

$$A = (V_x^2 + V_y^2)/a^2 + V_z^2/b^2$$

$$B = \frac{(2X_sV_x + 2Y_sV_y)}{a^2} + \frac{2Z_sV_z}{b^2}$$

$$C = \frac{X_s^2 + Y_s^2}{a^2} + Z_s^2/b^2 - 1$$

$$K = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

take short one!

$$K = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$h_0 = h$

$h_1 = h_0$

$a_0 = 6378137$

$f = 1/298.257223563$

$e^2 = 2f - f^2$

$b_0^2 = a_0^2 \cdot (1 - e^2)$

(iterate for height) 3x enough

$a = a_0 + h_1$

$b = b_0 + h_1$

solve quadratic for K

$$X_G = \begin{pmatrix} X_s \\ Y_s \\ Z_s \end{pmatrix} + K \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G \rightarrow \phi, \lambda, h_2$ h_2 different than h so correct h_1

$dh = h_2 - h_0$ (error)

$h_1 = h_1 - dh$

done $\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G$ (dh should $\rightarrow$ ZERO) l, s, h enter with these $\boxed{\pm 2G}$

read ephemeris data
read attitude data
fix constants eph_start_time, first_line_time, avg_line_rate,
dtl, dte, det_pitch, princ_dist

construct $\vec{V}_{sensor} = \begin{bmatrix} X_0 \\ Y_0 - s * det\_pitch \\ princ\_dist \end{bmatrix}$

$t_{line} = first\_line\_time + l \cdot dtl$

interpolate X_s, Y_s, Z_s Vel_x, Vel_y, Vel_z

interpolate q_i, q_j, q_k, q_s

unitize

compute rotation matrix from q's (M)

$\vec{V}_{ecf} = M \cdot \vec{V}_{sensor}$

correct for atmospheric refraction

$\vec{V}_{ecf\_cor1}$

correct for velocity aberration

$\vec{V}_{ecf\_cor2}$

intersect with ellipsoid at height h

$(X, Y, Z)_G$

convert to (enu) @ GCP reference

$\Delta e, \Delta n$ = ? (misclosure)