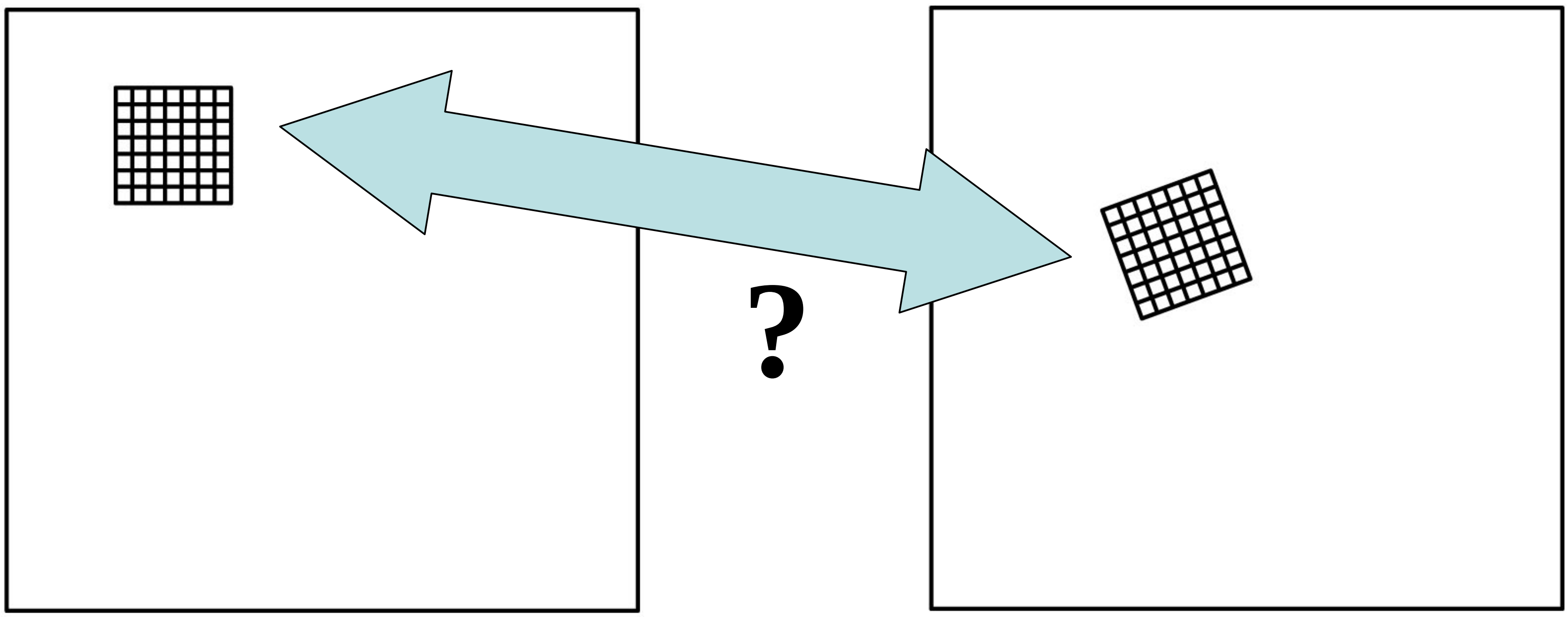
Aspects of Least Squares Matching Related to Finding Interest Points

How can we model the correspondence between small match windows in two overlapping images?



How about: gray level from left and gray level from right, at transformed location, are equal?

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$$F_{LSM} = I_L(x,y) - I_R(x',y') = 0$$

$$x' = a_0 + a_1 x + a_2 y$$

$$y' = b_0 + b_1 x + b_2 y$$

$$F_{LSM} = I_L(x,y) - I_R(a_0 + a_1 x + a_2 y, b_0 + b_1 x + b_2 y)$$

Observation (gray value, intensity) on the left image

Gray value or intensity on right image, computed via the geometric parameters a,b

Linearize,

$$F \approx F^0 + \frac{\partial F}{\partial a_0}\Delta a_0 + \frac{\partial F}{\partial a_1}\Delta a_1 + \frac{\partial F}{\partial a_2}\Delta a_2 + \frac{\partial F}{\partial b_0}\Delta b_0 + \frac{\partial F}{\partial b_1}\Delta b_1 + \frac{\partial F}{\partial b_2}\Delta b_2$$

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To get the derivatives, we must use the chain rule,

$$\frac{\partial F}{\partial p} = \frac{\partial F}{\partial I_R} \frac{\partial I_R}{\partial x'} \frac{\partial x'}{\partial p}$$

-1

Change in intensity along x or y, i.e. gradient, g_x or g_y

Coefficient from 6-parameter equations

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$$\frac{\partial F}{\partial a_0} = -g_x$$

$$\frac{\partial F}{\partial a_1} = -g_x x$$

$$\frac{\partial F}{\partial a_2} = -g_x y$$

$$\frac{\partial F}{\partial b_0} = -g_y$$

$$\frac{\partial F}{\partial b_1} = -g_y x$$

$$\frac{\partial F}{\partial b_2} = -g_y y$$

$$\begin{bmatrix} -g_x & -g_x x & -g_x y & -g_y & -g_y x & -g_y y \\ -g_x & -g_x x & -g_x y & -g_y & -g_y x & -g_y y \\ -g_x & -g_x x & -g_x y & -g_y & -g_y x & -g_y y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -g_x & -g_x x & -g_x y & -g_y & -g_y x & -g_y y \end{bmatrix}$$

B Matrix for least squares estimation of the 6 geometric parameters, with 2 columns highlighted, those for the 2 *shift parameters*, a_0 *and* b_0. Six columns for the six parameters, one row for each pixel in the match window.

Now form normal equations **N=B^T^B** considering only the 2 shift parameters

$$\begin{bmatrix} g_x g_x + g_x g_x + \ldots + g_x g_x & g_x g_y + g_x g_y + \ldots + g_x g_y \\ g_x g_y + g_x g_y + \ldots + g_x g_y & g_y g_y + g_y g_y + \ldots + g_y g_y \end{bmatrix} = \mathbf{B}^{\mathbf{T}}\mathbf{B} = \mathbf{N}$$

Covariance matrix of the shift parameters will be inverse of this

$$\Sigma_{\mathbf{shift}} = \left(\mathbf{B}^{\mathbf{T}}\mathbf{B}\right)^{-1} = \begin{bmatrix} \Sigma g_x^2 & \Sigma g_x g_y \\ \Sigma g_x g_y & \Sigma g_y^2 \end{bmatrix}^{-1}$$

Big idea: Shift Precision is related to the *Size* and *Shape* of the uncertainty regions defined by this covariance matrix

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Construct a standard error ellipse and look at

$$\text{Size}:\text{Area of ellipse}=\pi ab$$

$$\text{Shape}:\text{Eccentricity (informally)}=a/b$$

Compute these, or equivalent, quantities for every point in an image, then rank order and threshold for

•Small size, and

•Circular shape

•Highest ranking points will be the *Interest Points*

Examples

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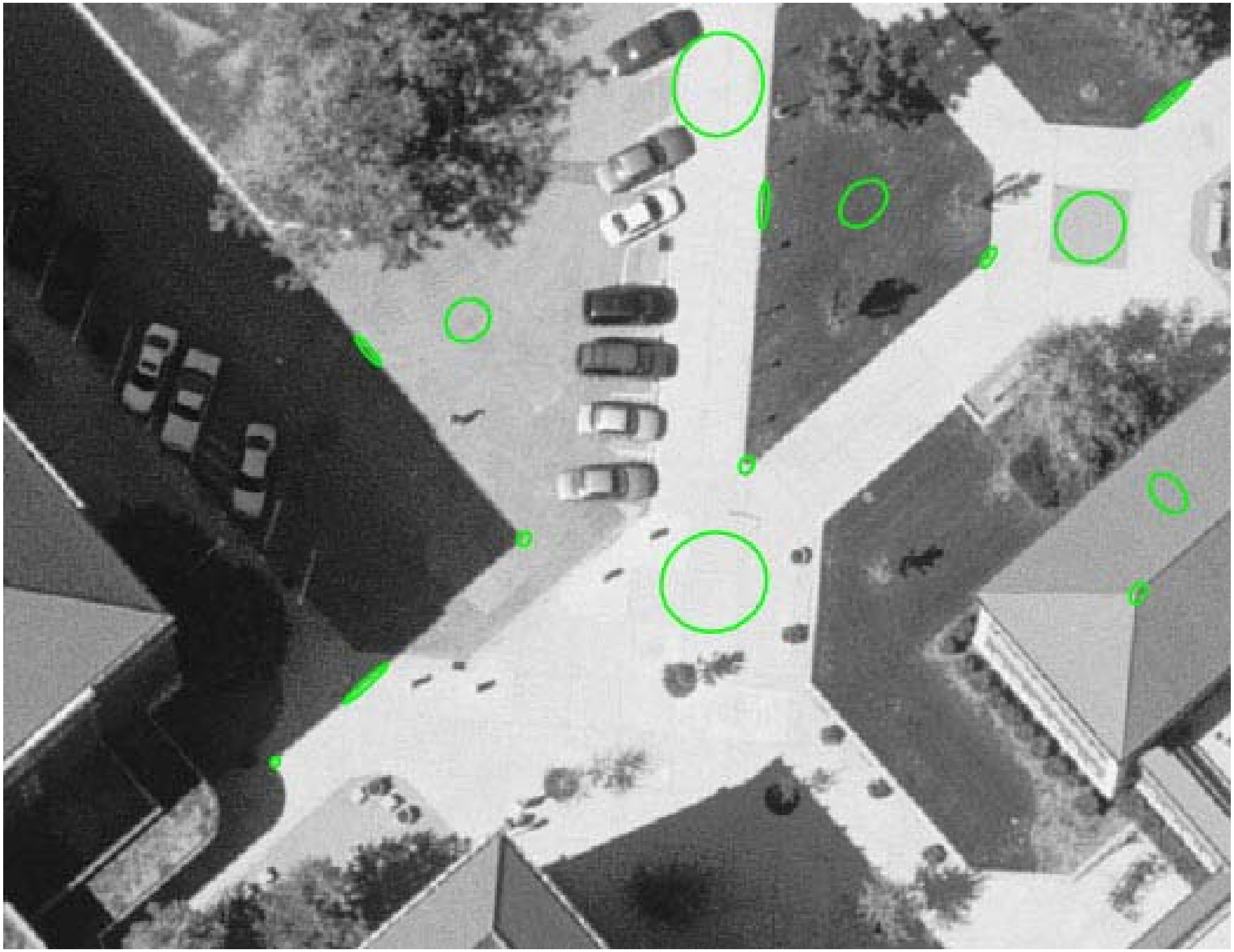

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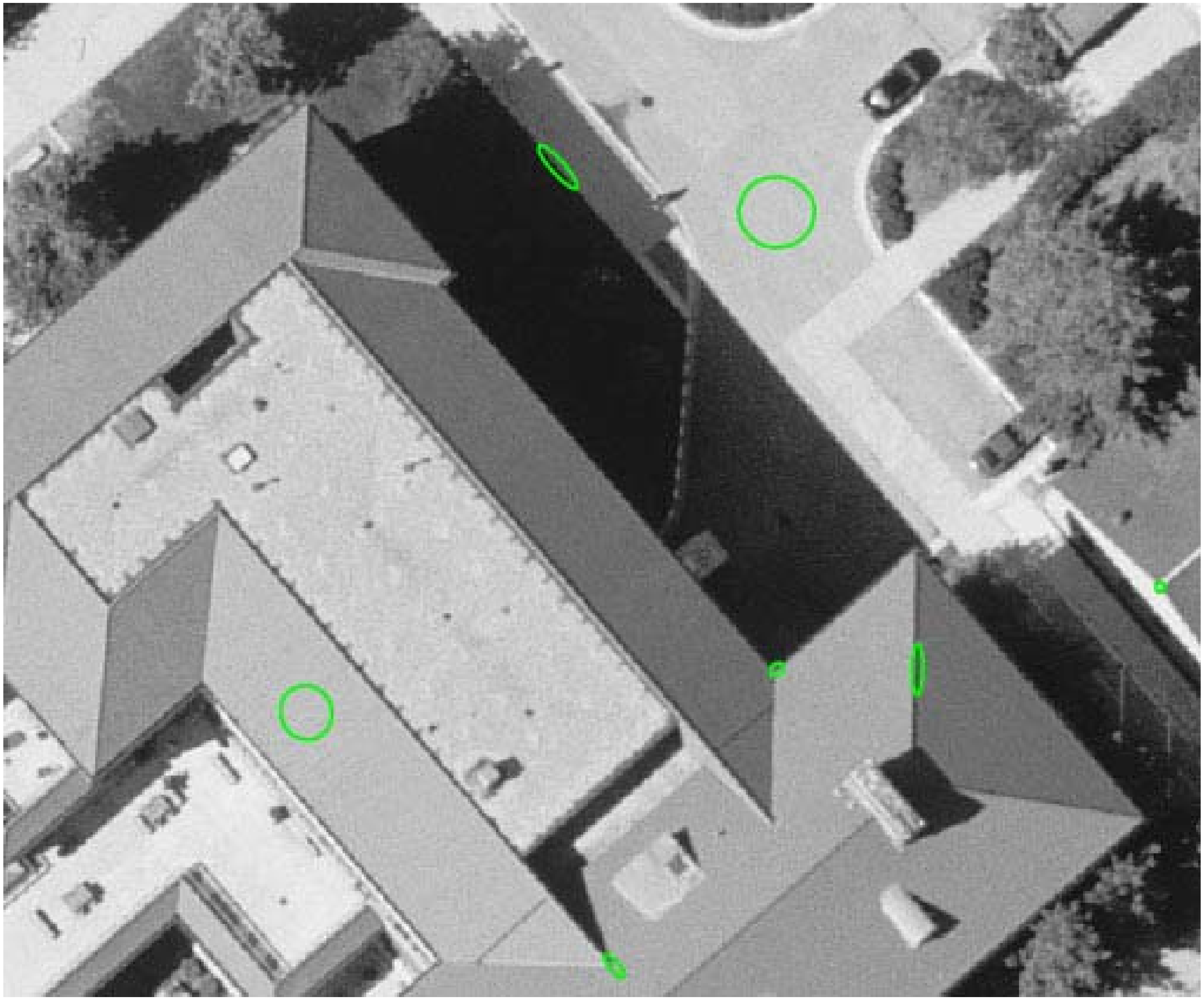

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