Nonlinear Example: Indirect Observations

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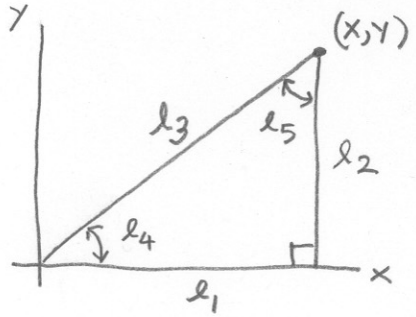

$n = 5$ Choose 2 parameters : (X, Y)

$n_0 = 2$ write $n = 5$ condition equations of form

$r = 3$ $F_i(l, x) = l_i + G_i(x) = 0$

Choose
$X^0 = 10.0$
$Y^0 = 20.0$

$l_1 = 10.0$ $\sigma = 0.1$
$l_2 = 20.0$ $\sigma = 0.1$
$l_3 = 22.3$ $\sigma = 0.1$
$l_4 = 63.0^\circ$ $\sigma = 0.1$ degree
$l_5 = 26.0^\circ$ $\sigma = 0.1$ degree

$l_1 = X$ $\quad l_1 - X = 0$

$l_2 = Y$ $\quad l_2 - Y = 0$

$l_3 = [X^2 + Y^2]^{1/2}$ $\quad l_3 - [X^2 + Y^2]^{1/2} = 0$

$l_4 = \tan^{-1}(Y/X)$ $\quad l_4 - \tan^{-1}(Y/X) = 0$

$l_5 = \tan^{-1}(X/Y)$ $\quad l_5 - \tan^{-1}(X/Y) = 0$

$$B = \frac{\partial F}{\partial X} = \begin{bmatrix} \partial F_1/\partial X & \partial F_1/\partial Y \\ \partial F_2/\partial X & \partial F_2/\partial Y \\ \partial F_3/\partial X & \partial F_3/\partial Y \\ \partial F_4/\partial X & \partial F_4/\partial Y \\ \partial F_5/\partial X & \partial F_5/\partial Y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ \frac{-X}{[X^2+Y^2]^{1/2}} & \frac{-Y}{[X^2+Y^2]^{1/2}} \\ \frac{Y}{X^2+Y^2} & \frac{-X}{X^2+Y^2} \\ \frac{-Y}{X^2+Y^2} & \frac{X}{X^2+Y^2} \end{bmatrix}_{(X^0, Y^0)}, \quad f = -\begin{bmatrix} l_1 - X \\ l_2 - Y \\ l_3 - [X^2+Y^2]^{1/2} \\ l_4 - \tan^{-1}(Y/X) \\ l_5 - \tan^{-1}(X/Y) \end{bmatrix}_{(X^0, Y^0)}$$

$\sigma_1^2 = .01$, $\sigma_2^2 = .01$, $\sigma_3^2 = .01$, $\sigma_4^2 = (.0017)^2 = 3 \times 10^{-6}$ Rad, $\sigma_5 = 3 \times 10^{-6}$

Choose $\sigma_0^2 = .01$

$w_i = \sigma_0^2 / \sigma_i^2$, $w_1 = 1$, $w_2 = 1$, $w_3 = 1$, $w_4 = .01/3 \times 10^{-6} = 3333$, $w_5 = .3333$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3333 & 0 \\ 0 & 0 & 0 & 0 & 3333 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -.4472 & -.8944 \\ .0400 & -.0200 \\ -.0400 & .0200 \end{bmatrix} \quad f = \begin{bmatrix} 0 \\ 0 \\ .0607 \\ .0076 \\ .0099 \end{bmatrix}$$

$\Delta = (B^T W B)^{-1} B^T W f$

$\Delta = \begin{bmatrix} -.0347 \\ -.0166 \end{bmatrix}$

$X^0 = X^0 + \Delta = \begin{bmatrix} 9.9653 \\ 19.9834 \end{bmatrix}$

do again

$\Delta = \begin{bmatrix} .00001831 \\ -.00003007 \end{bmatrix}$

$X^0 = X^0 + \Delta = \begin{bmatrix} 9.9653 \\ 19.9834 \end{bmatrix}$

stop.

$V = f - B\Delta$

$V = \begin{bmatrix} -.0347 \\ -.0166 \\ .0303 \\ .0086\,R \\ .0088\,R \end{bmatrix}$

$V_4 = 0.49$ deg

$V_5 = 0.50$ deg

$\hat{l} = l + V$

$\hat{l} = \begin{bmatrix} 9.9653 \\ 19.9834 \\ 22.3303 \\ 1.1082\,R \\ 0.4626\,R \end{bmatrix}$

$\hat{l}_4 = 63.49$ deg

$\hat{l}_5 = 26.50$ deg

Nonlinear Example : Observations Only
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same problem as previous.

$n = 5$ need 3 condition equations

$n_0 = 2$

$r = 3$

$$l_4 + l_5 = 90^\circ \qquad l_4 + l_5 - 90^\circ = 0$$

$$l_3 = \left[l_1^2 + l_2^2\right]^{1/2} \qquad l_3 - \left[l_1^2 + l_2^2\right]^{1/2} = 0$$

$$l_4 = \sin^{-1}\left(l_2/l_3\right) \qquad l_4 - \sin^{-1}\left(l_2/l_3\right) = 0$$

($l^\circ = l$ initially)

$$A = \left.\frac{\partial F}{\partial l}\right|_{l^\circ} = \begin{bmatrix} \partial F_1/\partial l_1 & \partial F_1/\partial l_2 & \partial F_1/\partial l_3 & \partial F_1/\partial l_4 & \partial F_1/\partial l_5 \\ \partial F_2/\partial l_1 & \partial F_2/\partial l_2 & \partial F_2/\partial l_3 & \partial F_2/\partial l_4 & \partial F_2/\partial l_5 \\ \partial F_3/\partial l_1 & \partial F_3/\partial l_2 & \partial F_3/\partial l_3 & \partial F_3/\partial l_4 & \partial F_3/\partial l_5 \end{bmatrix}_{l^\circ}$$

$$f = -F(l^\circ) - A(l - l^\circ)$$

$$f = -\begin{bmatrix} l_4 + l_5 - 90^\circ \\ l_3 - \left[l_1^2 + l_2^2\right]^{1/2} \\ l_4 - \sin^{-1}\left(l_2/l_3\right) \end{bmatrix}_{l^\circ} - A(l - l^\circ)$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ \dfrac{-l_1}{\left[l_1^2 + l_2^2\right]^{1/2}} & \dfrac{-l_2}{\left[l_1^2 + l_2^2\right]^{1/2}} & 1 & 0 & 0 \\ 0 & \dfrac{-1}{\left[l_3^2 - l_2^2\right]^{1/2}} & \dfrac{l_2/l_3}{\left[l_3^2 - l_2^2\right]^{1/2}} & 1 & 0 \end{bmatrix}_{l^\circ}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ -.4472 & -.8944 & 1 & 0 & 0 \\ 0 & -.1014 & .0909 & 1 & 0 \end{bmatrix}, \quad f = \begin{bmatrix} .0175 \\ .0607 \\ .0131 \end{bmatrix}$$

solve $\begin{bmatrix} -W & A^T \\ A & 0 \end{bmatrix}\begin{bmatrix} V \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$,

$$\begin{bmatrix} V \\ k \end{bmatrix} = \begin{bmatrix} -.0352 \\ -.0164 \\ .0303 \\ .0086 \\ .0088 \\ \text{------} \\ 29.3518 \\ .0786 \\ -.5319 \end{bmatrix}$$

new $l^\circ = l + v$

$\Delta l = v$,

do again

$$V = \begin{bmatrix} -.0347 \\ -.0166 \\ .0303 \\ .0086\ R \\ .0088\ R \end{bmatrix}$$

new $l^\circ = l + V$

$\Delta l = l^\circ_{new} - l^\circ_{prior} = \begin{bmatrix} .00049 \\ -.00019 \\ .00006 \\ .00000 \\ .00000 \end{bmatrix}$ Stop

$\hat{l} = l + v$

$$\hat{l} = \begin{bmatrix} 9.9653 \\ 19.9834 \\ 22.3303 \\ 1.1082\ R \\ 0.4626\ R \end{bmatrix}$$