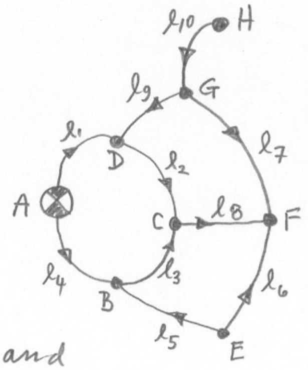
23 Oct 01 CE506 Exam 1 Fall 2001

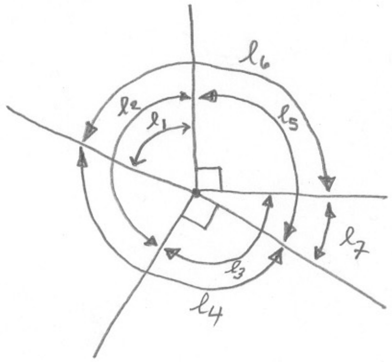
Name

YOU ARE ALLOWED ONE $8\frac{1}{2}'' \times 11''$ PAGE OF NOTES (BOTH SIDES)

1. Find n, n_0, and r in the level network shown in the sketch. Point A has a known, fixed height.



2.



Analyze the angle figure and determine n, n_0, r. Note that two of the angles shown are fixed at 90°. Write condition equations in matrix form as

(1) $v + B\Delta = f$

(2) $Av = f$

3. Given the following equations,

$$Y_1 = X_1 + 3X_3$$
$$Y_2 = 2X_1 - X_2 - 2X_3$$

$$\Sigma_{XX} = \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1X_2} & \sigma_{X_1X_3} \\ \sigma_{X_2X_1} & \sigma_{X_2}^2 & \sigma_{X_2X_3} \\ \sigma_{X_3X_1} & \sigma_{X_3X_2} & \sigma_{X_3}^2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

what is Σ_{YY} ?

what is the correlation coefficient $\gamma_{Y_1Y_2}$?

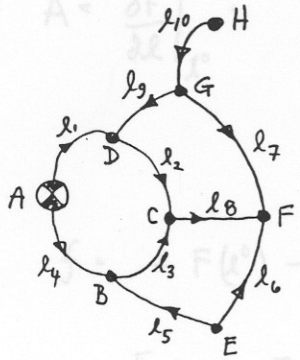
4. Show the linearized form of the following condition equations as $Av = f$ using the given values for the original observations, l, and the refined observations, l°.

$$F(\hat{l}) = \hat{l}_1 \sin \hat{l}_2 - \hat{l}_3 \hat{l}_4^2 = 0$$

$$l = \begin{bmatrix} 100.2 \\ 30.2 \text{ deg.} \\ 0.52 \\ 9.8 \end{bmatrix}, \quad l^\circ = \begin{bmatrix} 100.1 \\ 30.1 \text{ deg.} \\ 0.51 \\ 9.9 \end{bmatrix}$$

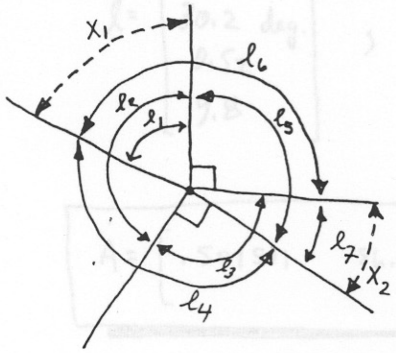
CE 506 Exam I Solution 23-Oct-01

1.



$n = 10$
$n_0 = 7$
$r = 3$

2.



parameters selected x_1, x_2

$n = 7$
$n_0 = 2$
$r = 5$

(1) $Av = f$

1. $\hat{\ell}_6 = \hat{\ell}_1 + 90^\circ$, $\hat{\ell}_1 - \hat{\ell}_6 = -90^\circ$
2. $\hat{\ell}_3 = \hat{\ell}_7 + 90^\circ$, $\hat{\ell}_3 - \hat{\ell}_7 = 90^\circ$
3. $\hat{\ell}_5 - \hat{\ell}_7 = 90^\circ$, $\hat{\ell}_5 - \hat{\ell}_7 = 90^\circ$
4. $\hat{\ell}_2 + \hat{\ell}_5 + 90^\circ = 0$, $\hat{\ell}_2 + \hat{\ell}_5 = -90^\circ$
5. $\hat{\ell}_4 + \hat{\ell}_6 + \hat{\ell}_7 = 360^\circ$, $\hat{\ell}_4 + \hat{\ell}_6 + \hat{\ell}_7 = 360^\circ$

$$\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} -90^\circ \\ 90^\circ \\ 90^\circ \\ -90^\circ \\ 360^\circ \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \\ \ell_5 \\ \ell_6 \\ \ell_7 \end{bmatrix}$$

$$A \cdot v = \underbrace{d - A \cdot \ell}_{f}$$

$\ell_1 + v_1 = x_1$, $v_1 - x_1 = -\ell_1$
$\ell_2 + v_2 = 360^\circ - 180^\circ - x_2$, $v_2 + x_2 = 180^\circ - \ell_2$
$\ell_3 + v_3 = 90^\circ + x_2$, $v_3 - x_2 = 90^\circ - \ell_3$
$\ell_4 + v_4 = 360^\circ - 90^\circ - x_1 - x_2$, $v_4 + x_1 + x_2 = 270^\circ - \ell_4$
$\ell_5 + v_5 = 90^\circ + x_2$, $v_5 - x_2 = 90^\circ - \ell_5$
$\ell_6 + v_6 = 90^\circ + x_1$, $v_6 - x_1 = 90^\circ - \ell_6$
$\ell_7 + v_7 = x_2$, $v_7 - x_2 = -\ell_7$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} + \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 1 & 1 \\ 0 & -1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 180^\circ \\ 90^\circ \\ 270^\circ \\ 90^\circ \\ 90^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \\ \ell_5 \\ \ell_6 \\ \ell_7 \end{bmatrix}$$

$$v + B \cdot \Delta = \underbrace{d - \ell}_{f}$$

3. $$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Y = A \cdot x$$

$$\Sigma_{YY} = A \Sigma_{xx} A^T = \underbrace{\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & -2 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 4 & 6 \\ 5 & -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 21 & -10 \\ -10 & 22 \end{bmatrix}$$

$$\Sigma_{YY} = \begin{bmatrix} 21 & -10 \\ -10 & 22 \end{bmatrix}$$

$$\rho_{Y_1Y_2} = \frac{\sigma_{Y_1Y_2}}{\sigma_{Y_1}\sigma_{Y_2}} = \frac{-10}{(4.582)(4.690)} = -.465$$

4. $$A = \left.\frac{\partial F}{\partial l}\right|_{l^\circ} : \begin{bmatrix} \frac{\partial F}{\partial l_1} & \frac{\partial F}{\partial l_2} & \frac{\partial F}{\partial l_3} & \frac{\partial F}{\partial l_4} \end{bmatrix}$$

$$\boxed{F(\hat{l}) = \hat{l}_1 \sin \hat{l}_2 - \hat{l}_3 \hat{l}_4^2 = 0}$$

$$\left.\begin{bmatrix} \sin l_2 & l_1 \cos l_2 & -l_4^2 & -2 l_3 l_4 \end{bmatrix}\right|_{l^\circ}$$

$$f = -F(l^\circ) - A(l - l^\circ)$$

$$l = \begin{bmatrix} 100.2 \\ 30.2 \text{ deg.} \\ 0.52 \\ 9.8 \end{bmatrix}, \quad l^\circ = \begin{bmatrix} 100.1 \\ 30.1 \\ 0.51 \\ 9.9 \end{bmatrix}, \quad l - l^\circ = \begin{bmatrix} 0.1 \\ 0.1 \text{ deg} \\ 0.01 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.001745 \text{ Rad} \\ 0.01 \\ -0.1 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} .501511 & 86.601657 & -98.01 & -10.098 \end{bmatrix}}$$

$$f = -\left[(100.1)\sin(30.1^\circ) - (0.51)(9.9)^2\right] - \begin{bmatrix} .501511 & 86.601657 & -98.01 & -10.098 \end{bmatrix} \begin{bmatrix} 0.1 \\ .001745 \text{ R} \\ .01 \\ -0.1 \end{bmatrix}$$

$$f = \begin{bmatrix} -0.216125 & - & .230971 \end{bmatrix}$$

$$\boxed{f = -.447096}$$

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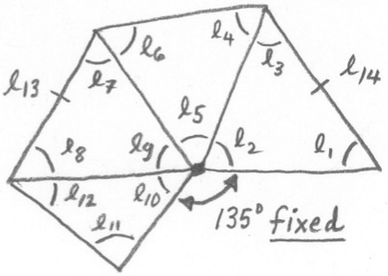
1. The size and shape of the figure are the model. Give n, n_0, and r.

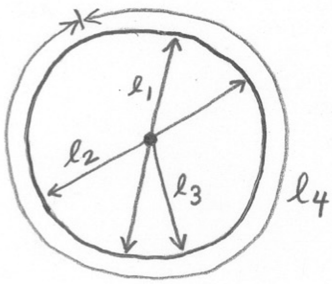


2. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$ relates coordinates (x, y), coordinates (X, Y), and transformation parameters (a, b).

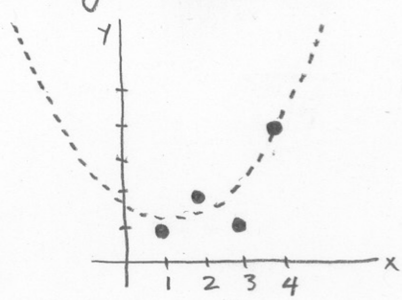
If $(X, Y) = (1, 1)$ are constants, and if $\Sigma_{\binom{a}{b}} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$,

What is $\Sigma_{\binom{x}{y}}$?

3. The size of a circle is the model. Observed are 2 diameters ℓ_1, ℓ_2, a radius, ℓ_3, and a circumference ℓ_4. Write the condition equations for adjustment of observations only, in matrix form, $Av = f$

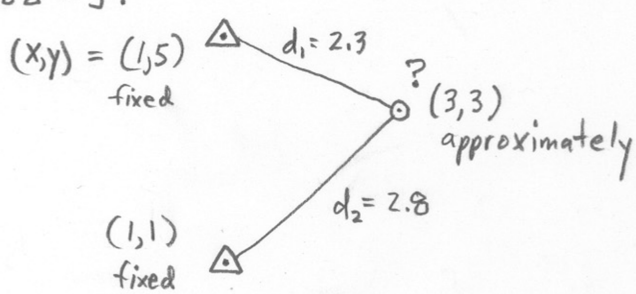


4. You wish to fit a parabola of the form $Y = a_0 + a_1 X + a_2 X^2$ to the following data. X's are constant, y's are observations. Write condition equations for adjustment of indirect observations in matrix form, $V + B\Delta = f$. What is an appropriate weight matrix ?



X	Y	σ_y
1	1	3
2	2	2
3	1	1
4	4	1

5. Two distance observations are made from two fixed points to an unknown point. Write condition equations to solve for coordinates of the unknown point by least squares, indirect observations, in matrix form, $v + B\Delta = f$.



CE 506 Exam I – Solution

1.

$n = 14$
$n_0 = 8$
$r = 6$

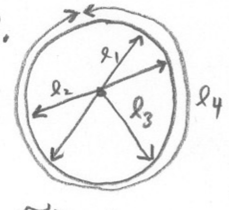
2. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$, must be in form $Y = AX$

constant matrix ↑ random vector with known cov. matrix

$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{matrix} x = a \cdot 1 + b \cdot 1 \\ y = -b \cdot 1 + a \cdot 1 \end{matrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$ That's what we need

$\Sigma_{\binom{x}{y}} = A \Sigma_{\binom{a}{b}} A^T$, $\Sigma_{\binom{x}{y}} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$

3.



$\pi D = C$

$n = 4$
$n_0 = 1$
$r = 3$

for observations only, need $c = r = 3$ condition equations

$\hat{l}_4 = \pi \hat{l}_1$	$\pi \hat{l}_1 - \hat{l}_4 = 0$	$\pi(l_1 + v_1) - l_4 - v_4 = 0$	$\pi v_1 - v_4 = -\pi l_1 + l_4$
$\hat{l}_1 = \hat{l}_2$	$\hat{l}_1 - \hat{l}_2 = 0$	$l_1 + v_1 - l_2 - v_2 = 0$	$v_1 - v_2 = -l_1 + l_2$
$\hat{l}_3 = \frac{1}{2}\hat{l}_1$	$\frac{1}{2}\hat{l}_1 - \hat{l}_3 = 0$	$\frac{1}{2}(l_1 + v_1) - l_3 - v_3 = 0$	$0.5 v_1 - v_3 = -.5 l_1 + l_3$

$$\underset{A}{\begin{bmatrix} \pi & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0.5 & 0 & -1 & 0 \end{bmatrix}} \underset{v}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}} = \underbrace{- \begin{bmatrix} \pi & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0.5 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix}}_{f}$$

4. $y = a_0 + a_1 x + a_2 x^2$

$y + v_y - a_0 - a_1 x - a_2 x^2 = \phi$

$\boxed{v_y - a_0 - a_1 x - a_2 x^2 = -y}$

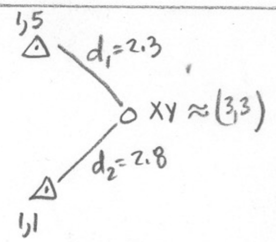
$n = 4$
$n_0 = 3$
$r = 1$

$v_1 - a_0 - a_1(1) - a_2(1^2) = -1$
$v_2 - a_0 - a_1(2) - a_2(2^2) = -2$
$v_3 - a_0 - a_1(3) - a_2(3^2) = -1$
$v_4 - a_0 - a_1(4) - a_2(4^2) = -4$

$$\underset{v}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}} + \underset{B}{\begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -4 \\ -1 & -3 & -9 \\ -1 & -4 & -16 \end{bmatrix}} \underset{\Delta}{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}} = \underset{f}{\begin{bmatrix} -1 \\ -2 \\ -1 \\ -4 \end{bmatrix}}$$

$$W = \begin{bmatrix} 1/3^2 & & & \\ & 1/2^2 & & \\ & & 1/1^2 & \\ & & & 1/1^2 \end{bmatrix} = \begin{bmatrix} 1/9 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.



$n = 2$
$n_0 = 2$
$r = 0$

indirect observations
$u = n_0 = 2$, params: X,Y
$c = n = 2$ condition equations

⇒ residuals will be zero!

$d_1 = \left[(x-1)^2 + (y-5)^2\right]^{1/2}$

$d_2 = \left[(x-1)^2 + (y-1)^2\right]^{1/2}$

$F_1 = d_1 - \left[(x-1)^2 + (y-5)^2\right]^{1/2} = 0$

$F_2 = d_2 - \left[(x-1)^2 + (y-1)^2\right]^{1/2} = 0$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \partial F_1/\partial x & \partial F_1/\partial y \\ \partial F_2/\partial x & \partial F_2/\partial y \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} -F_1^0 \\ -F_2^0 \end{bmatrix}$$

evaluated at $\underline{\underline{x^0, y^0 = 3,3}}$

$\frac{\partial F_1}{\partial x} = -\frac{1}{2}[\;]^{-1/2} \cdot 2(x-1) = -0.707$, $\frac{\partial F_1}{\partial y} = -\frac{1}{2}[\;]^{-1/2} \cdot 2(y-5) = +0.707$,

$\frac{\partial F_2}{\partial x} = -\frac{1}{2}[\;]^{-1/2} \cdot 2(x-1) = -0.707$, $\frac{\partial F_2}{\partial y} = -\frac{1}{2}[\;]^{-1/2} \cdot 2(y-1) = -0.707$

$F_1^0 = -0.52$
$F_2^0 = -0.03$

$$\underset{v}{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}} + \underset{B}{\begin{bmatrix} -0.707 & 0.707 \\ -0.707 & -0.707 \end{bmatrix}} \underset{\Delta}{\begin{bmatrix} \delta x \\ \delta y \end{bmatrix}} = \underset{f}{\begin{bmatrix} 0.52 \\ 0.03 \end{bmatrix}}$$