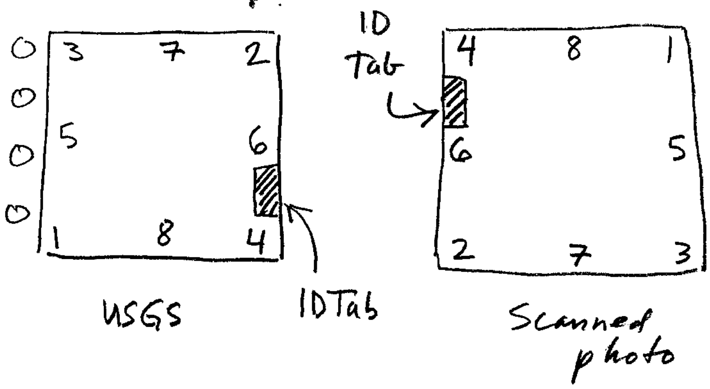
HW2 Solution

Fiducial Correspondence:



So, measurement in upper right corresponds to Fid. number 1, lower left is Fid. number 2, etc.

1. −105.994 −105.992
2. 106.006 106.002
⋮ ⋮

$$x = a_0 + a_1 X + a_2 Y$$
$$y = b_0 + b_1 X + b_2 Y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & X & Y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & X & Y \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{pmatrix} -1 & -X & -Y & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -X & -Y \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} \text{ or } \begin{pmatrix} -\text{line} \\ -\text{sample} \end{pmatrix}$$

results of LS

$a_0 = 4820.6$

$a_1 = -.10306$

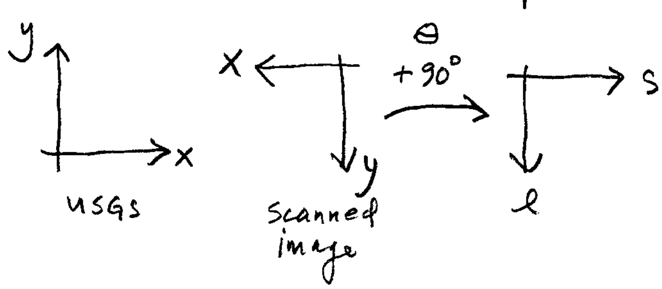
$a_2 = 41.662$

$b_0 = 4821.5$

$b_1 = -41.659$

$b_2 = .10299$

residuals all < 1 pixel



Scanned pixel size: $41.659 \times \text{mm} = \text{pixel}$

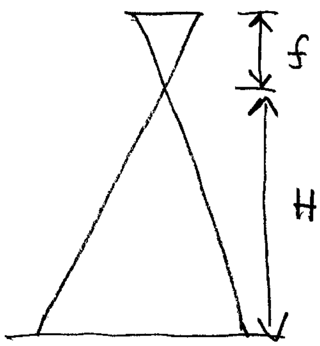
$$\frac{\text{mm}}{\text{pixel}} = \frac{1}{41.659} = .024 \ \text{mm/pixel}$$

This photo was downsampled by 2x

⇒ original scanned pixel was .012 mm (or 12 μm) fountain : image distance 176.1 mm, ground distance 441.1 m } photo scale is

$$\frac{.1761}{441.1} = .000399229 = \frac{1}{2505}$$

looks like target photo scale was $1/2500$



$$\frac{f}{H} = \text{scale} = \frac{1}{2500}$$

$f = 153.517$ (from report)

$$H = f/\text{scale} = .153517\,m \times 2500 = \underline{384\,m}$$

above ground level

local terrain ~ 200 m

⇒ flying height above sea level ≈ 584 m

relief displacement

$$h = \frac{dr}{r} \cdot H$$

$$h_{CE} = \frac{4.6982}{119.05} \times 383.8 = 15.1\,m$$

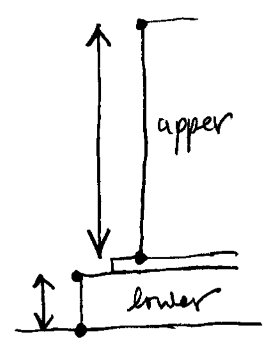
$$h_{CHEM} = \frac{4.0292}{107.87} \times 383.8 = 14.3\,m$$

} base of roof

$$h_{MATH}\ (\text{upper}) = \frac{5.3089}{63.056} \times 383.8 = 32.3\,m$$

$$h_{MATH}\ (\text{lower}) = \frac{0.730}{69.604} \times 383.8 = 4.0\,m$$

} h_{MATH} (total) $= 36.3\,m$



$$h_{TOWER}\ (\text{base of roof}) = \frac{10.32}{89.886} \times 383.8 = 44.1\,m$$

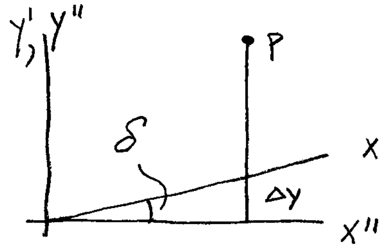
$$h_{TOWER}\ (\text{peak of roof}) = \frac{11.844}{92.78} \times 383.8 = 49.0\,m \quad (= 160.7\ ft.)$$

Wikipedia says Purdue Bell Tower is 160 ft. !

6 parameter transformation

first apply scales $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

next, non-orthogonality



δ = small, $\frac{x''}{x'} = \cos\delta \approx 1$, $x'' = x'$

$\frac{\Delta y}{x'} = \sin\delta \approx \delta$, $\Delta y = \delta x'$

$y'' = y' + \Delta y = y' + \delta x'$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x''' \\ y''' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

change to x, y & XY, multiply out

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} s_x(\cos\theta + \delta\sin\theta) & s_y \sin\theta \\ s_x(-\sin\theta + \delta\cos\theta) & s_y\cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

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given $a_0, a_1, a_2, b_0, b_1, b_2$ solve for S_x, S_y, θ, δ :

- ▶ $S_y = \sqrt{a_2^2 + b_2^2}$
- ▶ $\theta = \tan^{-1}(a_2/b_2)$

  use atan2 (a_2, b_2) for correct quadrant !

  $a_1 b_2 - b_1 a_2 = S_x S_y$
- ▶ $S_x = (a_1 b_2 - b_1 a_2) / S_y$

  $a_1 - S_x\cos\theta = S_x\,\delta\sin\theta$
- ▶ $\delta = \frac{a_1 - S_x\cos\theta}{S_x\sin\theta}$

$S_y = 41.662$

$S_x = 41.659$

$\theta = 1.5733\ R,\ = 90.142°$

$\delta = -1.6939 \times 10^{-6}\ R$

$= -9.705 \times 10^{-5}$ deg

```
innor5
6-parameter results
p =
       4820.6
     -0.10306
       41.662
       4821.5
      -41.659
     -0.10299
resid =
      0.27829
     0.013126
       0.1176
     -0.48224
       0.1867
     -0.11145
      0.12587
     -0.35746
     -0.55274
     -0.10585
      -0.3431
       0.5958
      0.14096
      0.34203
     0.046417
      0.10605
scale_y =
       41.662
theta =
       1.5733
theta_d =
       90.142
scale_x =
       41.659
alpha =
 -1.6939e-006
alpha_d =
 -9.7052e-005
transformed coordinates of data points
displ =
          101       100.34      -54.853
          102       104.43      -57.171
          103      -36.739      -97.269
          104      -37.137      -101.28
          105      -54.068      -20.286
          106      -58.984      -22.291
          107      -67.528      -13.551
          108      -68.247      -13.673
          109      -12.324      -78.606
          110      -13.859      -88.811
          111      -12.441      -79.975
          112      -14.164      -91.692
          113       48.264       -88.85
          114      -114.46       -21.54
ce bldg
dr =
       4.6982
r =
       119.05
chem bldg
dr =
       4.0292
r =
```

```
        107.87
ms1 bldg
dr =
        5.3089
r =
        63.056
ms2 bldg
dr =
       0.73007
r =
        69.604
bt1 bldg
dr =
         10.32
r =
        89.886
bt2 bldg
dr =
        11.844
r =
         92.78
fount bldg
dr =
         176.1
r =
        116.47
diary off
```

```
% innor5.m 15-nov-08
% surdex camera nga project over campus
% rc30 lens id tab upper left in photo, so photo rotated
% 180 from usgs sketch
% compute 6-parameter transformation for inner orientation
% and then transform measured data points
% write results to innor.out
% x = a0 + a1*X + a2*Y
% y = b0 + b1*X + b2*Y
% x = [ 1  X  Y  0  0  0 ] [ a0 ]
% y = [ 0  0  0  1  X  Y ] [ a1 ]
%                          [ a2 ]
%                          [ b0 ]
%                          [ b1 ]
%                          [ b2 ]

nfid=8;
calx=[-105.994;106.006;-105.993;106.005;-111.996;112.007;0.007;0.003];
caly=[-105.992;106.002;106.004;-105.992;0.010;0.007;112.004;-111.989];

% observed photoshop x,y
obsx=[9248;394;9226;416;9487;156;4810;4833];
obsy=[416;9226;9248;394;4832;4809;9487;155];
obsl=obsy;
obss=obsx;

% data points
% ce base, ce top
% chem base, chem top
% (1) m/s base, m/s top
% (2) m/s base, m/s top
% (1) b/t base. b/t top
% (2) b/t base, b/t top
% engr fountain
% beering fountain
% m/s = math science,  b/t = bell tower

ndat=14;
datx=[647;477;6362;6379;7076;7281;7636;7666;5343;5408;5348;5421;2820;9592];
daty=[2525;2428;772;605;3981;3898;4263;4258;1547;1122;1490;1002;1114;3935];
datl=daty;
dats=datx;

f=zeros(2*nfid,1);
B=zeros(2*nfid,6);
for i=1:nfid
  ii=i*2-1;
  ii1=i*2;
  f(ii)=obsl(i);
  f(ii1)=obss(i);
  B(ii,:)=[1 calx(i) caly(i) 0 0 0];
  B(ii1,:)=[0 0 0 1 calx(i) caly(i)];
end

disp('6-parameter results');
p=inv(B'*B)*B'*f
resid=f - B*p

% compute the nonlinear parameters
% scale-x, scale-y, genral rotation (theta), non-orthogonality (alpha)
a0=p(1);
a1=p(2);
a2=p(3);
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```
b0=p(4);
b1=p(5);
b2=p(6);

% scale_x=sqrt(a1^2 + b1^2)
% scale_y=sqrt(a2^2 + b2^2)
% theta=atan2(-b1,a1)
% thetad=theta*(180/pi)
% alpha=atan((a1*a2+b1*b2)/(a1*b2-a2*b1))
% alphad=alpha*(180/pi)
% theta seems wrong above, below ok

% use solutions from 2007 notes-7 7-7 through 7-10]
% note the atan2 is important
scale_y=sqrt(a2^2 + b2^2)
theta=atan2(a2,b2)
theta_d=theta*(180/pi)
scale_x=(a1*b2-b1*a2)/scale_y
alpha=(a1-scale_x*cos(theta)) / (scale_x*sin(theta))
alpha_d=alpha*(180/pi)

% invert and apply to measurements (l,s) -> (x,y)
% [l;s]=[a1 a2; b1 b2]*[calx; caly] + [a0; b0];
% [calx; caly]= inv( [a1 a2; b1 b2] ) * ( [l;s] - [a0;b0] );

caldat=zeros(ndat,2);
M=inv([a1 a2;b1 b2]);
for i=1:ndat
  vec=[datl(i)-a0; dats(i)-b0];
  cal=M*vec;
  caldat(i,:)=cal';
  end

% label the data files
lab=[101;102;103;104;105;106;107;108;109;110;111;112;113;114];
displ=[lab caldat];
disp('transformed coordinates of data points');
displ
disp('ce bldg');
i=1;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('chem bldg');
i=3;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('ms1 bldg');
i=5;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('ms2 bldg');
i=7;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('bt1 bldg');
i=9;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('bt2 bldg');
i=11;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
disp('fount bldg');
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```
i=13;
dr=sqrt((caldat(i,1)-caldat(i+1,1))^2 + (caldat(i,2)-caldat(i+1,2))^2)
r=sqrt(caldat(i+1,1)^2 + caldat(i+1,2)^2)
```