28-1

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\,\delta(n-k)$$

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\,h(n-k)$$

for $k = n-2, n-1, n$

$$y(n) = x(n-2)\,h(2) + x(n-1)\,h(1) + x(n)\,h(0)$$

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28-2

$\longrightarrow$ time

$x(n-2)\ h(0)$ $x(n-2)\ h(1)$

$x(n-1)\ h(0)$

$x(n-2)\ h(2)$

$x(n-1)\ h(1)$ ..

$x(n)\ h(0)$ ...

$y(n)$

"time reversal"

two ways of deriving the convolution sum

1D imagery: 2D

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euler's equation

28-3

$e^{i\theta} = \cos\theta + i\sin\theta \quad i = \sqrt{-1}$

$e^{i\omega t} = \cos\omega t + i\sin\omega t$

ω: rad/sec

$e^{i2\pi ft} = \cos 2\pi ft + i\sin 2\pi ft$

f: cycles/sec

ωt: kx, $2\pi fx$

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Fourier Transform (Discrete) 1D

28-4

$$\text{DFT}: F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)\, e^{-i2\pi xu/N}$$

$$\text{IDFT}: f(x) = \sum_{u=0}^{N-1} F(u)\, e^{+i2\pi xu/N}$$

2D

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)\, e^{-i2\pi(ux/N + vy/M)}$$

$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{M-1} F(u,v)\, e^{+i2\pi(ux/N + vy/M)}$$

DFT

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28-5

If you can arrange for # elements to be power of 2, then you can evaluate by FFT.

DFT : possible to state as matrix vector product
clever factorization/decomposition of matrix
product done very quickly : FFT

famous property of F.T.
convolution in space/time domain $\Leftrightarrow$
multiplication in freq domain.

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28-6

convolution $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$

correlation $\left.\begin{array}{l} f(t) * g(-t) \\ f(t) \star g(t) \end{array}\right\} = \int_{-\infty}^{\infty} f(\tau)\, g(t+\tau)\, d\tau$

conv: $f(t) * g(t) \Leftrightarrow F(\omega) \cdot G(\omega)$ (mult)
(element by element mult)

corr: $f(t) * g(-t)$
$f(t) \star g(t)$ $\Leftrightarrow F(\omega) \cdot G^*(\omega)$
$*$ is complex conjugate

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conv: $f(t) * g(t) = \mathcal{F}^{-1}(F(\omega) \cdot G(\omega))$ 28-7

Corr $f(t) \star g(t) = \mathcal{F}^{-1}(F(\omega) \cdot G^*(\omega))$

is data has length 2^n use FFT

Steps: 1. F.T. signal/image to F.D. (include conjugate)

2. element by element mult

3. inverse F.T. back in space/time domain

4. have CC response map

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Matlab has 28-8

DFT, IDFT (1D)

FFT, IFFT

FFT2, IFFT2 (2D)

.* ⇒ element wise multiplication

Least Squares Matching

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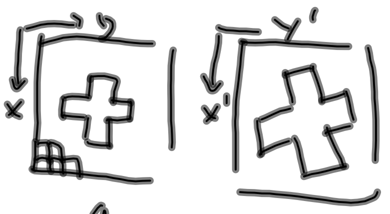
28-9



g h

unknowns

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \\ k_1 \\ k_2 \end{bmatrix}$$

condition equation: $g(x,y) = h(x',y')$

$$\left.\begin{aligned} x' &= a_1 x + a_2 y + a_3 \\ y' &= b_1 x + b_2 y + b_3 \end{aligned}\right\} \text{6 geometric parameters}$$

revised cond. eqn: $g(x,y) = \underbrace{k_1}_{\text{gain}} h(x',y') + \underbrace{k_2}_{\text{offset}}$

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28-10

$$\boxed{F = g(x,y) - k_1 h(x',y') - k_2 = 0}$$

$x' = a_1 x + \cdots$

$y' = b_1 x + \cdots$

based on our assumption that 2 images are approximately aligned: init param. approx:

$a_1 \approx 1,\ a_2 \approx 0,\ a_3 \approx 0$

$b_1 \approx 0,\ b_2 \approx 1,\ b_3 \approx 0$

$$\frac{\partial F}{\partial a_1} = -k_1 \underbrace{\frac{\partial h}{\partial a_1}}$$

$$= -k_1 \underbrace{\frac{\partial h}{\partial x'}}_{\substack{\text{image}\\ \text{gradient}\\ h_x}} \cdot \underbrace{\frac{\partial x'}{\partial a_1}}_{x}$$

$B = \left[\frac{\partial F}{\partial a_1} \quad \frac{\partial F}{\partial a_2} \quad \frac{\partial F}{\partial a_3} \cdots \right]$

assumption $k_1 = 1,\ k_2 = 0$

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28-11

$$\frac{\partial F}{\partial a_1} = -\underline{\underline{h_x}}\, x$$

$$h_x = \frac{\partial h}{\partial x} \approx \frac{\Delta h}{\Delta x}$$

$$\left.\begin{aligned} h_x &= \frac{h(x'+1, y') - h(x'-1, y')}{2} \\ h_y &= \frac{h(x', y'+1) - h(x', y'-1)}{2} \end{aligned}\right\} \text{computing gradient}$$

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28-12

$$\frac{\partial F}{\partial a_1} = -h_x \cdot x \qquad \frac{\partial F}{b_1} = -h_y x$$

$$\frac{\partial F}{\partial a_2} = -h_x \cdot y \qquad \frac{\partial F}{\partial b_2} - h_y \cdot y$$

$$\frac{\partial F}{\partial a_3} = -h_x \qquad \frac{\partial F}{\partial b_3} = -h_y$$

$$\frac{\partial F}{\partial k_1} = -h(x', y'), \quad \frac{\partial F}{\partial k_2} = -1$$

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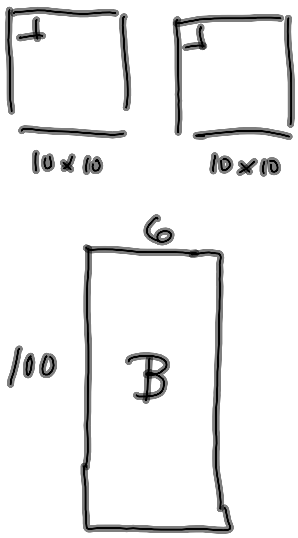

Solve for 6 params,
resample one image
repeat until
converges

when converged then
images are aligned

28-13

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