due Wed. 5 March

Photo 1 – Homework 4 (Lens Distortion & Rectification)

1. make a "measured to refined" function :

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = fm2r(x, y, p)$$

(x, y) : measured , (x'', y'') : refined

$$p = [x_0 \ y_0 \ f \ k_1 \ k_2 \ k_3 \ P_1 \ P_2 \ c_1 \ c_2 \ c_3]^T$$

$$x' = x - x_0$$
$$y' = y - y_0$$
$$r = \sqrt{(x')^2 + (y')^2}$$
$$dr = c_1 k_1 r^3 + c_2 k_2 r^5 + c_3 k_3 r^7$$
$$dx_r = (dr/r) x'$$
$$dy_r = (dr/r) y'$$
$$dx_d = c_1 P_1 (r^2 + 2(x')^2) + 2 c_1 P_2 x' y'$$
$$dy_d = 2 c_1 P_1 x' y' + c_1 P_2 (r^2 + 2(y')^2)$$

$$x'' = x' + dx_r + dx_d$$
$$y'' = y' + dy_r + dy_d$$

2. in order to invert the function in #1, we need a version which is nominally zero :

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = fm2r\_0(x'', y'', x, y, p)$$

which is just,

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} x'' \\ y'' \end{bmatrix} - fm2r(x, y, p)$$

3. Now we can construct the inverse function "refined to measured":

$$\begin{bmatrix} x \\ y \end{bmatrix} = fr2m(x'', y'', p)$$

initial approximations $x^\circ = x''$, $y^\circ = y''$

$$\begin{bmatrix} \frac{\partial F_x}{\partial x} \\ \frac{\partial F_y}{\partial x} \end{bmatrix} = \left[fm2r\_0(x'', y'', x^\circ + dx, y^\circ, p) - \overset{*}{fm2r\_0(x'', y'', x^\circ, y^\circ, p)} \right] / dx$$

$$\begin{bmatrix} \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial y} \end{bmatrix} = \left[fm2r\_0(x'', y'', x^\circ, y^\circ + dy, p) - \overset{*}{fm2r\_0(x'', y'', x^\circ, y^\circ, p)} \right] / dy$$

$$\begin{bmatrix} F_x^\circ \\ F_y^\circ \end{bmatrix} = \overset{*}{fm2r\_0(x'', y'', x^\circ, y^\circ, p)}$$

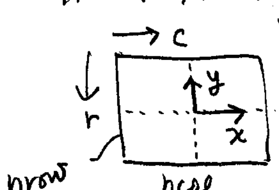
(* all are the same)

solve for $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$ using:

$$\begin{bmatrix} \partial F_x/\partial x & \partial F_x/\partial y \\ \partial F_y/\partial x & \partial F_y/\partial y \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -F_x^\circ \\ -F_y^\circ \end{bmatrix}$$

$$\begin{bmatrix} x^\circ \\ y^\circ \end{bmatrix} = \begin{bmatrix} x^\circ \\ y^\circ \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$, iterate until $|\Delta x| < .001$, $|\Delta y| < .001$

4. Auxiliary Computations



$x = c - ncol/2$ $\qquad$ $c = x + ncol/2$

$y = -(r - nrow/2)$ $\qquad$ $r = -y + nrow/2$

$$maxr = \sqrt{\left(\frac{nrow}{2}\right)^2 + \left(\frac{ncol}{2}\right)^2}$$

$c_1 = 1/maxr^2$, $\quad c_2 = 1/maxr^4$, $\quad c_3 = 1/maxr^6$

5. Verify inverse function:

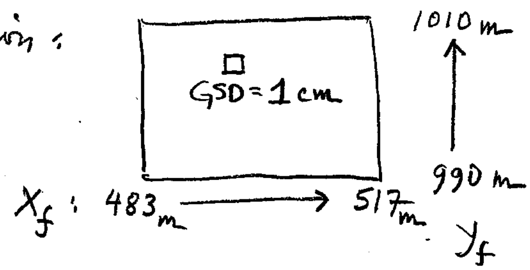
- pick any (r, c) in the image
- convert to x, y
- compute x'', y'' using fm2r
- compute x, y using fr2m
- convert to (r, c)

<u>compare</u>

Produce a Rectified Image of Basketball Court:

1. select any landscape photo, from calibration set

2. limits & resolution:
(in X_f, Y_f system)



3. relation to BBA reference system

$$X = -X_f + 1000\,\text{m}$$
$$Y = -Y_f + 2000\,\text{m}$$
$$Z = Z_f$$

BBA system   Rectification System

4. use constant elevation $Z = Z_f = 10\,\text{m}$

5. use the <u>flow chart</u> given in class Friday, 21 Feb. using bilinear interpolation.