Homework 2 - Photo 2 - Atmospheric Refraction

- See the accompanying sketch for a discrete geometric model of the atmosphere to use for computing refraction from low earth orbit (in 2D).
- Height of the satellite is 500 km, angle alpha is 20 degrees, dH is 0.1 km. Use the accompanying Matlab function "atmdens.m" in order to compute a density (rho in kg/m^3) for the input value for height (in meters!). This model comes from the NASA Glenn earth atmosphere model, as seen in accompanying pages. From density compute the refractive index by $n^2 = 1 + 2C\rho$, where $C = 0.00022667$. The equation is from Simon Newcomb (1906) and the constant is from G. Schut (1969).
- For each refracted ray, intersect with circle defining the next layer boundary. Be sure the surface normal used in Snell's law ($n_1 \sin\theta_1 = n_2 \sin\theta_2$) is normal to the curved surface.
- Use the following relations between bearing and unit vector components:

$$B = \text{bearing of line}$$

$$v_x = \cos(B)$$

$$v_y = \sin(B)$$

$$B = \operatorname{atan2}(v_y, v_x)$$

- Final result should be d-alpha as shown on the sketch.
- Compare this with the "conventional" approach using the formula of Saastamoinen, Eq. 3.149 in the Manual of Photogrammetry, Ed. 5, also shown on the next page.

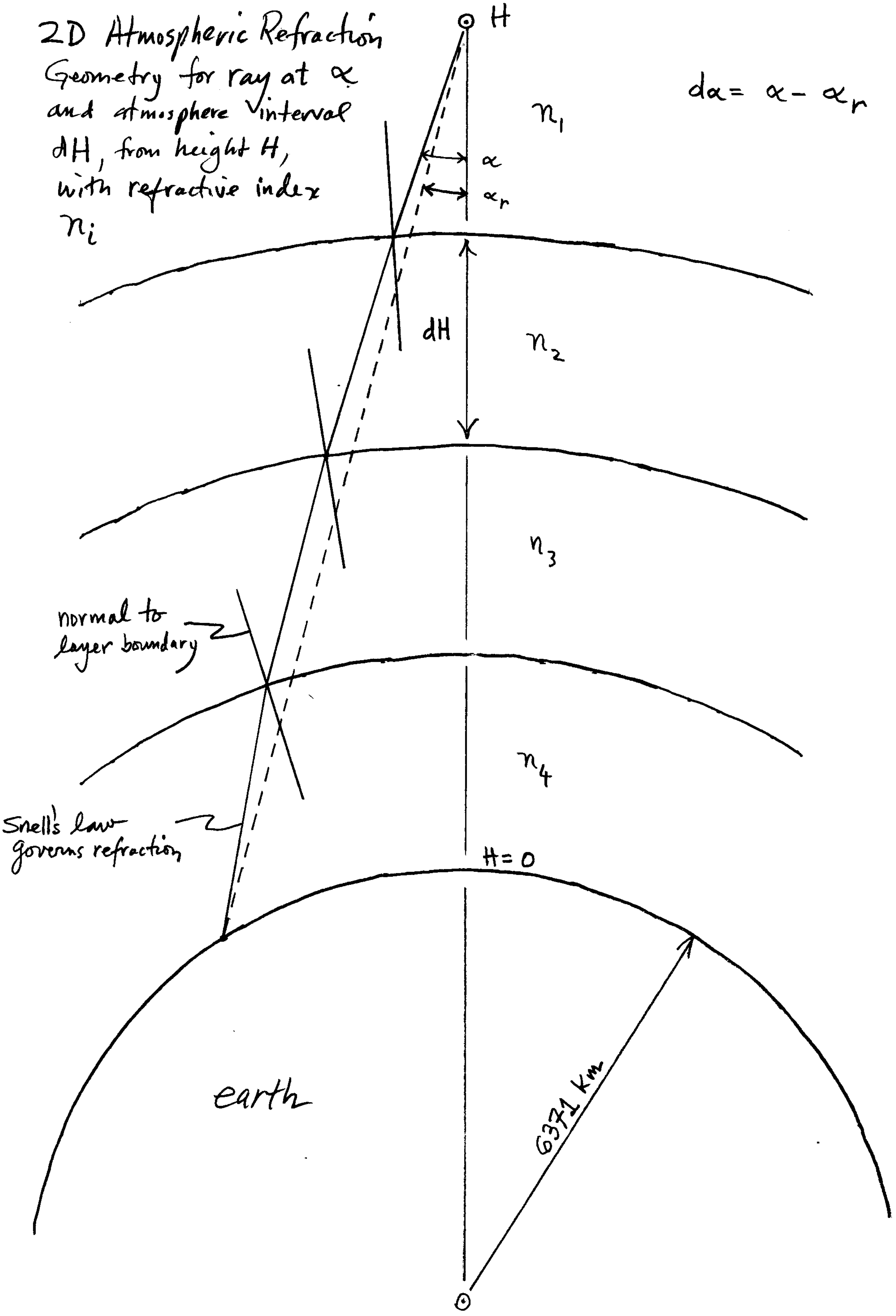
$$K = \left[\frac{2335}{H-h}\left(1-0.02257h\right)^{5.256} - 0.8540^{H-11}\left(82.2-\frac{521}{H-h}\right)\right]\times 10^{-6}$$

$$d\alpha = K \tan \alpha$$

$$d\alpha \text{ in radians}$$

Also recall that the dot product of two unit vectors is the cosine of the angle between them.

What is the equivalent ground displacement of this d-alpha? Is it significant for ground sample distance (GSD) of 0.5 meter?

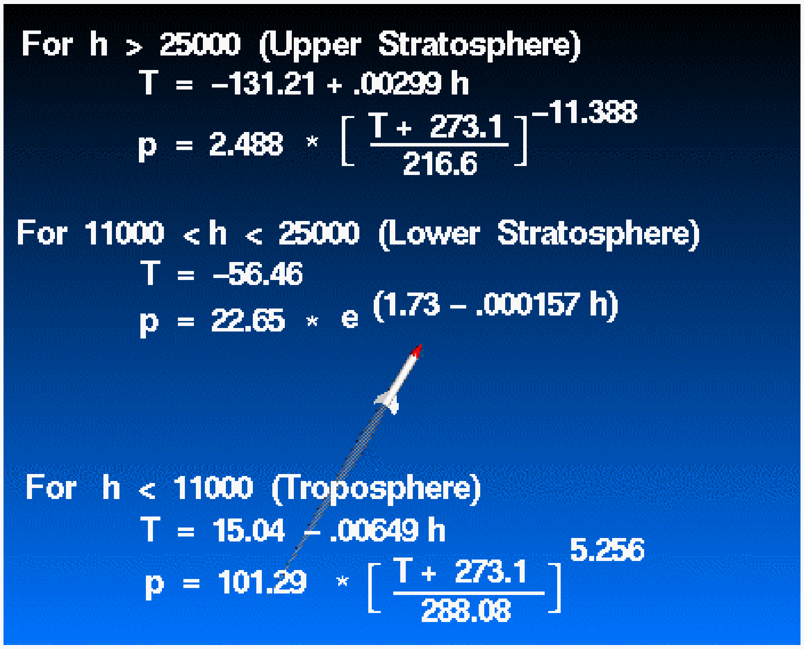


2D Atmospheric Refraction Geometry for ray at α and atmosphere interval dH, from height H, with refractive index n_i

$$d\alpha = \alpha - \alpha_r$$

Earth Atmosphere Model

Metric Units



For h > 25000 (Upper Stratosphere)

$$T = -131.21 + .00299\ h$$

$$p = 2.488 * \left[\frac{T + 273.1}{216.6}\right]^{-11.388}$$

For 11000 < h < 25000 (Lower Stratosphere)

$$T = -56.46$$

$$p = 22.65 * e^{(1.73 - .000157\ h)}$$

For h < 11000 (Troposphere)

$$T = 15.04 - .00649\ h$$

$$p = 101.29 * \left[\frac{T + 273.1}{288.08}\right]^{5.256}$$

ρ = density (kg/cu m)

p = pressure (K-Pa)

$\rho = p \,/\, (.2869 * (T + 273.1))$

T = temperature (^{0}C)

h = altitude (m)

The Earth's atmosphere is an extremely thin sheet of air extending from the surface of the Earth to the edge of space. If the Earth were the size of a basketball, a tightly held pillowcase would represent the thickness of the atmosphere. Gravity holds the atmosphere to the Earth's surface. Within the atmosphere, very complex chemical, thermodynamic, and fluid dynamics effects occur. The atmosphere is not uniform; fluid properties are constantly changing with time and place. We call this change the weather.

Variations in air properties extend upward from the surface of the Earth. The sun heats the surface of the Earth, and some of this heat goes into warming the air near the surface. The heated air is then diffused or convected up through the atmosphere. Thus the air temperature is highest near the surface and decreases as altitude increases. The speed of sound depends on the temperature and also decreases with increasing altitude. The pressure of the air can be related to the weight of the air over a given location. As we increase altitude through the atmosphere, there is some air below us and some air above us. But there is always less air above us than was present at a lower altitude. Therefore, air pressure decreases as we increase altitude. The air density depends on both the temperature and the pressure through the equation of state and also decreases with increasing altitude.

Aerodynamic forces directly depend on the air density. To help rocket designers, it is useful to define a **standard atmosphere** model of the variation of properties through the atmosphere. There are actually several different models available--a standard or average day, a hot day, a cold day, and a tropical day. The models are updated every few years to include the latest atmospheric data. The model was developed from atmospheric measurements that were averaged and curve fit to produce the given equations. The model assumes that the pressure and temperature change only with altitude. The particular model shown here was developed in the early sixties, and the curve fits are given in Metric units. Curve fits are also available in English units.

```
% atmdens.m  25-jan-11
% curve fits from nasa glenn
% H meters, T celsius, P kilopascals, rho kg/m^3

function rho=atmdens(H)

rho=0.0;
if(H < 11000)
  T=15.04 - 0.00649*H;
  P=101.29 * ((T+273.1)/288.08)^5.256;
  rho=P/(0.2869 * (T+273.1));
elseif((H >= 11000) & (H < 25000))
  T=-56.46;
  P=22.65 * exp(1.73 - 0.000157*H);
  rho=P/(0.2869 * (T+273.1));
else
  T=-131.21 + 0.00299*H;
  P=2.488 * ((T+273.1)/216.6)^-11.388;
  rho=P/(0.2869 * (T+273.1));
  end
end
```