

# Terrestrial/Close-range Block – Handheld, Ricoh 35mm, f~50mm



chme01



chme02



chme03



chme04



chme05



chme06

Issues: sufficient CP/PP to connect each photo to block, “gimbal lock”, ray intersection angle

# Terrestrial Block



chme07



chme08



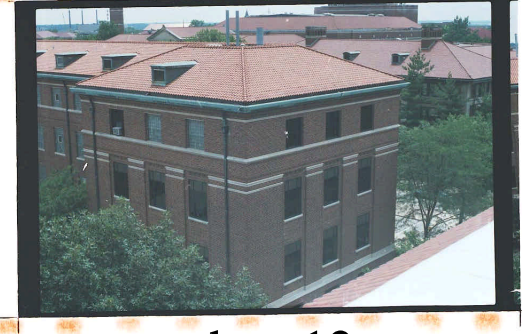
chme09



chme10



chme11



chme12

Applications: architecture, restoration, 3D model building/visualization, ge positioning of points not easily seen on “vertical” imagery

# Application: Restoration of University Hall Tower after Wind Damage in 1999





Surface damage, structure weakened, and the whole tower moved

Use close-range photogrammetry to record “as-built” shape of tower for reconstruction”. Use hydraulic lift for camera access, for 360 deg. coverage

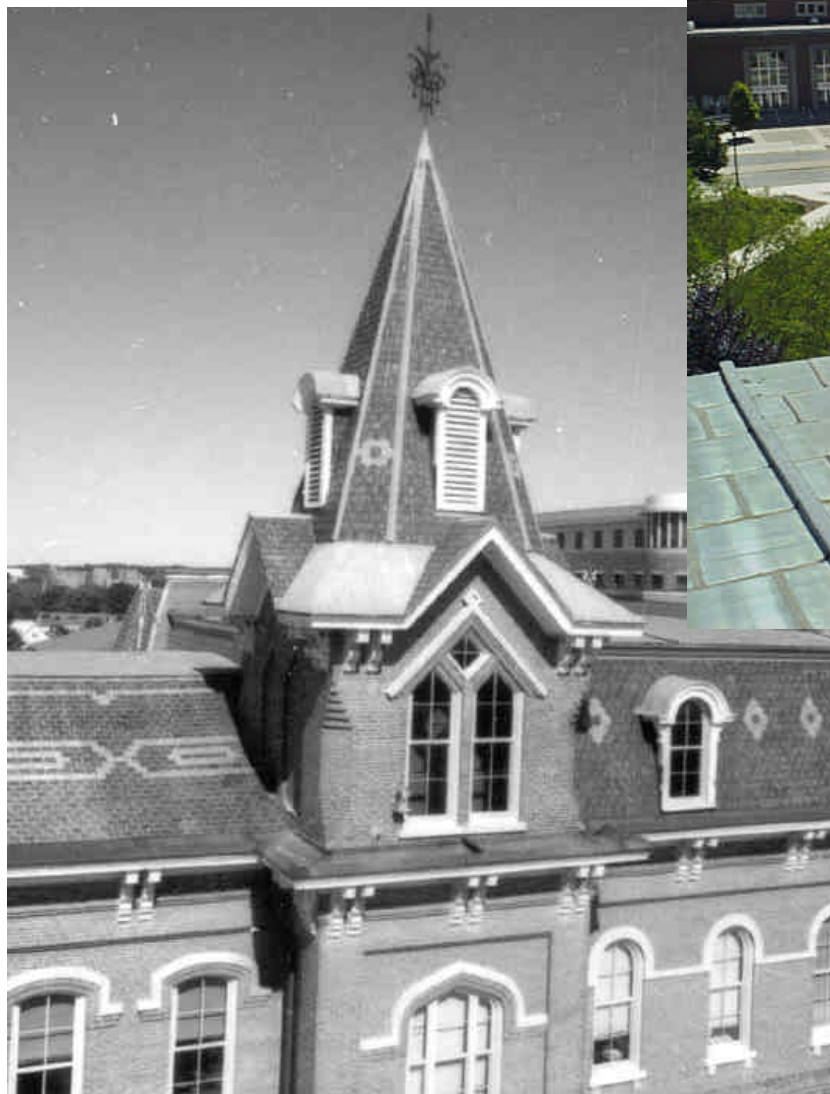




Camera used was non-metric Hasselblad (70mm film) with self-calibration in the bundle adjustment



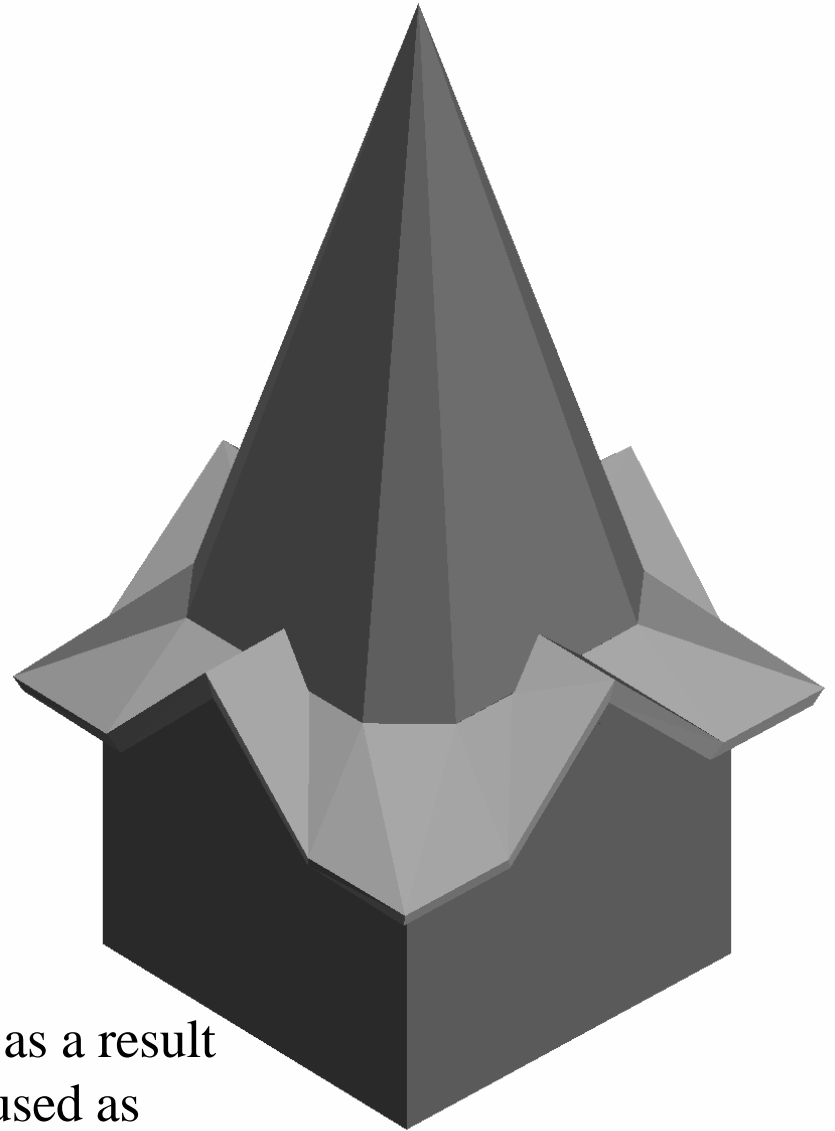
One of the ~15 B&W photos used in the bundle adj.



Tying photogrammetric survey to the reference network. Painted target locations determined by leveling and theodolite triangulation from the ground – also visible in the photogrammetric block.



Taping to a control point



3D CAD model produced as a result of the photogrammetry – used as guide for dimensions of new tower

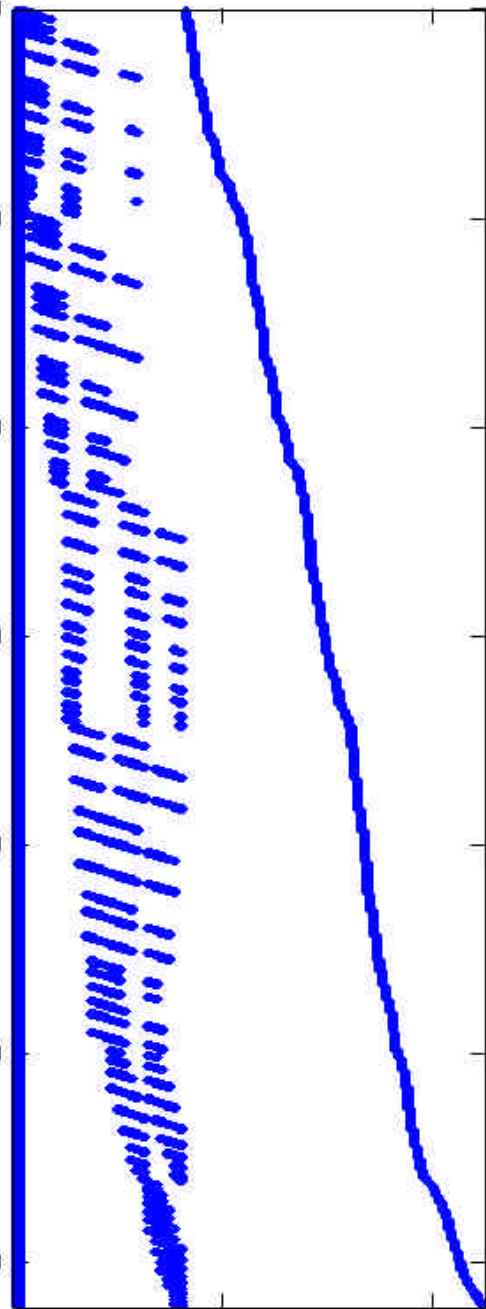
## Simulated Block

26 photos, 93 pass points, 3 control points

1242 equations, 450 unknowns

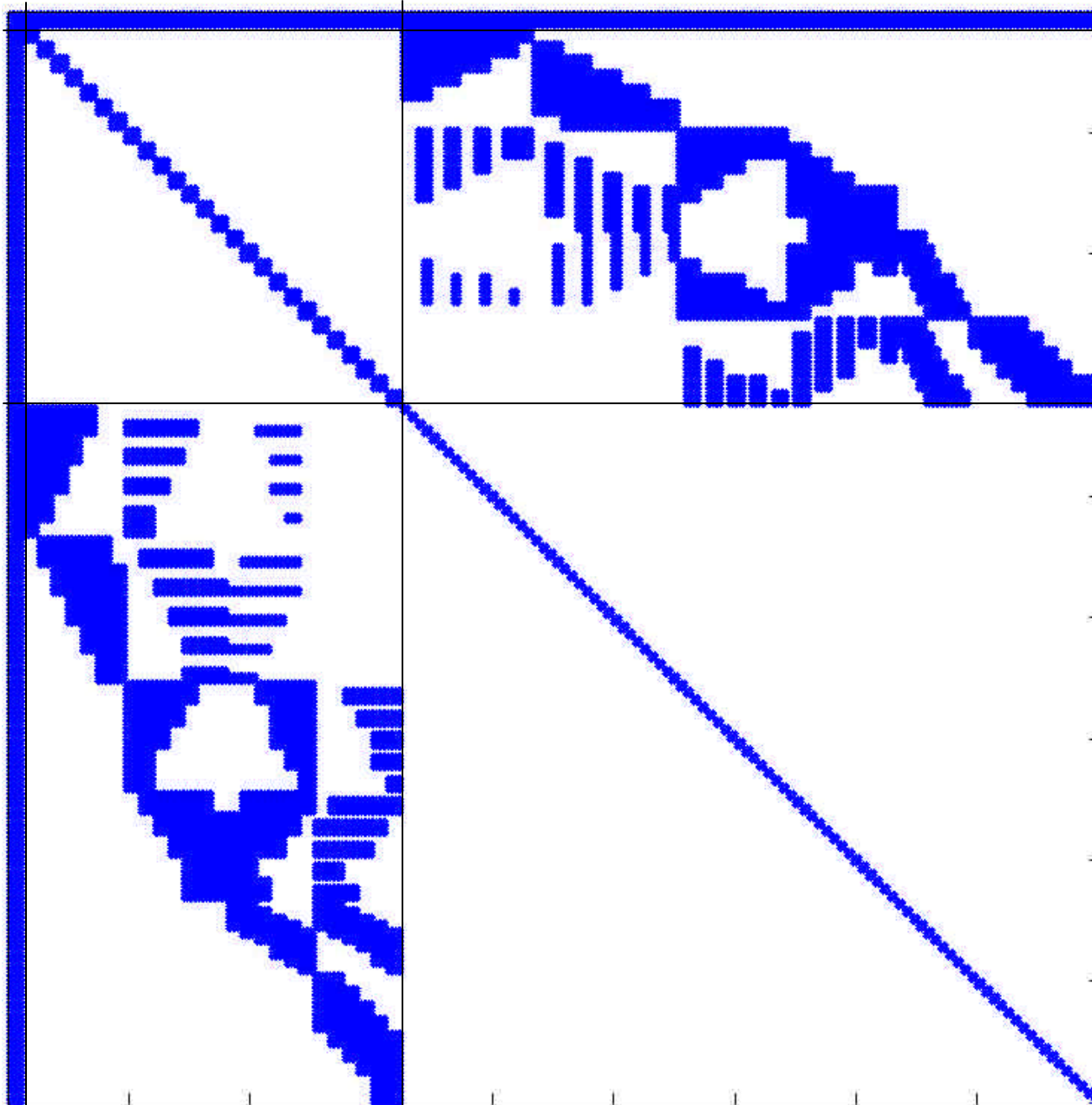
Photo & point layout very similar to the  
Purdue block

Pba.m on sun ultra-10: ~1/2 minute per  
iteration (all numerical partials, and full  
matrix inverse)



← B Matrix, from  
command: spy(B)





Simulated Block –  
normal equations,  
450x450, 3 partitions  
shown: camera internal  
parameters (6), photo  
exterior orientation  
parameters (6 per  
photo), and ground  
points (3 per point)

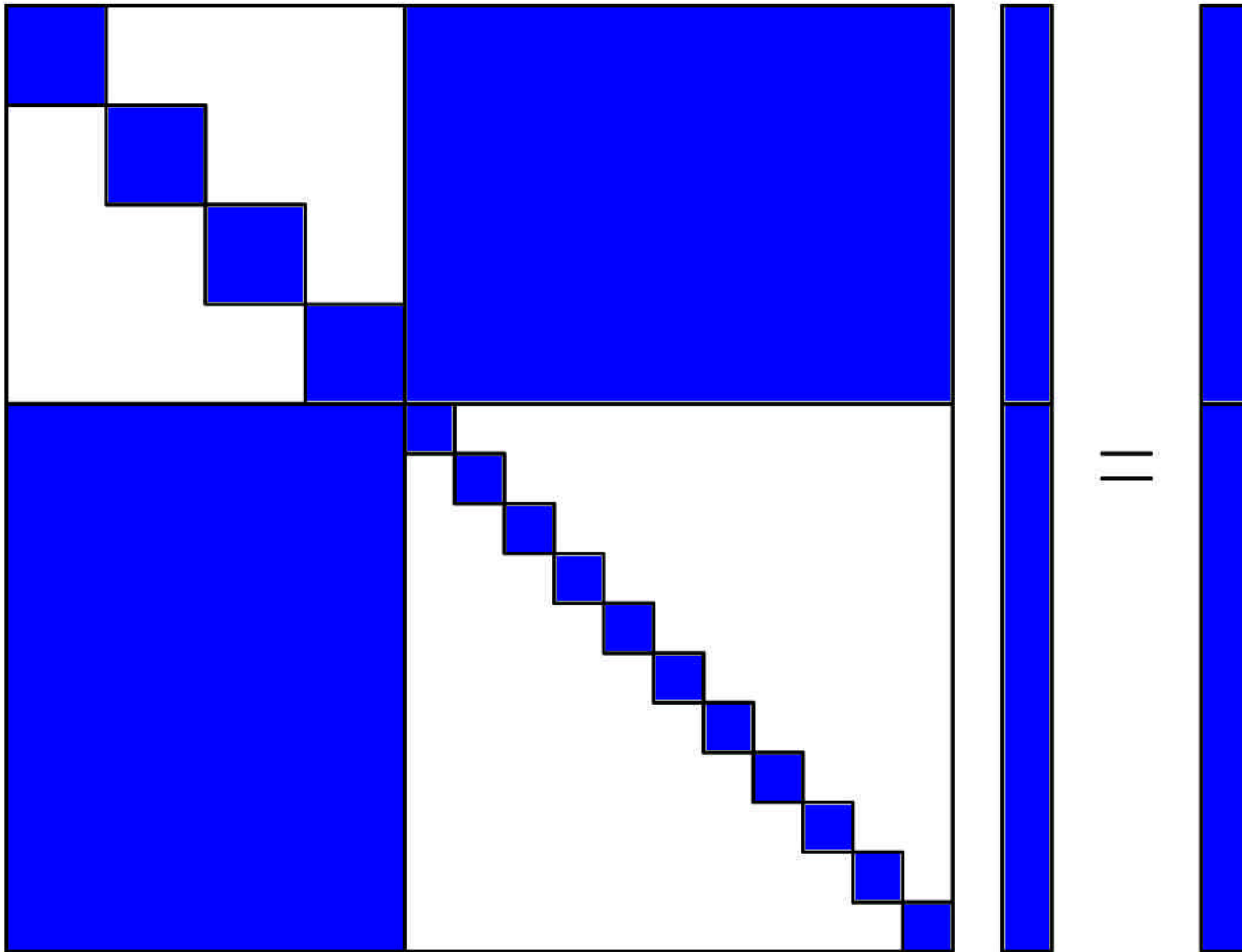
The off diagonal block  
is nonzero if that point  
occurs on that photo

The matrix is about  
85% zeros.

Let's look at an efficient  
block gauss elimination  
method to derive a set  
of *reduced normal  
equations* from the full  
normal equations

Figure from: spy(N)

Technique to move from full normals to reduced normals: use block gauss elimination to eliminate one point at a time

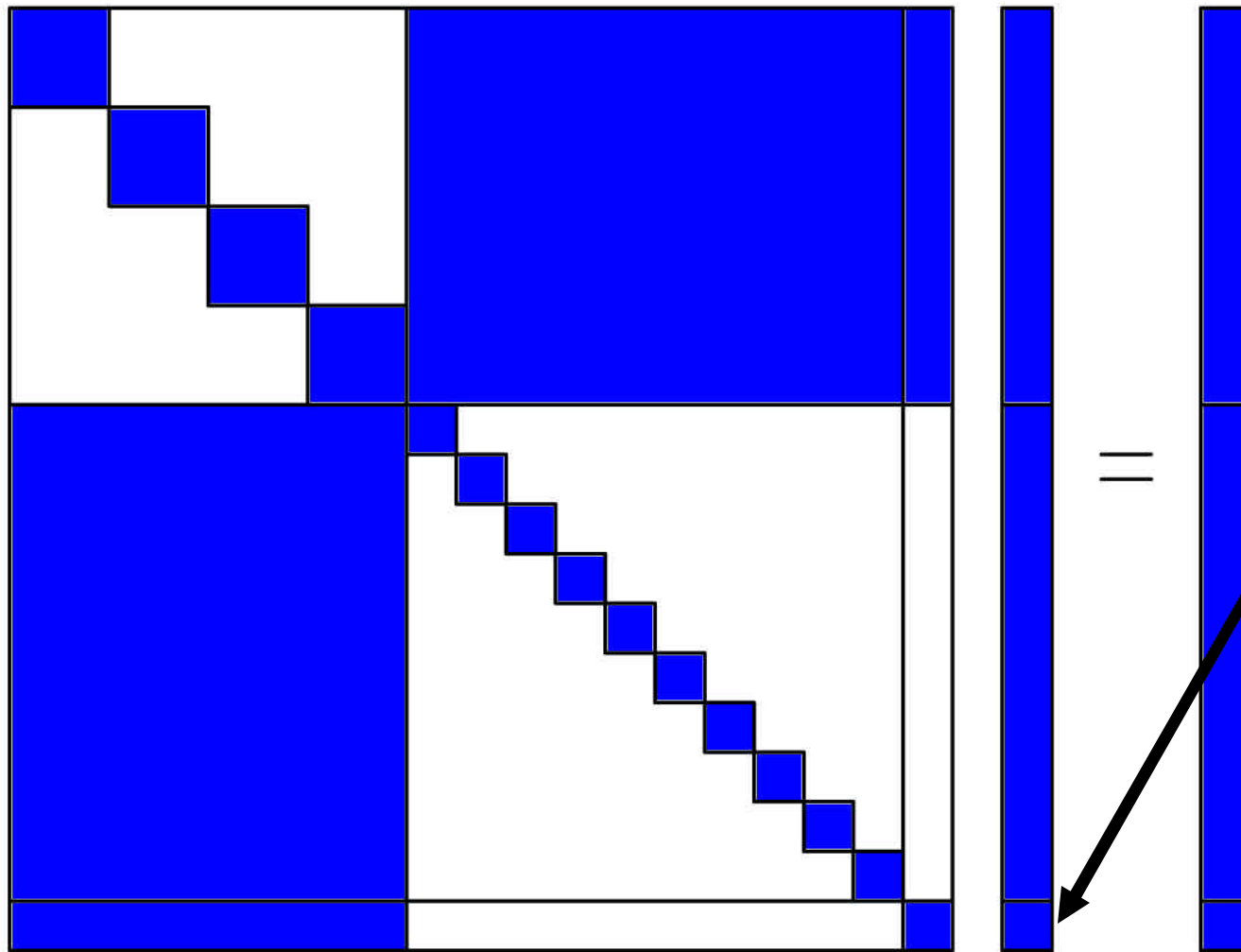


This figure is a schematic representation of

$$\mathbf{N}\Delta = \mathbf{t}$$

The off-diagonal partition is often sparse but we will assume full

# Block Gauss Elimination

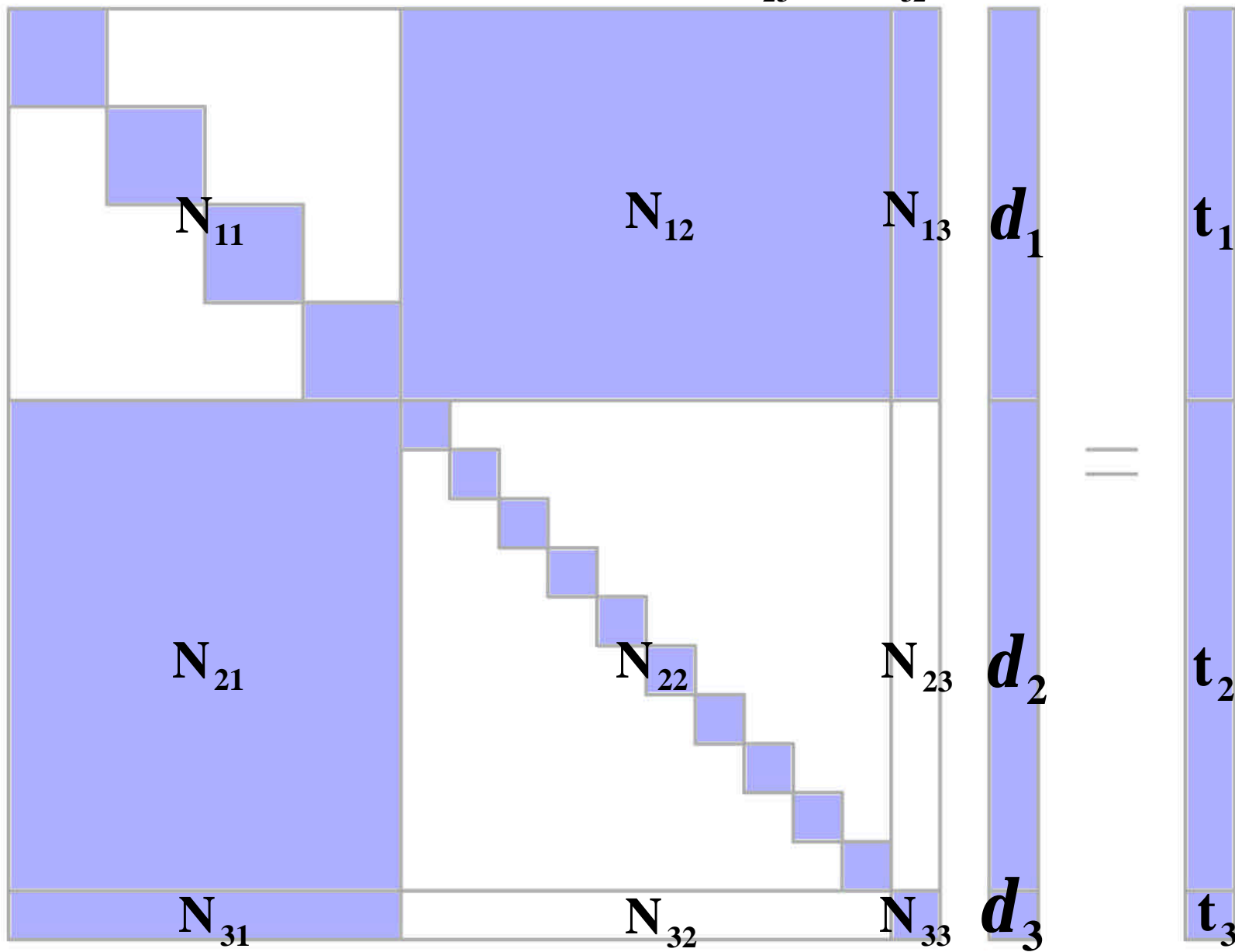


Make a new partition corresponding the unknown(s) that we wish to eliminate

This is the parameter vector partition that we want to eliminate

Recall: forward elimination followed by back substitution for complete solution

Label the partitions. Note that  $N_{23}$  and  $N_{32}$  are zero.



## Elimination Step

$$\mathbf{N}_{11}\mathbf{d}_1 + \mathbf{N}_{12}\mathbf{d}_2 + \mathbf{N}_{13}\mathbf{d}_3 = \mathbf{t}_1$$

$$\mathbf{N}_{21}\mathbf{d}_1 + \mathbf{N}_{22}\mathbf{d}_2 + \mathbf{N}_{23}\mathbf{d}_3 = \mathbf{t}_2$$

$$\mathbf{N}_{31}\mathbf{d}_1 + \mathbf{N}_{32}\mathbf{d}_2 + \mathbf{N}_{33}\mathbf{d}_3 = \mathbf{t}_3$$

Eliminate  $\mathbf{d}_3$  from eqn. 3, remember  
that  $\mathbf{N}_{32} = \mathbf{0}$

$$\mathbf{d}_3 = \mathbf{N}_{33}^{-1}(\mathbf{t}_3 - \mathbf{N}_{31}\mathbf{d}_1)$$

Now substitute that expression into the  
first two equations

$$(\mathbf{N}_{11} - \mathbf{N}_{13}\mathbf{N}_{33}^{-1}\mathbf{N}_{31})\mathbf{d}_1 + \mathbf{N}_{12}\mathbf{d}_2 = \mathbf{t}_1 - \mathbf{N}_{13}\mathbf{N}_{33}^{-1}\mathbf{t}_3$$

$$\mathbf{N}_{21}\mathbf{d}_1 + \mathbf{N}_{22}\mathbf{d}_2 = \mathbf{t}_2$$

Remember to save the  $\mathbf{N}$ 's  
and the  $\mathbf{t}$  from this step so  
that after we have solved for  
delta-1 we can come back  
and solve for delta-3

Note only changes are  
in the  $\mathbf{N}_{11}$  partition and  
in the  $\mathbf{t}_1$  partition, that  
is why it is so efficient

Notice that the elimination of a point does not involve  
the other points – hence they can be accumulated and  
eliminated right away, never form full  $\mathbf{N}_{22}$

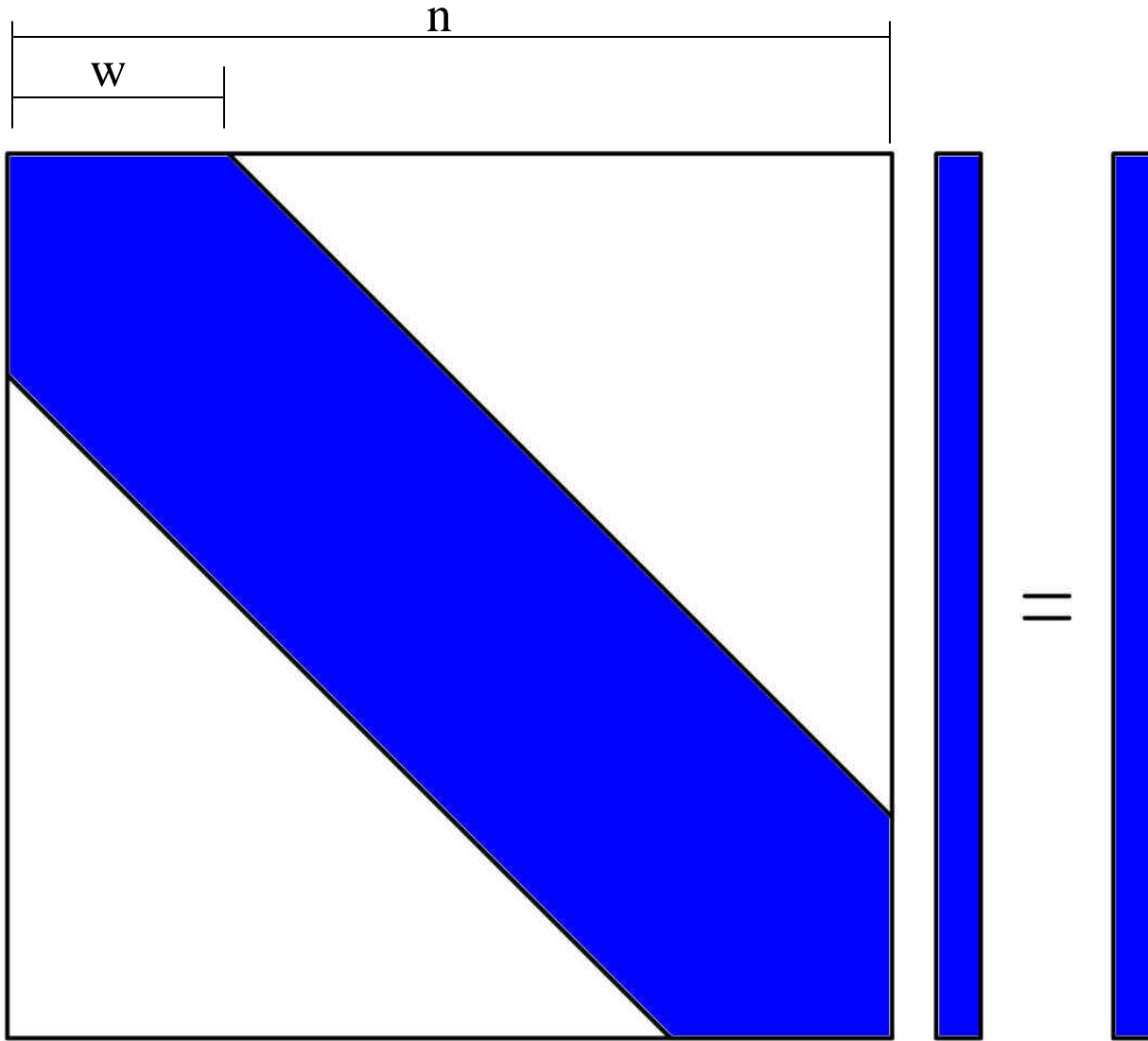
## After Eliminating All of the Points

When all points have been eliminated, one by one, by the procedure just described, we are left with the *reduced normal equations* in which now the only remaining unknowns are the photo and camera parameters. The rules, now, for when a block is nonzero: (a) the diagonal blocks are nonzero as before, (b) any off-diagonal block (corresponding to two photos) is nonzero if those two photos share a common point.

With large blocks and parallel flight lines, even this reduced normal equation matrix is still *sparse*. Its structure is banded (or banded-bordered). A similar partitioning plan can be used to successively further reduce this until it is full (number of steps is related to the *bandwidth*).

A famous photogrammetrist, Duane Brown, did much work on the efficient solution of large photogrammetric blocks, and referred to this procedure as *recursive partitioning*.

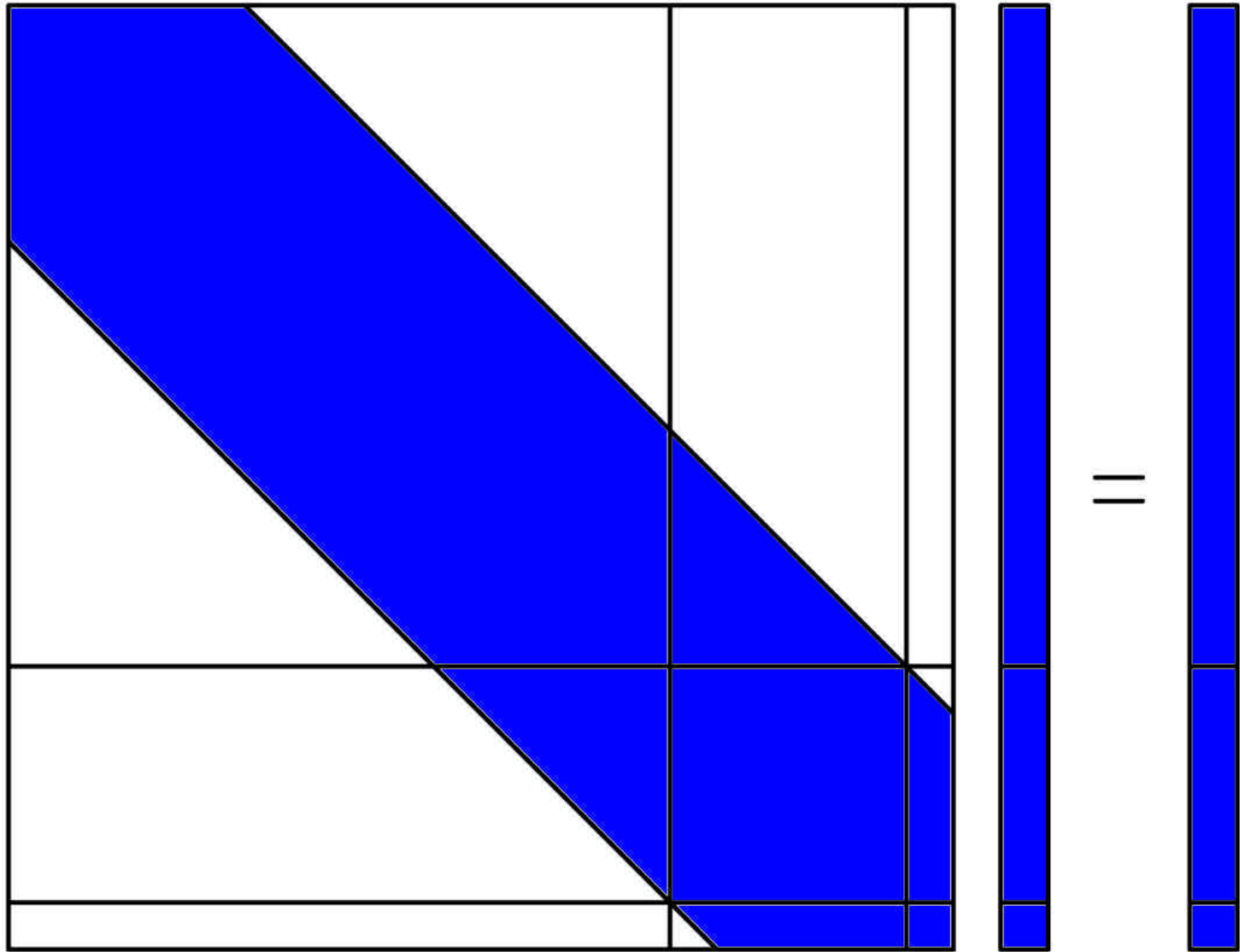
# Band Matrices



Solution of System with full matrix takes on the order of  $n^3$  operations. Solution of band system takes on the order of  $w^2n$  – that can be *much* less for large  $n$  and small  $w$ .

Do it by special partitioning to create a zero partition.

Make the partition so that  $\mathbf{N}_{13}$  is zero. Then we can efficiently eliminate  $\delta_3$





## Similar to previous elimination step

$$\mathbf{N}_{11}\mathbf{d}_1 + \mathbf{N}_{12}\mathbf{d}_2 + \mathbf{N}_{13}\mathbf{d}_3 = \mathbf{t}_1$$

$$\mathbf{N}_{21}\mathbf{d}_1 + \mathbf{N}_{22}\mathbf{d}_2 + \mathbf{N}_{23}\mathbf{d}_3 = \mathbf{t}_2$$

$$\mathbf{N}_{31}\mathbf{d}_1 + \mathbf{N}_{32}\mathbf{d}_2 + \mathbf{N}_{33}\mathbf{d}_3 = \mathbf{t}_3$$

Eliminate  $\mathbf{d}_3$  from eqn. 3, remember that  $\mathbf{N}_{31} = \mathbf{0}$

$$\mathbf{d}_3 = \mathbf{N}_{33}^{-1}(\mathbf{t}_3 - \mathbf{N}_{32}\mathbf{d}_2)$$

Now substitute that expression into the first two equations

$$\mathbf{N}_{11}\mathbf{d}_1 + \mathbf{N}_{12}\mathbf{d}_2 = \mathbf{t}_1$$

$$\mathbf{N}_{21}\mathbf{d}_1 + \left(\mathbf{N}_{22} - \mathbf{N}_{23}\mathbf{N}_{33}^{-1}\mathbf{N}_{32}\right)\mathbf{d}_2 = \mathbf{t}_2 - \mathbf{N}_{23}\mathbf{N}_{33}^{-1}\mathbf{t}_3$$

Notice all activity takes place within the band

This is the forward elimination step. Do it many times, saving intermediate results. Then do back substitution, recalling saved results until the full banded system is solved. That gets you the photo parameters, then do back substitution for the eliminated points, and you are done – for *this iteration* !

## Solution Strategy for Block Adjustment

- Process data by point
- When all contributions (equations) for a point have been constructed, eliminate it, and save intermediate results
- After all points have been processed and eliminated, you have left only the camera and photo parameters, which are banded or band-bordered
- Use band matrix processing to efficiently solve for camera/photo parameters
- Retrieve saved intermediate results and solve for all of the ground points
- Finished this iteration – keep going until you converge!