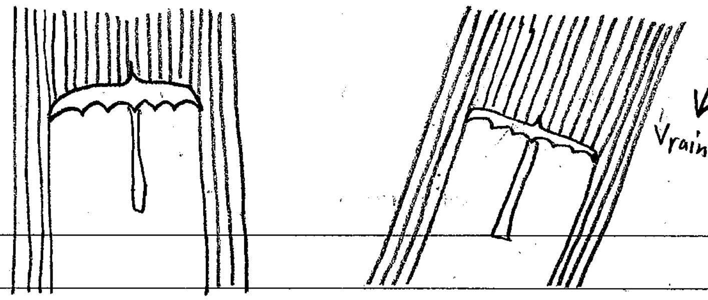
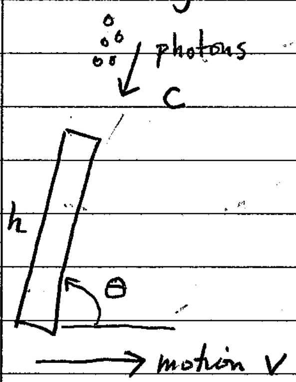
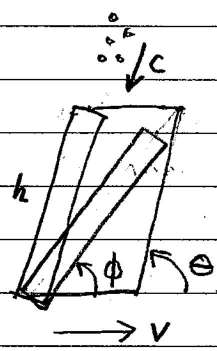


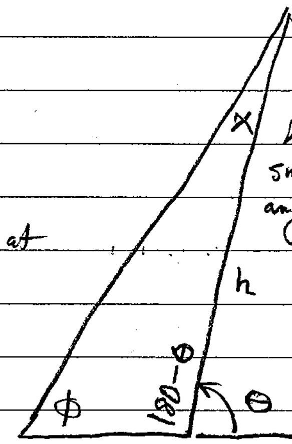
if you move you must tilt the umbrella in the direction of motion to use umbrella as a sensor to detect direction of rain motion, you must tilt it back.

stationary — moving $\longrightarrow V_{motion}$





must tilt telescope in direction of motion to detect photons coming at original angle θ



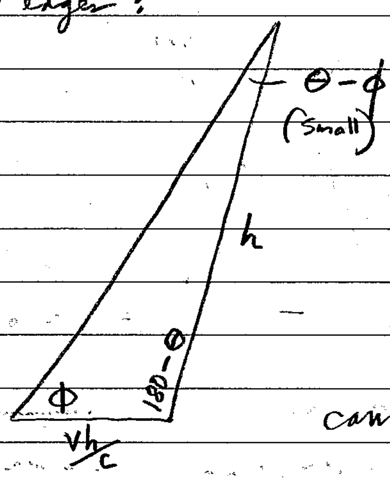
$x = \theta - \phi$ small angle

$d = v.t$, $t = d/v$, $c = 3\times10^8$ m/s

photon enters tube (length = h), takes $t = h/c$ sec. to reach the detector; telescope moving at speed $= v$ travels distance: $v\cdot t = v\cdot h/c$. label triangle edges:

$\cancel{180} - \theta + \phi + x = \cancel{180}$

$x = \theta - \phi$ (small angle)



photon traverses tube while telescope moves vh/c distance.

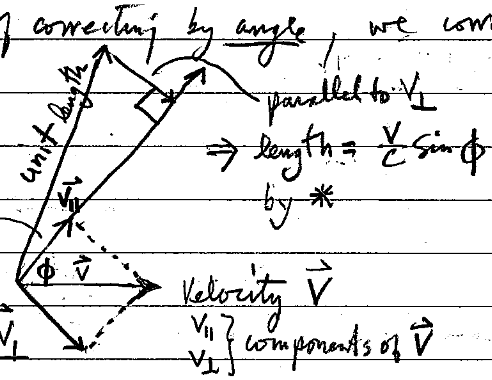
law of sines: $\dfrac{vh/c}{\sin(\theta-\phi)} = \dfrac{h}{\sin\phi}$

small angle $\theta - \phi$: $\dfrac{vh/c}{\theta-\phi} = \dfrac{h}{\sin\phi}$

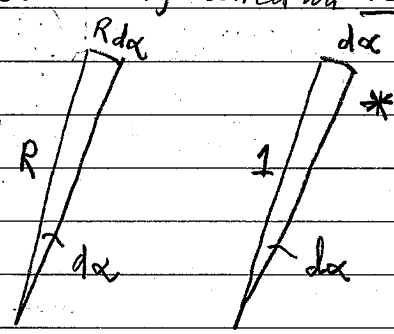
cancel h: $\dfrac{v/c}{\theta-\phi} = \dfrac{1}{\sin\phi}$

$\boxed{\theta - \phi = \frac{v}{c}\sin\phi}$ correction angle to tilt telescope.

instead of correcting by angle, we correct view vector by correction vector



correction angle $\theta - \phi = \frac{v}{c}\sin\phi$



$$\text{correction angle} = \frac{v}{c}\sin\phi = \frac{|\vec{V}|}{c}\sin\phi \quad |\vec{V}_\perp| = |\vec{V}|\sin\phi \Bigg\} \text{angle} = \frac{|\vec{V}_\perp|}{c} \Rightarrow \underset{(R=1)}{\text{arc length}} = \frac{|\vec{V}_\perp|}{c}$$

(in direction $\vec{V}_\perp$)

$$\Rightarrow \text{vector correction} = \frac{\vec{V}_\perp}{c}$$

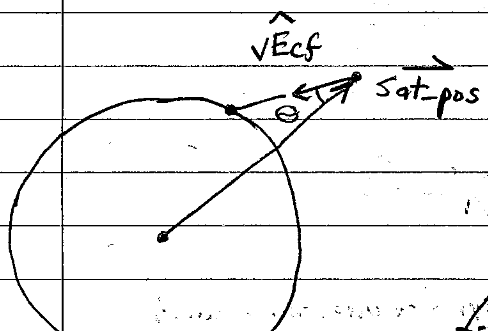
(our algorithm deals with relative velocities not absolute velocities)

$\boxed{5}$
$$\boxed{\overrightarrow{\text{vEcfCorr}} = \widehat{\text{vEcf}} - \frac{\overrightarrow{\text{vRel}_\perp}}{c}}$$

correction vector negative since we undo the tilt of the telescope.

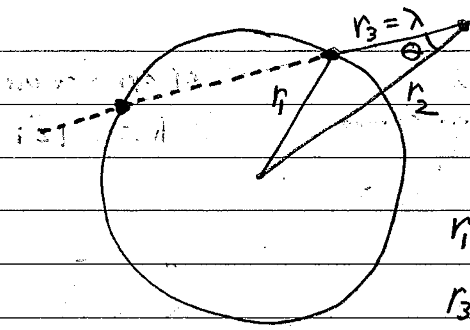
corrected view vector — original view vector

$\hat{\square}$: implies unit vector



$$\boxed{\cos\Theta = \widehat{\text{vEcf}} \cdot \frac{-\overrightarrow{\text{sat\_pos}}}{|\text{sat\_pos}|}} \quad \boxed{1}$$

$$= \widehat{\text{vEcf}} \cdot -\widehat{\text{sat\_pos}}$$



use law of cosines for the triangle

$$r_1^2 = r_2^2 + r_3^2 - 2r_2r_3\cos\Theta$$

$r_3 = \lambda$, r_1 = mean radius of earth

$r_1 = r_e = 6371$ km

$$r_e^2 = r_2^2 + \lambda^2 - 2r_2\lambda\cos\theta$$

$$r_2^2 + \lambda^2 - r_e^2 - 2r_2\lambda\cos\theta = 0$$

use quadratic formula to solve for λ

$$A\lambda^2 + B\lambda + C = 0, \quad \lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

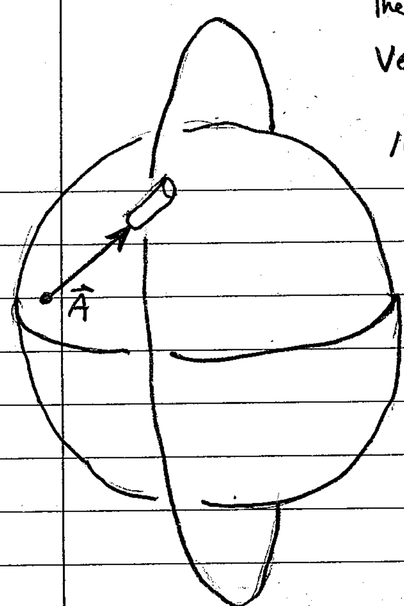
[choose minus since we want smaller solution for λ]

$$A = 1, \quad B = -2r_2\cos\theta, \quad C = r_2^2 - r_e^2, \quad r_2 = |\overrightarrow{\text{sat\_pos}}|$$

$$\lambda = \frac{2r_2\cos\theta - \left[4r_2^2\cos^2\Theta - 4r_2^2 + 4r_e^2\right]^{1/2}}{2}$$

$$\lambda = r_2\cos\theta - \left[r_2^2\cos^2\Theta + r_e^2 - r_2^2\right]^{1/2}$$

$\boxed{2}$
$$\boxed{\lambda = |\overrightarrow{\text{sat\_pos}}|\cos\theta - \left[|\overrightarrow{\text{sat\_pos}}|^2\cos^2\Theta + r_e^2 - |\text{sat\_pos}|^2\right]^{1/2}}$$



The following relation works for $\vec{A}$ either position or velocity, with $\frac{d\vec{A}}{dt}$ respectively velocity or acceleration

in our case $\vec{A}$ is position, $\frac{d\vec{A}}{dt}$ is velocity

the equation relates <u>fixed</u> and <u>rotating</u> reference frames.

$$\boxed{\left.\frac{d\vec{A}}{dt}\right|_{fixed} = \left.\frac{d\vec{A}}{dt}\right|_{rotating} + \underset{\uparrow}{\vec{\omega}} \times \vec{A}}$$

mean sidereal earth rotation rate vector

$\vec{A}$: relative position

$\frac{d\vec{A}}{dt}$: relative velocity

$$\vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ 7.292115833e{-}05 \end{bmatrix} \text{Rad/sec}$$

$$\left.\frac{d\vec{A}}{dt}\right|_{fixed} = \begin{bmatrix} Vx \\ Vy \\ Vz \end{bmatrix} + \vec{\omega} \times \lambda(-v\hat{E}cf)$$

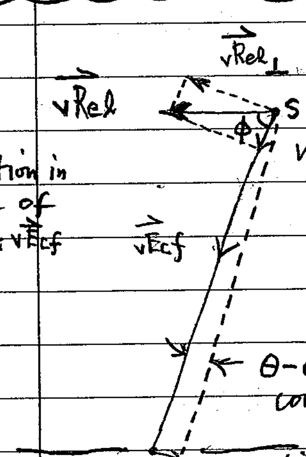
relative velocity fixed

interpolated velocity vector from ephemeris (rotating)

$v\hat{E}cf$ opposite direction from $\vec{A}$

λ from $\boxed{2}$

$\boxed{3}$ $$\boxed{\overrightarrow{Satvelrel} = \begin{bmatrix} Vx \\ Vy \\ Vz \end{bmatrix} - \lambda\left(\vec{\omega} \times v\hat{E}cf\right)}$$



$$\overrightarrow{vRel} = \overrightarrow{vRel}_{\parallel} + \overrightarrow{vRel}_{\perp}$$

$$\overrightarrow{vRel}_{\perp} = \overrightarrow{vRel} - \overrightarrow{vRel}_{\parallel}$$

$$\overrightarrow{vRel}_{\parallel} = \left(\overrightarrow{vRel} \cdot v\hat{E}cf\right) v\hat{E}cf \quad \text{(projection)}$$

$$\boxed{\overrightarrow{vRel}_{\perp} = \overrightarrow{vRel} - \left(\overrightarrow{vRel} \cdot v\hat{E}cf\right) v\hat{E}cf} \quad \boxed{4}$$

Summary of 5 equations found in <u>vac.m</u>

1 $\cos\Theta = \hat{vEcf} \cdot \frac{-\vec{sat\_pos}}{|\vec{sat\_pos}|}$

2 $\lambda = |\vec{sat\_pos}|\cos\Theta - \left[|\vec{sat\_pos}|^2 \cos^2\Theta + r_e^2 - |sat\_pos|^2 \right]^{1/2}$

3 $\vec{vRel} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \lambda \left(\vec{\omega} \times \hat{vEcf} \right)$

4 $\vec{vRel}_\perp = \vec{vRel} - \left(\vec{vRel} \cdot \hat{vEcf} \right) \hat{vEcf}$

5 $vEcfCorr = \hat{vEcf} - \left[\frac{\vec{vRel}_\perp}{c} \right]$

```
function vEcfCorr = vac( vEcf, satPos, satVel );
 meanSiderealRotationRate = 7.292115833000E-05;
 omega = [0 0 meanSiderealRotationRate]'; % earth rotational rate
 speedOfLight = 299792458; % meters/second
 meanEarthRadius = 6371000; % Mean Earth radius (in meters)
 normSatPos = norm(satPos);

1  cosTheta = dot( vEcf, -satPos / normSatPos );

2  lambda = normSatPos*cosTheta - ( normSatPos^2 * cosTheta^2 ...
            + meanEarthRadius^2 - normSatPos^2 )^0.5;

3  satVelRelative = satVel - lambda * cross(omega, vEcf);

4  velPerp = satVelRelative - (satVelRelative'*vEcf) * vEcf;

5  vEcfCorr = vEcf - velPerp/speedOfLight;

   vEcfCorr = vEcfCorr/norm(vEcfCorr);
```