# Coherent-Image Reconstruction Using Convolutional Neural Networks

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**Abstract:** We describe a technique for incorporating convolutional-neural-network models into a comprehensive approach for coherent-image reconstruction in the presence of noise and phase errors using the consensus equilibrium framework. © 2019 The Author(s)

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### 1. Introduction

In coherent imaging, we seek to reconstruct a scene's real-valued reflectance, r, from complex-valued measurements,  $y = A_{\phi}g + w$ , where  $A_{\phi}$  is the sensor's measurement matrix with unknown phase errors,  $\phi$ , and w is additive white Gaussian noise with variance  $\sigma_w^2$  [1,2]. Here, g is the reflection coefficient, a zero-mean, circularly-symmetric complex normal random variable with variance  $E[|g_s|^2|r] = r_s$ . Due to the nonlinear relationship between the data, y, and reflectance, r, images are typically formed according to  $\hat{r} = A^H y$ , leading to speckled images with amplified noise. Here, the superscript, H, indicates the conjugate transpose. Additional post-processing is then applied to remove the effects of  $\phi$ . In [1], we proposed a reconstruction framework, known as the Model-Based Iterative Reconstruction (MBIR) algorithm, to jointly compute the maximum a posteriori (MAP) estimates of r and  $\phi$ . While this approach reduces the effects of noise, speckle variation, and phase errors, it uses a simple image model which can lead to undesirable image artifacts.

Recently, convolutional neural networks (CNNs) have shown significant gains in learning image models. Unfortunately, these CNNs tend to be narrowly focused on a single reconstruction task, e.g. estimating missing pixels, recovering high-spatial-frequency variations, and removing unwanted noise. Furthermore, they fail to incorporate fundamental knowledge of the physical measurement process which can aid in reconstruction. Therefore, these CNNs are not able to address the combination of issues that arise in coherent image reconstruction, namely measurement noise with unknown  $\sigma_w^2$ , speckle variations, and phase errors.

## 2. Method

In this work, we use the consensus equilibrium (CE) framework [3] to incorporate image models, learned using a CNN, into a comprehensive reconstruction algorithm and to balance those models with a physics-based measurement model for coherent imaging. For our approach, we use a sensor agent, which ensures our image estimate,  $\hat{r}$ , is consistent with the measured data using a physics-based model, and an image agent, which ensures  $\hat{r}$  is constant with our CNN-based image model. CE allows us to balance the influence of these two agents. Furthermore, we incorporate the estimation of  $\phi$  into the framework.

Our sensor agent is given by

$$F_s(x) = \underset{r}{\operatorname{argmin}} \left\{ q_r(r, r', \phi') + d(r - x, \sigma_s) \right\} \tag{1}$$

where  $q_r(r,r',\phi')=E_g\left[g^HD(r)^{-1}g|y,r',\phi'\right]-\log|D(r)|$  is a surrogate for the negative log likelihood of the data given the reflectance [1],  $d(r-x,\sigma)=\frac{1}{2\sigma^2}||r-x||^2$  is a damping function that retards the mapping with strength controlled by  $\sigma$  [3], and r' and  $\phi'$  are the current estimates of r and  $\phi$ , repsectively. Here,  $E_g[\cdot|\cdot]$  indicates a conditional expectation with respect to g and  $\mathscr{D}(\cdot)$  is an operator that produces a diagonal matrix from its vector argument. Note that our approach is inspired by the MAP algorithm in [1]. As such, we use a surrogate for the data log-likelihood function since the actual function is difficult to evaluate. The sensor agent maps an input, x, onto a space which is consistent with the measured data and the coherent imaging model.

Our image agent is given by

$$F_i(x) = \underset{r}{\operatorname{argmin}} \left\{ d(r - x, \sigma_i) - \log p(r) \right\}$$
 (2)

where p(r) is the prior model for the reflectance. The image agent maps an input, x, onto a space which is consistent with the prior model. Since Eq. (2) can be interpreted as a Gaussian-denoising operation [4], we use a DnCNN trained with noise  $\sigma_i$  as our image agent [5]. We therefore implicitly incorporate the image model learned by a CNN into a comprehensive reconstruction framework for coherent imaging. Note that since  $\sigma_i$  may vary depending on the desired regularization, we trained 60 different DnCNNs for  $\sigma_i \in (0,1)$ .

Using the above agents, we apply the Douglas-Rachford algorithm to obtain an iterative approach for solving the CE framework [3]. We also incorporate the estimation of  $\phi$  into the iterative process. Figure 1 shows the resulting algorithm, which we call CE for coherent imaging (CECI). In the algorithm,  $\rho \in (0,1)$  controls the the rate of convergence,  $q_{\phi}(\phi,r',\phi')=E_g\left[\frac{1}{\sigma_w^2}\left|\left|y-A_{\phi}g\right|\right|^2|y,r',\phi'\right]$ , and  $p(\phi)$  is a simple prior model for the phase errors defined in [1]. We also use a change of variables given by r=(w+v)/2 [3]. Note that we initialize the algorithm according to  $\phi^0=0$  and  $w^0=A_{\phi^0}^Hy$ .

#### 3. Results and Conclusions

Using the coherent-imaging simulation environment from [1], we compared the CECI algorithm to the MBIR algorithm over four different turbulence strengths, characterized by  $D/r_0$ . Here, D is the pupil diameter and  $r_0$  is the plane-wave coherence length [6]. High values of  $D/r_0$  correspond to strong phase errors,  $\phi$ . For each turbulence strength, we simulated data for 10 different images taken from the "standard" set commonly used in literature: camera man, peppers, Lena, etc. For the CECI algorithm, we set  $\rho=0.6$ ,  $\sigma_s=0.04$ , and  $\sigma_i=0.03$ . For each reconstruction, we measured the image quality using the peak-signal-to-noise ratio (PSNR). Additionally, we used peak Strehl ratio,  $S_p \in (0,1)$ , to measure the quality of our estimate for  $\phi$ , where  $S_p=1$  indicates a perfect estimate [2]. Figure 2 summarizes the results of our work and Fig. 3 shows example reconstruction for a single image. These results demonstrate that the CECI algorithm is able to achieve higher image quality during reconstruction when compared to MBIR. Furthermore, CECI produces more-accurate estimate of  $\phi$  compared to MBIR.

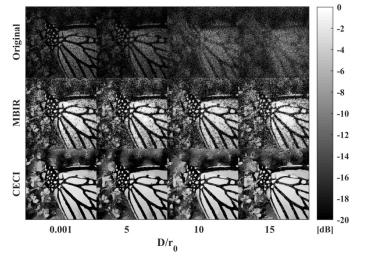
Fig. 3. Example results shown on a log scale.

Fig. 1. Update steps for the CECI Algorithm

$$\begin{aligned} v &\leftarrow 2F_s(w) - w \\ w &\leftarrow (1 - \rho)w + \rho (2F_i(v) - v) \\ \phi &\leftarrow \underset{\phi}{\operatorname{argmin}} \left\{ q_{\phi}(\phi, r', \phi') - \log p(\phi) \right\} \end{aligned}$$

Fig. 2. Mean PSNR /  $S_p$  over 10 images

MBIR	CECI
17.0 / 0.90	18.8 / 0.93
16.8 / 0.68	19.5 / 0.79
16.2 / 0.28	18.9 / 0.44
15.6 / 0.15	17.8 / 0.21
	17.0 / 0.90 16.8 / 0.68 16.2 / 0.28



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