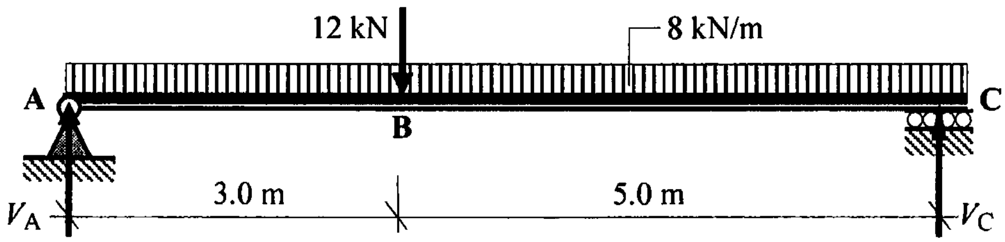
Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.1



Support Reactions

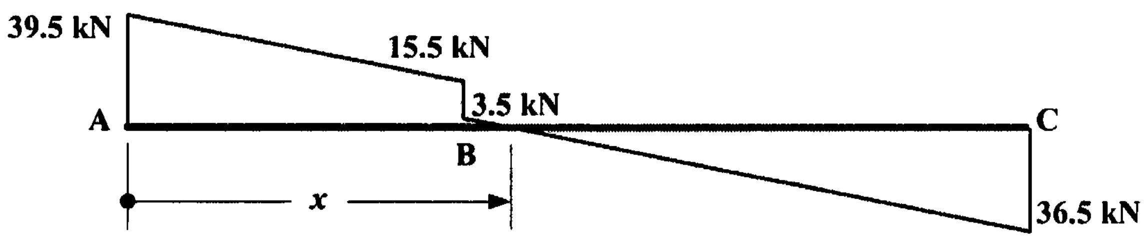
Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

$+ (12.0 \times 3.0) + (8.0 \times 8.0)(4.0) - (V_C \times 8.0) = 0$ $\therefore$ $V_C = +$ **36.5 kN** ↑

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$+ V_A - 12.0 - (8.0 \times 8.0) + V_C = 0$ $\therefore$ $V_A = +$ **39.5 kN** ↑

Shear Force Diagram

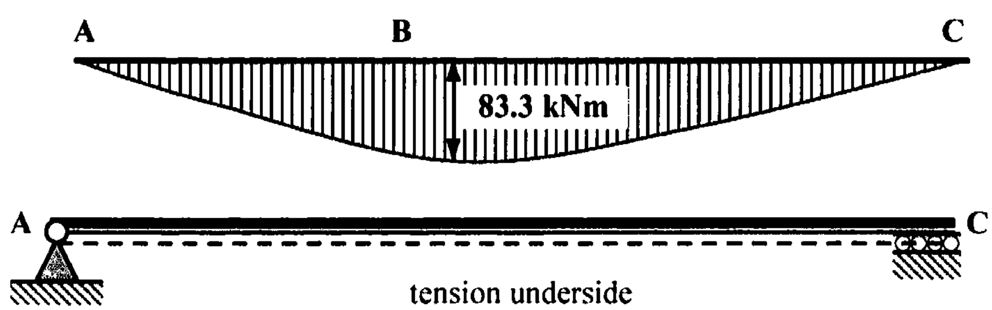


Position of zero shear force $x = [3.0 + (3.5/ 8.0)] = 3.438$ m
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

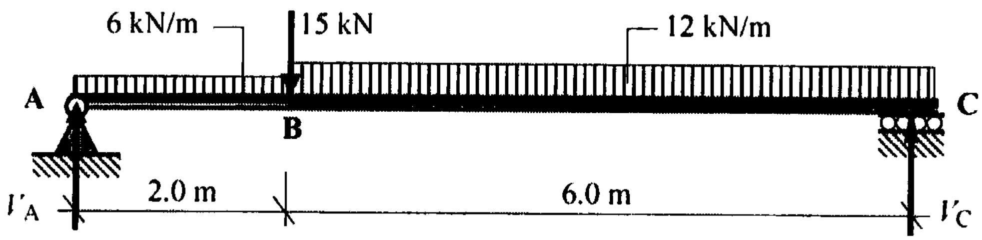
$M_x = + (39.5 \times 3.438) - (8.0 \times 3.438^2/2.0) - (12.0 \times 0.438) = +$ **83.3 kNm**

Alternatively, calculating the area under the shear force diagram:
$M_x = + [0.5(39.5 + 15.5)(3.0)] + (0.5 \times 0.438 \times 3.5) = + 83.3$ kNm



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.2 **Page No. 1**



Support Reactions

Consider the rotational equilibrium of the beam: $+\text{ve} \circlearrowright \Sigma M_A = 0$

$+ (6.0 \times 2.0)(1.0) + (15.0 \times 2.0) + (12.0 \times 6.0)(2.0 + 3.0) - (V_C \times 8.0) = 0$

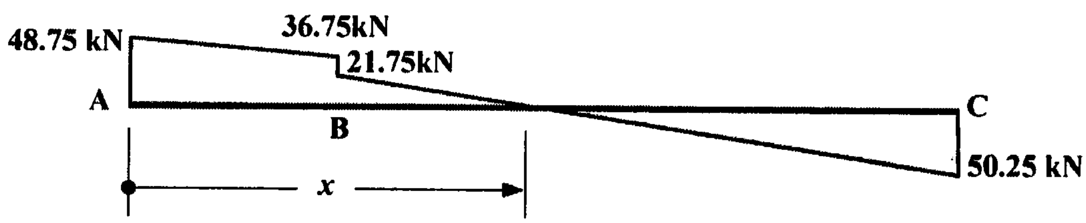
$\therefore$ $V_C = + \mathbf{50.25\ kN} \uparrow$

Consider the vertical equilibrium of the beam: $+\text{ve} \uparrow \Sigma F_y = 0$

$+ V_A - (6.0 \times 2.0) - 15.0 - (12.0 \times 6.0) + V_C = 0$

$\therefore$ $V_A = + \mathbf{48.75\ kN} \uparrow$

Shear Force Diagram



Position of zero shear force $x = [2.0 + (21.75/12.0)] = 3.813$ m

(This corresponds with the position of the maximum bending moment in the beam.)

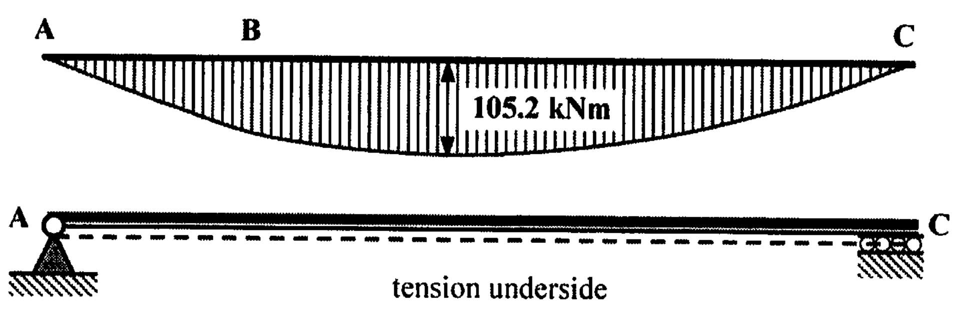
Bending Moment Diagram

$M_x = + (48.75 \times 3.813) - (6.0 \times 2.0)(3.813 - 1.0) - (15.0 \times 1.813) - (12.0 \times 1.813^2/2)$

$= + \mathbf{105.2\ kNm}$

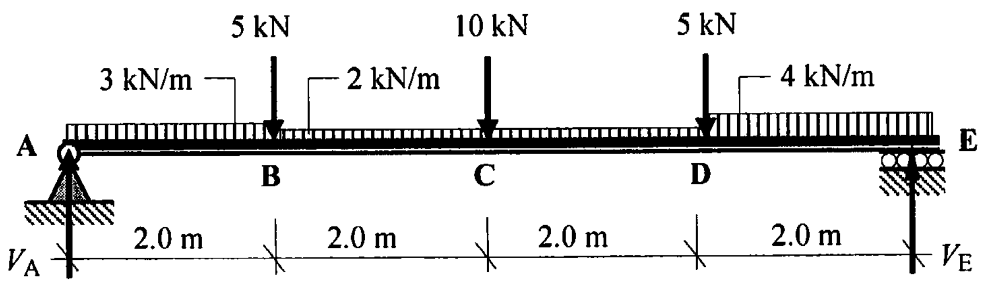
Alternatively, calculating the area under the shear force diagram:

$M_x = + [0.5(48.75 + 36.75)(2.0)] + (0.5 \times 1.813 \times 21.75) = + 105.2$ kNm



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.3 **Page No. 1**



Support Reactions

Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

$$+(3.0 \times 2.0)(1.0) + (5.0 \times 2.0) + (2.0 \times 4.0)(4.0) + (10.0 \times 4.0) + (5.0 \times 6.0) + (4.0 \times 2.0)(7.0) - (V_E \times 8.0) = 0$$

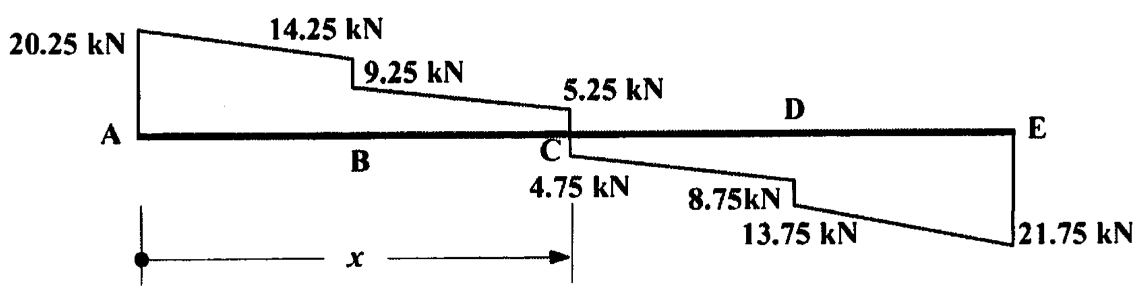
$\therefore$ **$V_E = +21.75$ kN** ↑

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$$+V_A - (3.0 \times 2.0) - 5.0 - (2.0 \times 4.0) - 10.0 - 5.0 - (4.0 \times 2.0) + V_E = 0$$

$\therefore$ **$V_A = +20.25$ kN** ↑

Shear Force Diagram

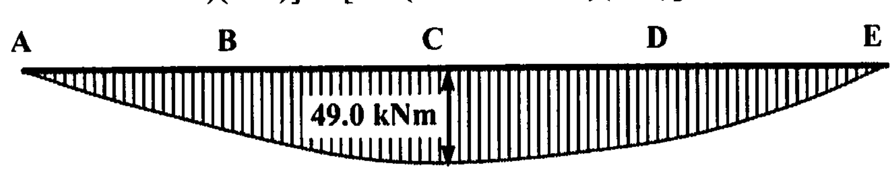


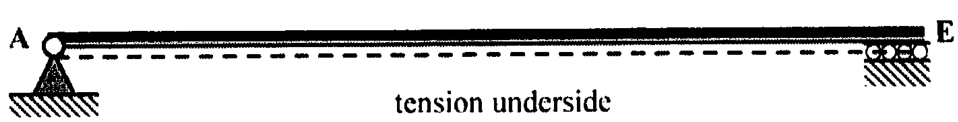
Position of zero shear force $x = 4.0$ m
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

$M_x = +(20.25 \times 4.0) - (3.0 \times 2.0)(3.0) - (5.0 \times 2.0) - (2.0 \times 2.0)(1.0) = +$ **49.0 kNm**

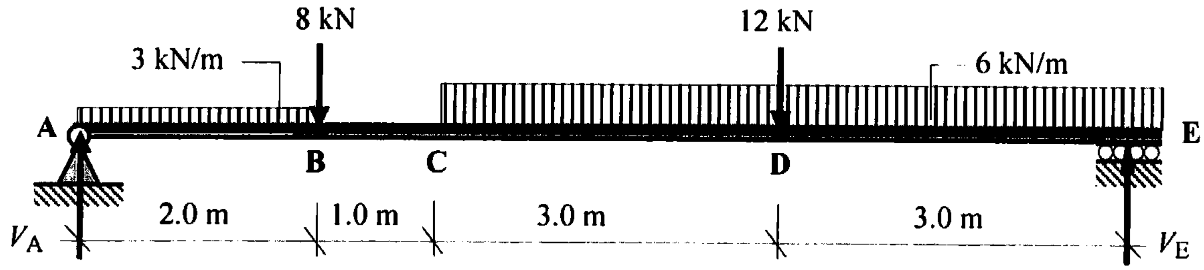
Alternatively, calculating the area under the shear force diagram:
$M_x = +[0.5(20.25 + 14.25)(2.0)] + [0.5(9.25 + 5.25)(2.0)] = +49.0$ kNm





Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.4 **Page No. 1**



Support Reactions

Consider the rotational equilibrium of the beam: $+\text{ve} \circlearrowright \Sigma M_A = 0$

$$+(3.0 \times 2.0)(1.0) + (8.0 \times 2.0) + (6.0 \times 6.0)(6.0) + (12.0 \times 6.0) - (V_E \times 9.0) = 0$$

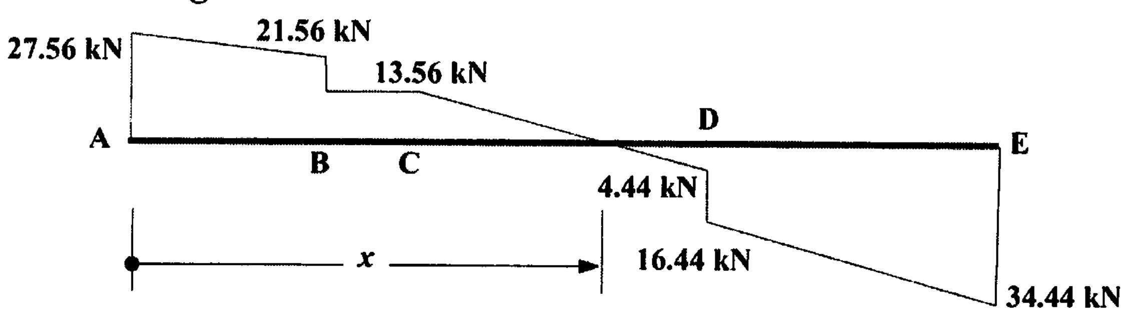
$\therefore$ $\mathbf{V_E = +34.44}$ **kN** ↑

Consider the vertical equilibrium of the beam: $+\text{ve} \uparrow \Sigma F_y = 0$

$$+V_A - (3.0 \times 2.0) - 8.0 - (6.0 \times 6.0) - 12.0 + V_E = 0$$

$\therefore$ $\mathbf{V_A = +27.56}$ **kN** ↑

Shear Force Diagram



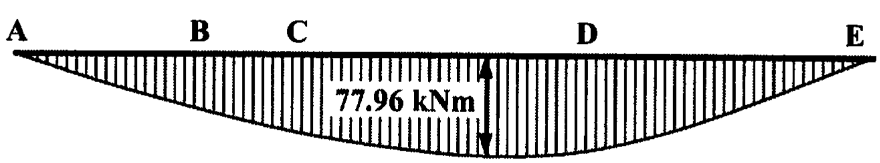
Position of zero shear force $x = [3.0 + (13.56/6.0)] = 5.26$ m (3.74 m from E)
(This corresponds with the position of the maximum bending moment in the beam.)

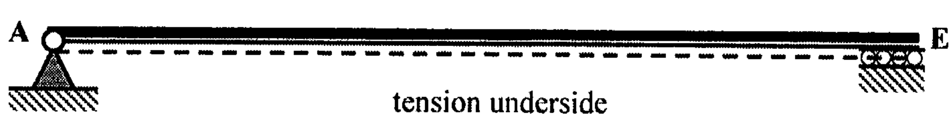
Bending Moment Diagram

$M_x = +(34.44 \times 3.74) - (6.0 \times 3.74^2/2) - (12.0 \times 0.74) = +$ **77.96 kNm**

Alternatively, calculating the area under the shear force diagram:

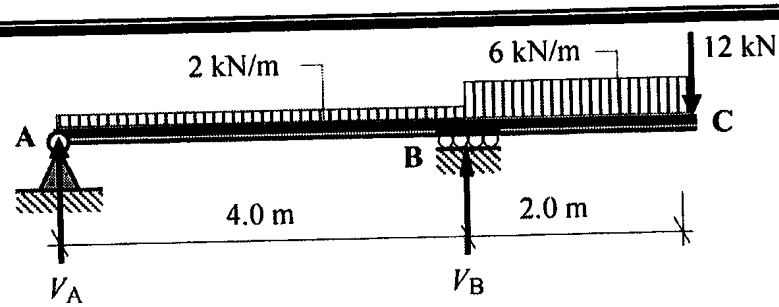
$M_x = +[0.5(34.44 + 16.44)(3.0)] + (0.5 \times 0.74 \times 4.44) = +77.96$ kNm





Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.5 **Page No. 1**



Support Reactions
Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

$$+(2.0 \times 4.0)(2.0) + (6.0 \times 2.0)(5.0) + (12.0 \times 6.0) - (V_B \times 4.0) = 0$$

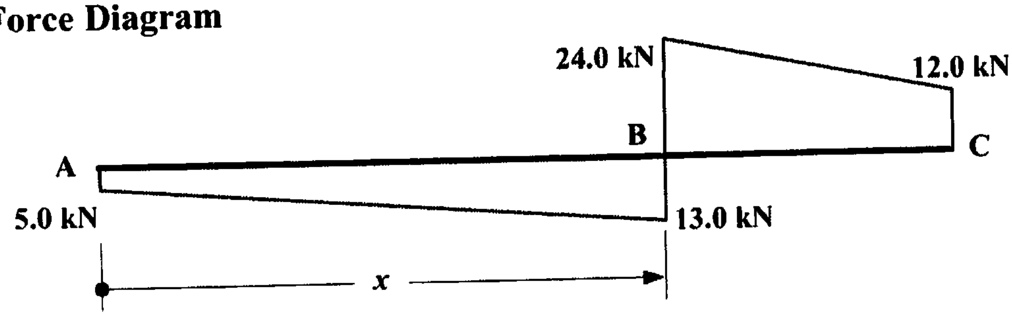
$\therefore$ $\mathbf{V_B = +37.0\ kN}$ ↑

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$$+V_A - (2.0 \times 4.0) - (6.0 \times 2.0) - 12.0 + V_B = 0$$

$\therefore$ $\mathbf{V_A = -5.0\ kN}$ ↓

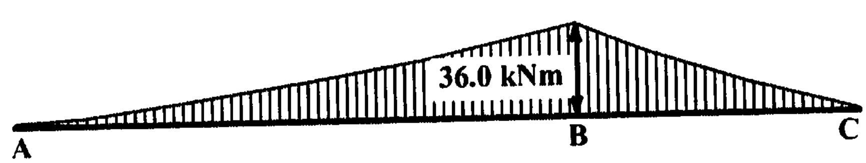
Shear Force Diagram

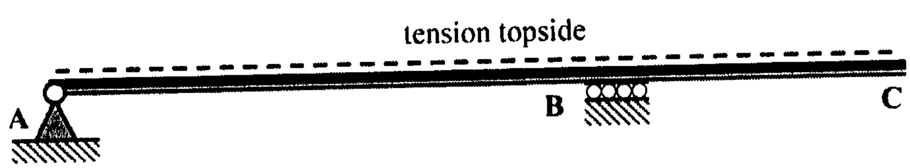


Position of zero shear force $x = 4.0$ m
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram
$M_x = -(5.0 \times 4.0) - (2.0 \times 4.0^2/2) = \mathbf{-36.0\ kNm}$

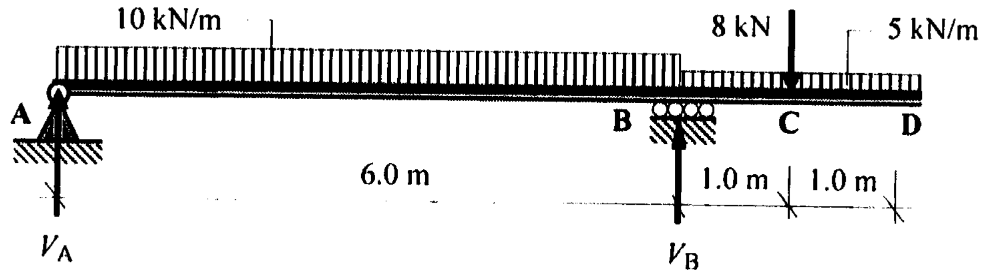
Alternatively, calculating the area under the shear force diagram:
$M_x = -[0.5(5.0 + 13.0)(4.0)] = -36.0$ kNm





Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.6



Support Reactions

Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

$+ (10.0 \times 6.0)(3.0) + (5.0 \times 2.0)(7.0) + (8.0 \times 7.0) - (V_B \times 6.0) = 0$

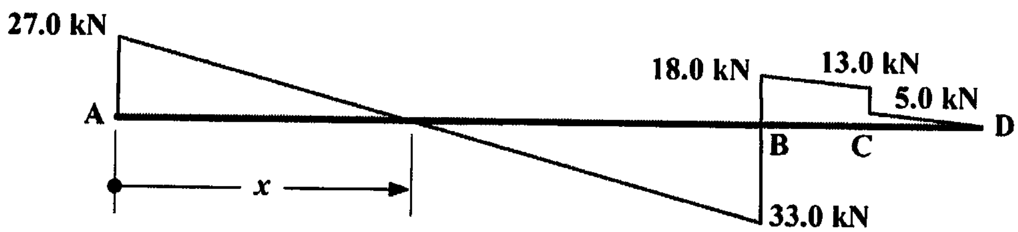
$\therefore$ $V_B = +$ **51.0 kN** ↑

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$+ V_A - (10.0 \times 6.0) - (5.0 \times 2.0) - 8.0 + V_B = 0$

$\therefore$ $V_A = +$ **27.0 kN** ↑

Shear Force Diagram



Positions of zero shear force: $x = (27.0/ 10.0) = 2.7$ m and $x = 6.0$ m
(These correspond with the positions of the maximum bending moments in the beam.)

Bending Moment Diagram

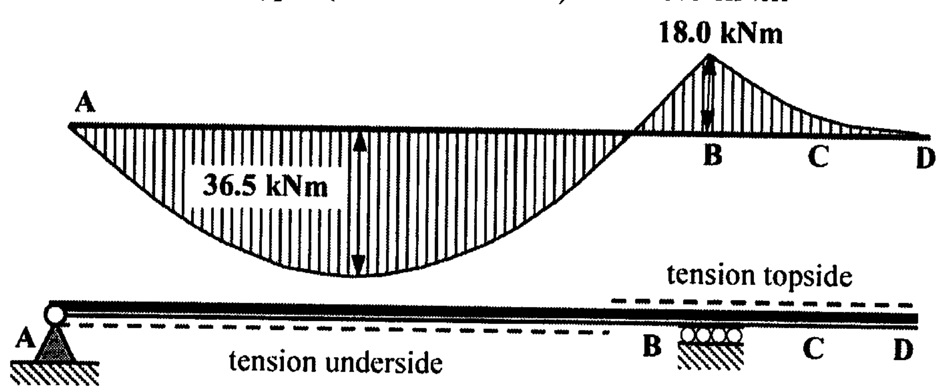
$M_x = + (27.0 \times 2.7) - (10.0 \times 2.7^2/2) = +$ **36.5 kNm**

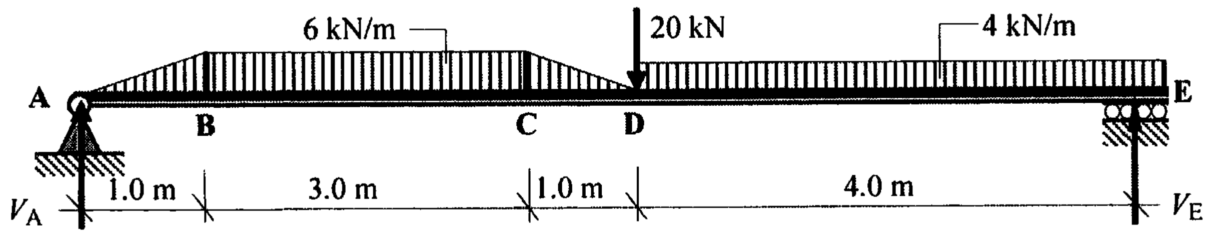
$M_B = - (5.0 \times 2.0)(1.0) - (8.0 \times 1.0) = -$ **18.0 kNm**

Alternatively, calculating the area under the shear force diagram:

$M_x = - (0.5 \times 2.7 \times 27.0) = + 36.5$ kNm

$M_B = - [0.5(18.0 + 13.0)(1.0)] + (0.5 \times 1.0 \times 5.0) = + 18.0$ kNm





Load between A and B = (0.5 × 1.0 × 6.0) = 3.0 kN: centre of gravity is 0.67 m from A
Load between B and C = (6.0 × 3.0) =18.0 kN:centre of gravity is 2.50 m from A
Load between C and D = (0.5 x 1.0 x 6.0) = 3.0 kN: centre of gravity is 4.33 m from A

Support Reactions

Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

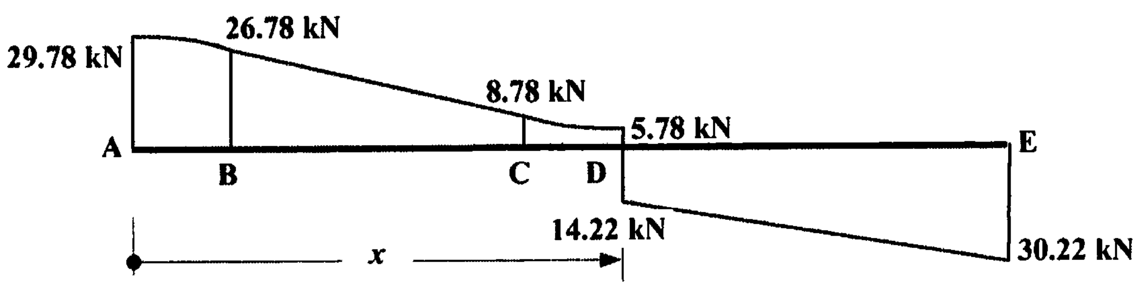
$+ (3.0 \times 0.67) + (18.0 \times 2.5) + (3.0 \times 4.33) + (20.0 \times 5.0) + (4.0 \times 4.0)(7.0)$
$- (V_E \times 9.0) = 0$

$\therefore$ **$V_E = + 30.22$ kN ↑**

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$+ V_A - 3.0 - 18.0 - 3.0 - 20.0 - (4.0 \times 4.0) + V_E = 0$ $\therefore$ **$V_A = + 29.78$ kN ↑**

Shear Force Diagram (**Note:** the diagram is curved from A to B and from C to D)



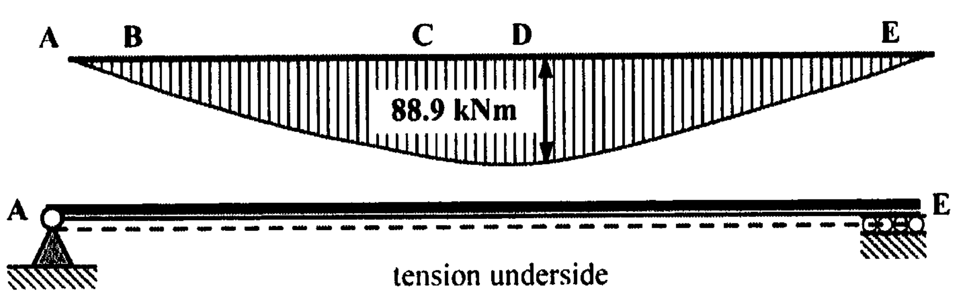
Position of zero shear force $x = 5.0$ m
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram: (consider the right-hand side)

$M_x = + (30.22 \times 4.0) - (4.0 \times 4.0^2/2) = +$ **88.9 kNm**

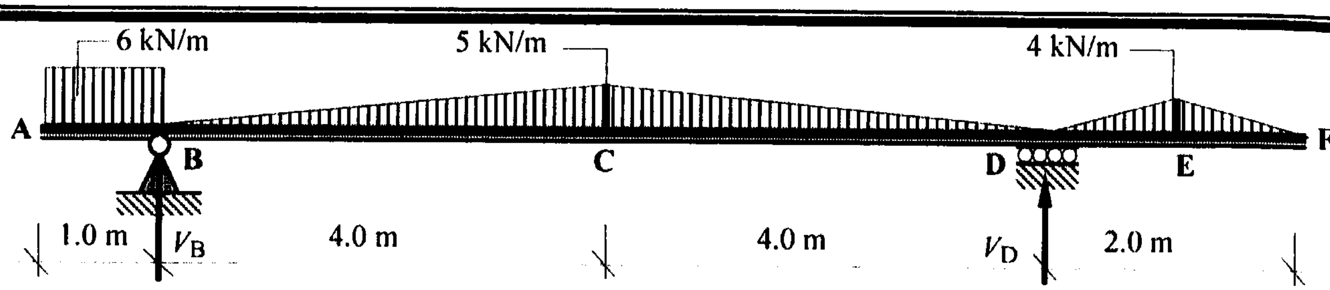
Alternatively, calculating the area under the shear force diagram:

$M_x = + 0.5(14.22 + 30.22)(4.0) = + 88.9$ kNm



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.8 **Page No. 1**



Support Reactions

Consider the rotational equilibrium of the beam: $+ve \circlearrowright \Sigma M_B = 0$

$$-(6.0 \times 1.0)(0.5) + (0.5 \times 8.0 \times 5.0)(4.0) + (0.5 \times 2.0 \times 4.0)(9.0) - (V_D \times 8.0) = 0$$

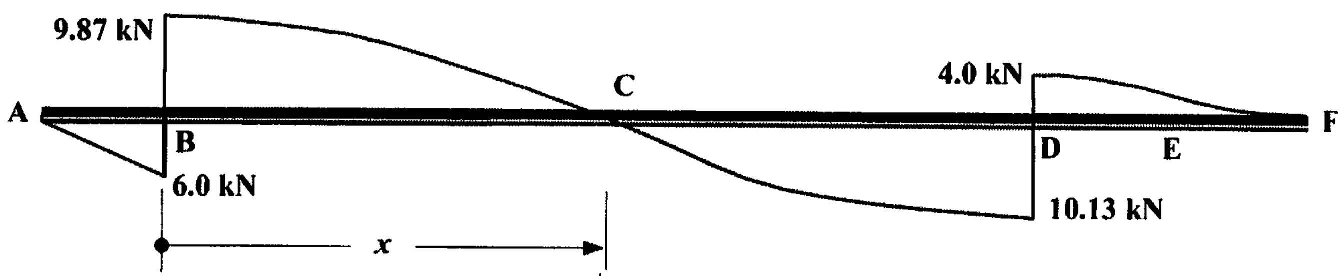
$\therefore$ $\mathbf{V_D = +\ 14.13\ kN}$ ↑

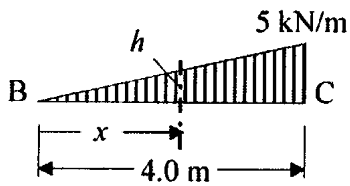
Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$-(6.0 \times 1.0) + V_B - (0.5 \times 8.0 \times 5.0) - (0.5 \times 2.0 \times 4.0) + V_D = 0$$

$\therefore$ $\mathbf{V_B = +\ 15.87\ kN}$ ↑

Shear Force Diagram





$(h/x) = (5.0/4.0)$ $\therefore h = 1.25x$

Force over length $x = (0.5 \times x \times 1.25x) = 0.625x^2$

This force must equal 9.78 for zero shear at x

Position of zero shear force x: $9.78 = 0.625x^2$ $\therefore x = 3.956$ m from B

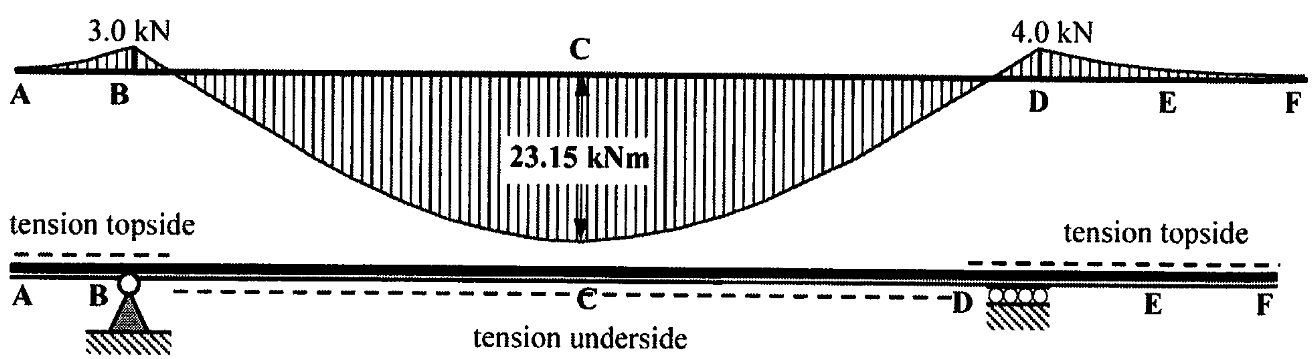
$\therefore h = (1.25 \times 3.956) = 4.945$

Bending Moment Diagram

$M_x = -(6.0 \times 1.0)(4.456) + (15.87 \times 3.956) - [(0.625 \times 3.956^2)(3.956/3.0)]$

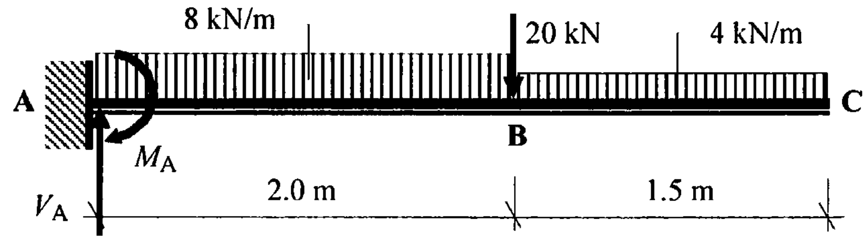
$= \mathbf{+\ 23.15\ kNm}$

$M_B = -(6.0 \times 1.0^2)/2 = \mathbf{-\ 3.0\ kNm}$; $M_D = -(0.5 \times 2.0 \times 4.0)(1.0) = \mathbf{+\ 4.0\ kNm}$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.9 **Page No. 1**



Support Reactions

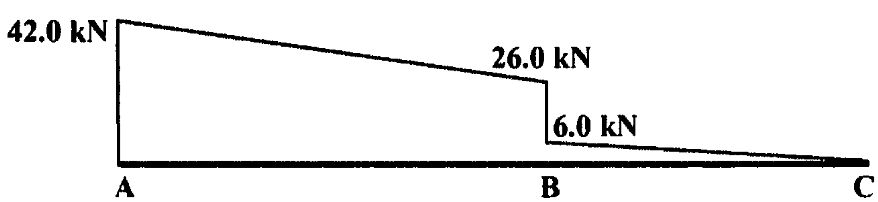
Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_A = 0$

$M_A + (8.0 \times 2.0)(1.0) + (20.0 \times 2.0) + (4.0 \times 1.5)(2.75) = 0$ $\therefore$ **$M_A = -72.5$ kNm** ↻

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$+V_A - (8.0 \times 2.0) - 20.0 - (4.0 \times 1.5) = 0$ $\therefore$ **$V_A = +42.0$ kN** ↑

Shear Force Diagram



Bending Moment Diagram

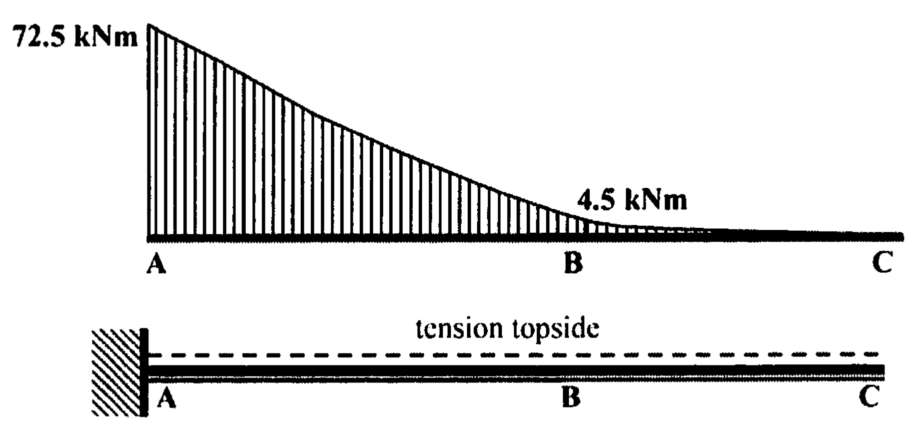
$M_A = = -$ **72.5 kNm**

$M_B = -(4.0 \times 1.5)(0.75) = -$ **4.5 kNm**

Alternatively, calculating the area under the shear force diagram:

$M_A = -[0.5(42.0 + 26.0)(2.0)] - (0.5 \times 1.5 \times 6.0) = -72.5$ kNm

$M_B = -(0.5 \times 1.5 \times 6.0) = -4.5$ kNm



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
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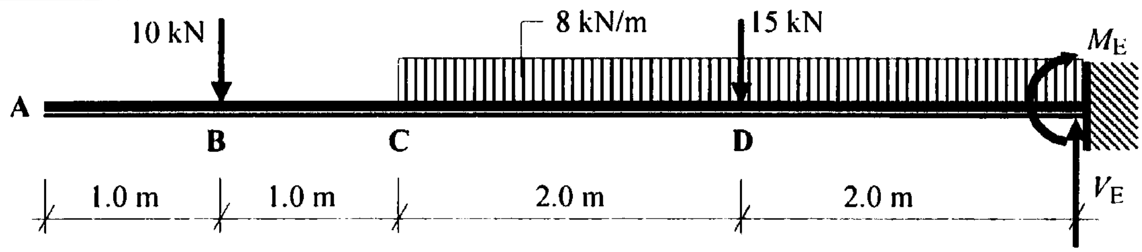


Support Reactions

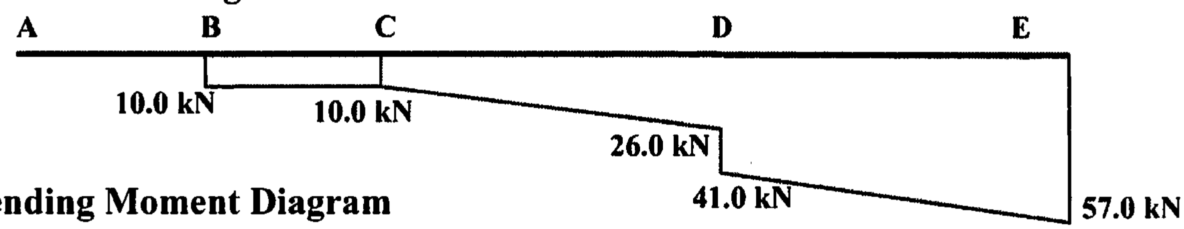
Consider the rotational equilibrium of the beam: +ve ↻ $\Sigma M_E = 0$

$-(10.0 \times 5.0) - (8.0 \times 4.0)(2.0) - (15.0 \times 2.0) + M_E = 0$ $\quad \therefore\ \mathbf{M_E = +\,144.0\ kN}$ ↺

Consider the vertical equilibrium of the beam: +ve ↑ $\Sigma F_y = 0$

$-10.0 - (8.0 \times 4.0) - 15.0 + V_E = 0$ $\quad \therefore\ \mathbf{V_E = +\,57.0\ kN}$ ↑

Shear Force Diagram



Bending Moment Diagram

$M_A = M_B =$ **zero**

$M_C = -(10.0 \times 1.0) = $ **– 10.0 kNm**

$M_D = -(10.0 \times 3.0) - (8.0 \times 2.0^2/2) = $ **– 46.0 kNm**

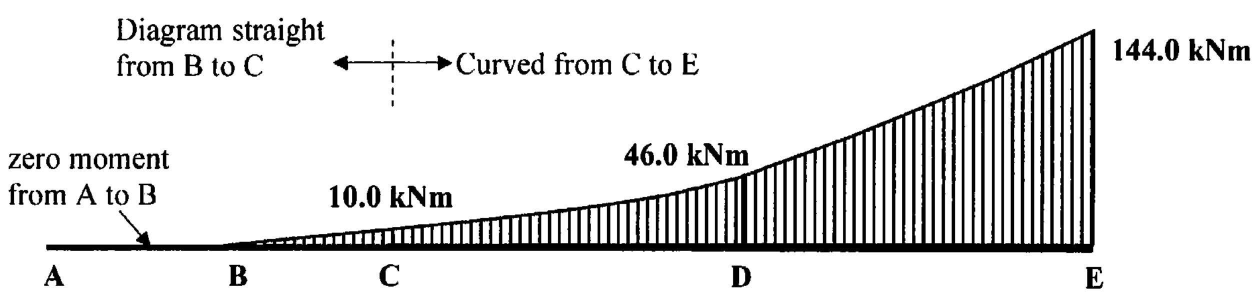
$M_E = $ **– 144.0 kNm**

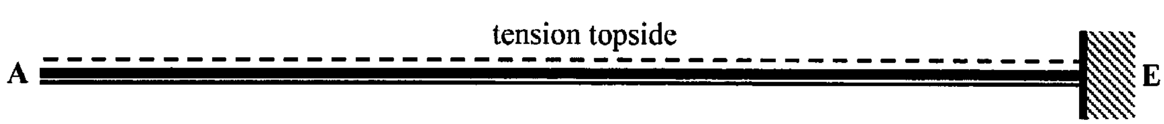
Alternatively, calculating the area under the shear force diagram:

$M_C = -(10.0 \times 1.0) = -\,10.0$ kNm

$M_D = -(10.0 \times 1.0) - [0.5(10.0 + 26.0)(2.0)] = -\,46.0$ kNm

$M_E = -(10.0 \times 1.0) - [0.5(10.0 + 26.0)(2.0)] - [0.5(41.0 + 57.0)(2.0)] = -\,144.0$ kNm





4.4.5 Example 4.10: Superposition – Beam 5

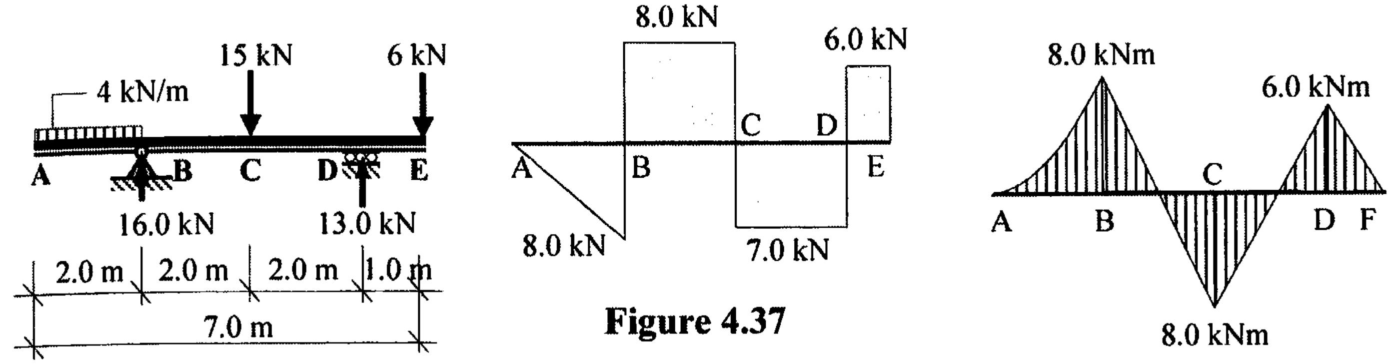


Figure 4.37

Using superposition this beam can be represented as the sum of:

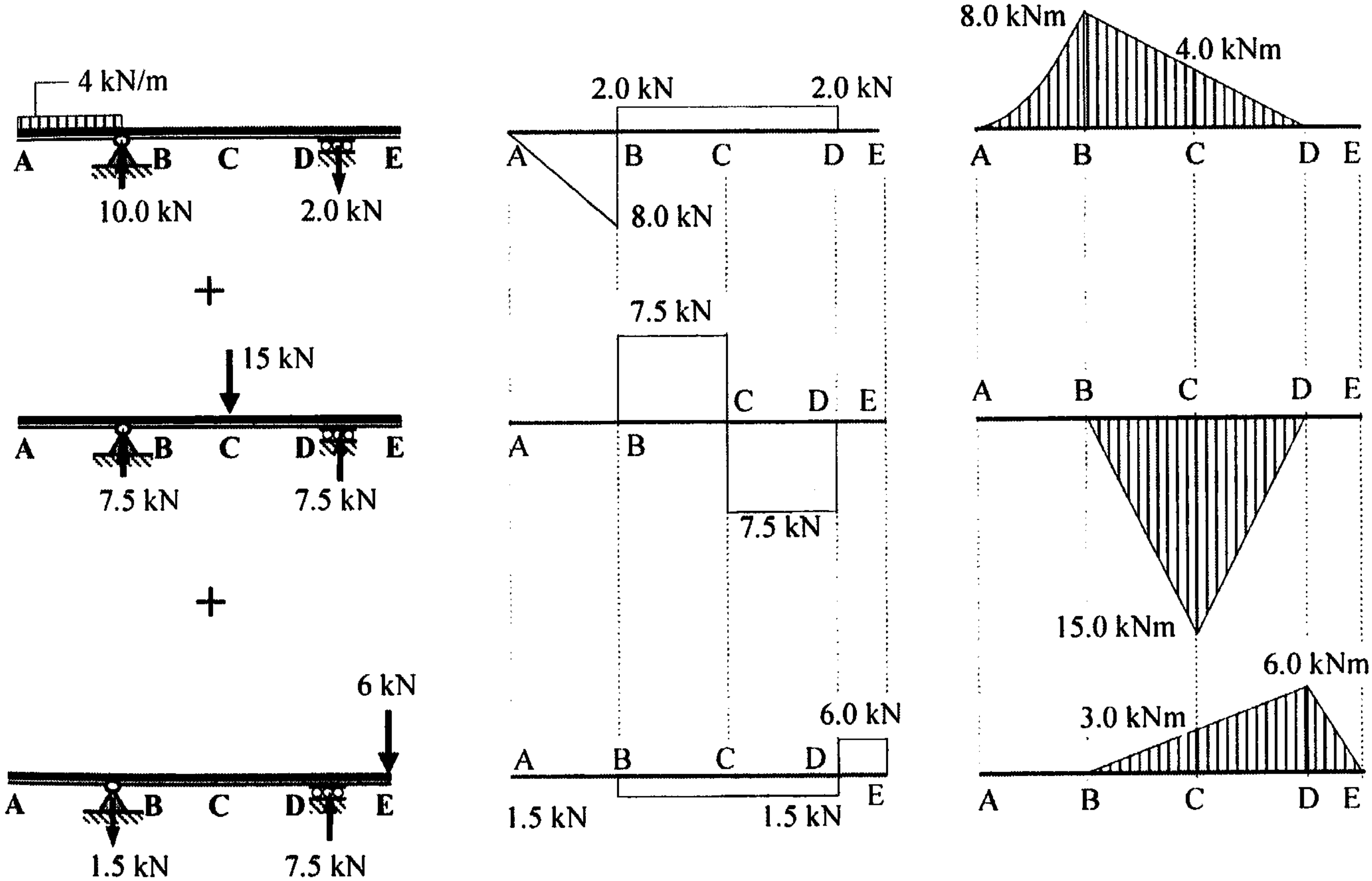


Figure 4.38

V_B = (+ 10.0 + 7.5 – 1.5) = 16.0 kN;
V_D = (– 2.0 + 7.5 + 7.5) = 13.0 kN;
Shear Force at B $_{\text{left-hand side}}$ = – 8.0 kN
Shear Force at B $_{\text{right-hand side}}$ = (+ 2.0 + 7.5 – 1.5) = + 8.0 kN
Shear Force at C $_{\text{left-hand side}}$ = (+ 2.0 + 7.5 – 1.5) = + 8.0 kN
Shear Force at C $_{\text{right-hand side}}$ = (+ 2.0 – 7.5 – 1.5) = – 7.0 kN
Shear Force at D $_{\text{left-hand side}}$ = (+ 2.0 – 7.5 – 1.5) = – 7.0 kN
Shear Force at D $_{\text{right-hand side}}$ = + 6.0 kN
Shear Force at E = + 6.0 kN
Bending Moment at B = – 8.0 kNm
Bending Moment at C = (– 4.0 + 15.0 – 3.0) = + 8.0 kNm
Bending Moment at D = – 6.0 kNm

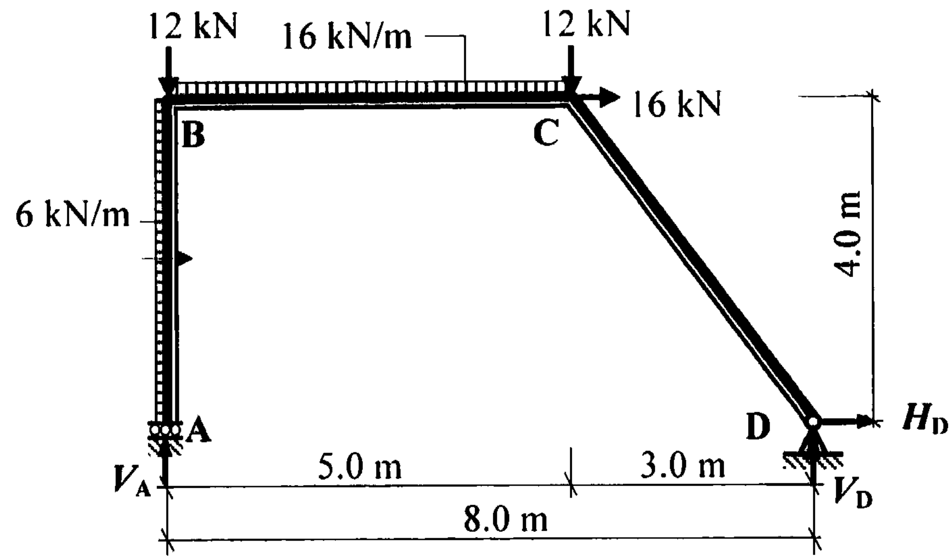


Figure 5.3

Solution:

Apply the three equations of static equilibrium to the force system

+ve ↑ $\Sigma F_y = 0$ $\quad V_A - 12.0 - (16.0 \times 5.0) - 12.0 + V_D = 0$ Equation (1)

+ve → $\Sigma F_x = 0$ $\quad (6.0 \times 4.0) + 16.0 + H_D = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$ $\quad (6.0 \times 4.0)(2.0) + (16.0 \times 5.0)(2.5) + (12.0 \times 5.0) + (16.0 \times 4.0)$
$- (V_D \times 8.0) = 0$ Equation (3)

From equation (2): $\quad 40.0 + H_D = 0$ $\quad \therefore$ $\boldsymbol{H_D = -40.0}$ **kN** ←

From equation (3): $\quad 372.0 - 8.0V_D = 0$ $\quad \therefore$ $\boldsymbol{V_D = +46.5}$ **kN** ↑

From equation (1): $\quad V_A - 104.0 + 46.5 = 0$ $\quad \therefore$ $\boldsymbol{V_A = +57.5}$ **kN** ↑

Assuming positive bending moments induce tension **inside** the frame:

$M_B = -(6.0 \times 4.0)(2.0) = -48.0$ kNm

$M_C = +(46.5 \times 3.0) - (40.0 \times 4.0) = -20.50$ kNm

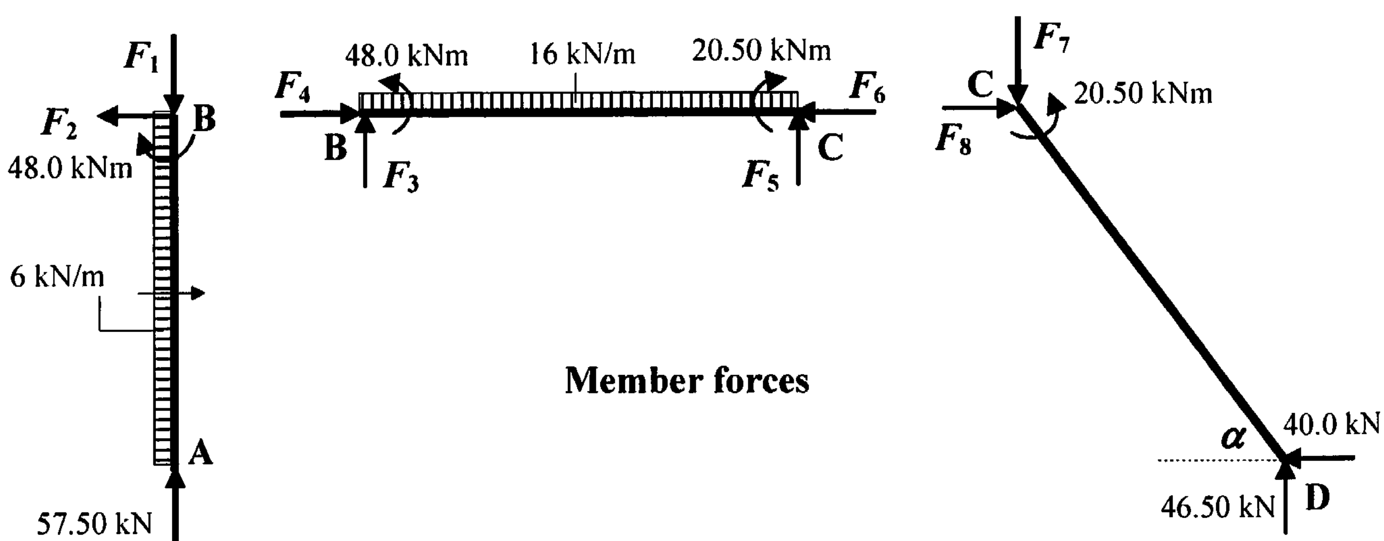


Figure 5.4

The values of the end-forces F_1 to F_8 can be determined by considering the equilibrium of each member and joint in turn.

Consider member AB:

+ve ↑ $\Sigma F_y = 0$ $\quad + 57.50 - F_1 = 0$ $\quad \therefore F_1 = 57.50$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad + (6.0 \times 4.0) - F_2 = 0$ $\quad \therefore F_2 = 24.0$ kN ←

Consider joint B:

+ve ↑ $\Sigma F_y = 0$ There is an applied vertical load at joint B = 12 kN ↓

$- F_1 + F_3 = - 12.0$ $\quad \therefore F_3 = 45.50$ kN ↑

+ve → $\Sigma F_x = 0$

$- F_2 + F_4 = 0$ $\quad \therefore F_4 = 24.0$ kN →

Consider member BC:

+ve ↑ $\Sigma F_y = 0$ $\quad + 45.5 - (16.0 \times 5.0) + F_5 = 0$ $\quad \therefore F_5 = 34.5$ kN ↑

+ve → $\Sigma F_x = 0$ $\quad + 24.0 - F_6 = 0$ $\quad \therefore F_6 = 24.0$ kN ←

Consider member CD:

+ve ↑ $\Sigma F_y = 0$ $\quad + 46.5 - F_7 = 0$ $\quad \therefore F_7 = 46.5$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad - 40.0 + F_8 = 0$ $\quad \therefore F_8 = 40.0$ kN →

Check joint C:

+ve ↑ ΣF_y There is an applied vertical load at joint C = 12 kN ↓

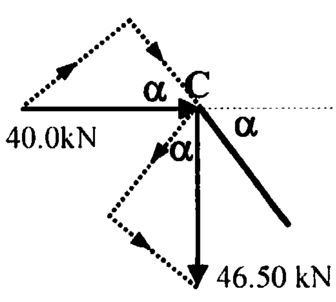
$+ F_5 - F_7 = + 34.5 - 46.5 = - 12.0$

+ve → ΣF_x There is an applied horizontal at joint C = 16 kN →

$- F_6 + F_8 = - 24.0 + 40.0 = + 16.0$

The axial force and shear force in member CD can be found from:
Axial load = +/– (Horizontal force × Cosα) +/– (Vertical force × Sinα)
Shear force = +/– (Horizontal force × Sinα) +/– (Vertical force × Cosα)
The signs are dependent on the directions of the respective forces.

Member CD:



$\alpha = \tan^{-1}(4.0/3.0) = 53.13°$

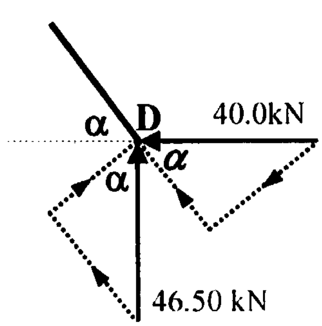
Cos $\alpha = 0.60$; Sin $\alpha = 0.80$

Assume axial compression to be positive.

At joint C

Axial force = + (40.0 × Cosα) + (46.50 × Sinα) = + 61.2 kN

Shear force = + (40.0 × Sinα) – (46.50 × Cosα) = + 4.10 kN



Similarly at joint D

Axial force = + 61.2 kN

Shear force = + 4.10 kN

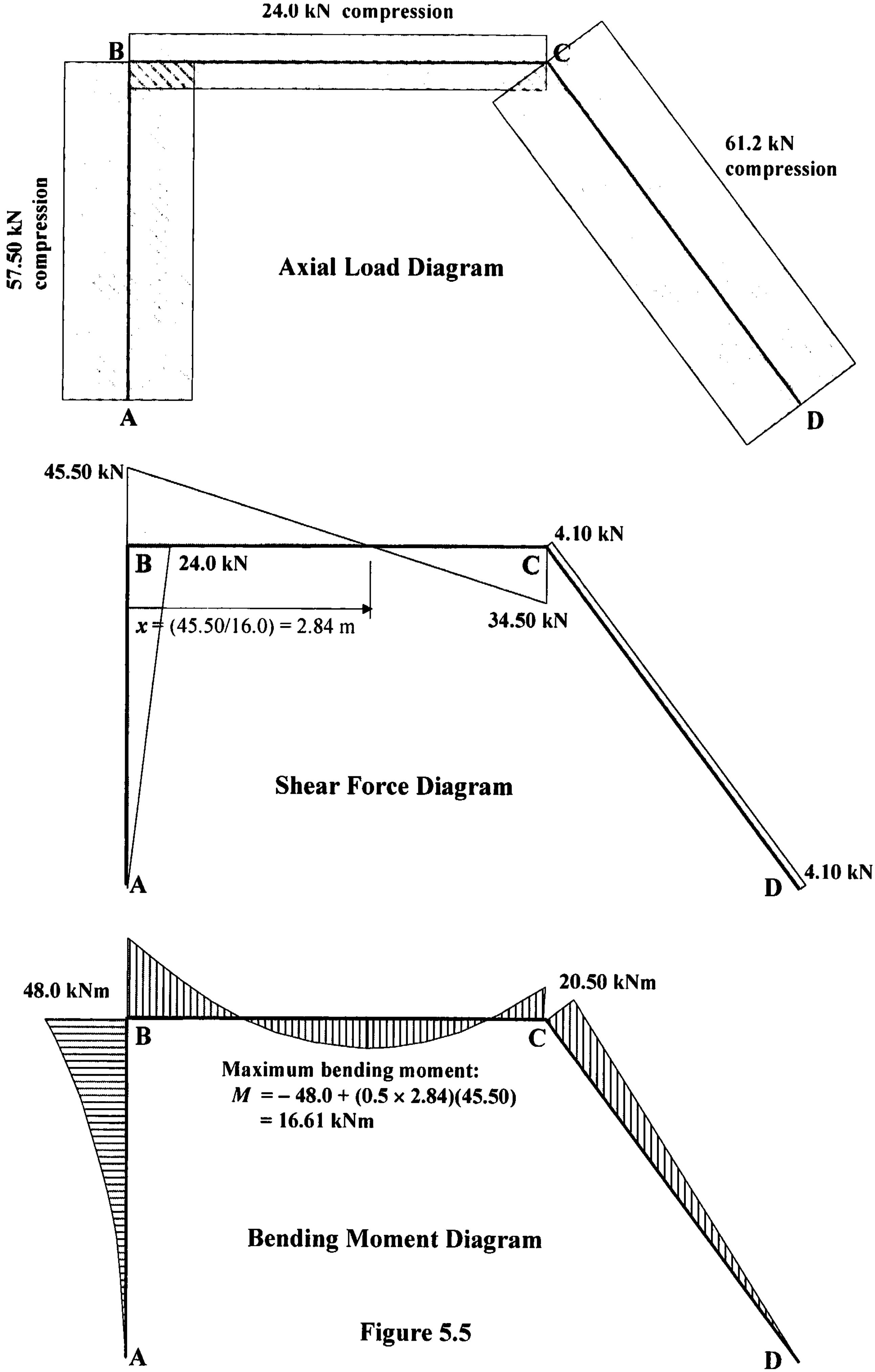

24.0 kN compression
B
C
61.2 kN
compression
57.50 kN
compression
Axial Load Diagram
A
D
45.50 kN
4.10 kN
B
24.0 kN
C
x = (45.50/16.0) = 2.84 m
34.50 kN
Shear Force Diagram
A
D
4.10 kN
48.0 kNm
20.50 kNm
B
C
Maximum bending moment:
M = – 48.0 + (0.5 × 2.84)(45.50)
= 16.61 kNm
Bending Moment Diagram
A
D

Figure 5.5

5.1.2 Example 5.2 Statically Determinate Rigid-Jointed Frame 2

A pitched-roof portal frame is pinned at supports A and H and members CD and DEF are pinned at the ridge as shown in Figure 5.6. For the loading indicated:

i) determine the support reactions and

ii) sketch the axial load, shear force and bending moment diagrams.

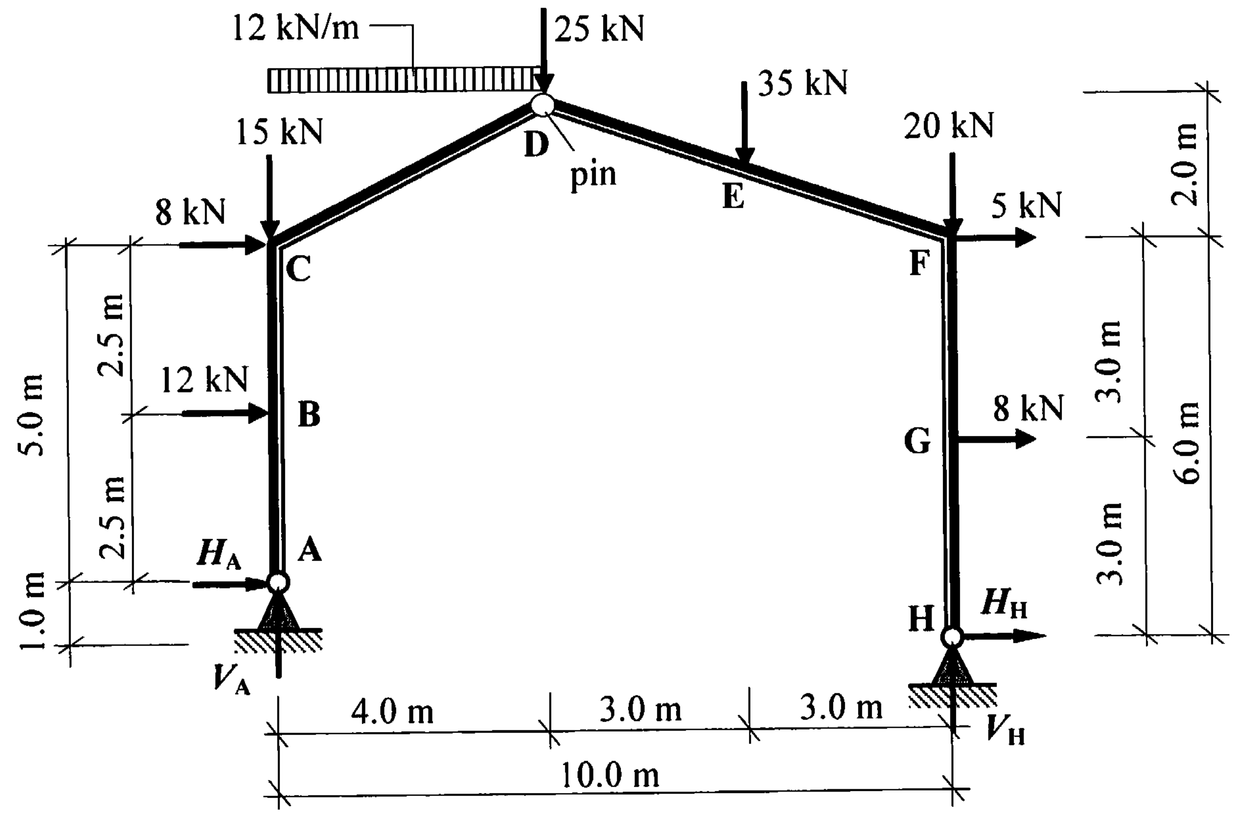


Figure 5.6

Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

+ve ↑ $\Sigma F_y = 0$

$V_A - 15.0 - (12.0 \times 4.0) - 25.0 - 35.0 - 20.0 + V_H = 0$ Equation (1)

+ve → $\Sigma F_x = 0$

$H_A + 12.0 + 8.0 + 5.0 + 8.0 + H_H = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$

$(12.0 \times 2.5) + (8.0 \times 5.0) + (12.0 \times 4.0)(2.0) + (25.0 \times 4.0) + (35.0 \times 7.0)$

$+ (20.0 \times 10.0) + (5.0 \times 5.0) + (8.0 \times 2.0) - (H_H \times 1.0) - (V_H \times 10.0) = 0$ Equation (3)

+ve ↻ $\Sigma M_{pin} = 0$ (right-hand side)

$+ (35.0 \times 3.0) + (20.0 \times 6.0) - (5.0 \times 2.0) - (8.0 \times 5.0) - (H_H \times 8.0) - (V_H \times 6.0) = 0$

Equation (4)

From Equation (3): $+ 752.0 - H_H - 10.0V_H = 0$ Equation (3a)

From Equation (4): $+ 175.0 - 8.0H_H - 6.0V_H = 0$ Equation (3b)

Solve equations 3(a) and 3(b) simultaneously: $V_H = +\ 78.93\ \text{kN}\ \uparrow$ $H_H = -\ 37.30\ \text{kN}\ \leftarrow$

From Equation (2): $H_A + 33.0 + H_H = 0$ $H_A = +\ 4.30\ \text{kN}\ \rightarrow$

From Equation (1): $V_A - 143.0 + V_H = 0$ $V_A = +\ 64.07\ \text{kN}\ \uparrow$

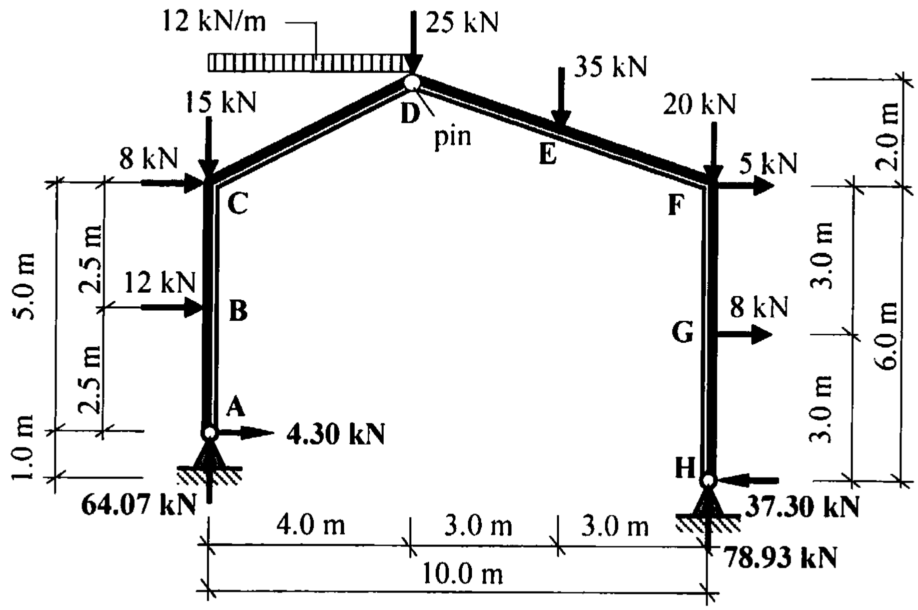


Figure 5.7

Assuming positive bending moments induce tension **inside** the frame:

$M_B = -\ (4.30 \times 2.5) = -\ 10.75\ \text{kNm}$

$M_C = -\ (4.30 \times 5.0) - (12.0 \times 2.5) = -\ 51.50\ \text{kNm}$

$M_D = \text{zero}$ (pin)

$M_E = -\ (20.0 \times 3.0) + (5.0 \times 1.0) + (8.0 \times 4.0) - (37.3 \times 7.0) + (78.93 \times 3.0)$

$= -\ 47.31\ \text{kNm}$

$M_F = +\ (8.0 \times 3.0) - (37.30 \times 6.0) = -\ 199.80\ \text{kNm}$

$M_G = -\ (37.30 \times 3.0) = -\ 111.90\ \text{kNm}$

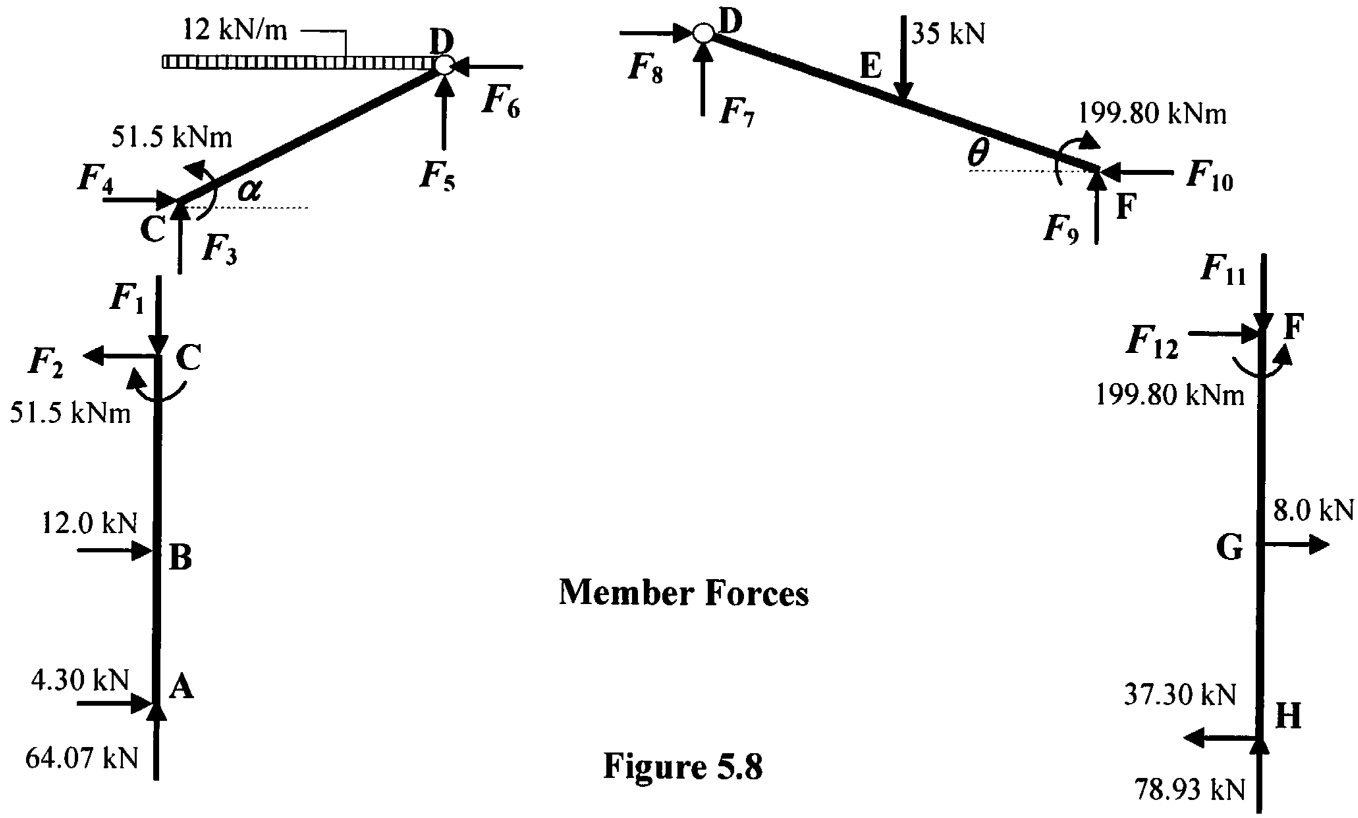


Figure 5.8

The values of the end–forces F_1 to F_{12} can be determined by considering the equilibrium of each member and joint in turn.

Consider member ABC:

+ve ↑ $\Sigma F_y = 0$ $\quad + 64.07 - F_1 = 0$ $\quad \therefore F_1 = 64.07$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad + 4.30 + 12.0 - F_2 = 0$ $\quad \therefore F_2 = 16.30$ kN ←

Consider joint C:

+ve ↑ $\Sigma F_y = 0$ There is an applied vertical load at joint C = 15 kN ↓

$- F_1 + F_3 = - 15.0$ $\quad \therefore F_3 = 49.07$ kN ↑

+ve → $\Sigma F_x = 0$ There is an applied horizontal load at joint C = 8 kN →

$- F_2 + F_4 = + 8.0$ $\quad \therefore F_4 = 24.30$ kN →

Consider member CD:

+ve ↑ $\Sigma F_y = 0$ $\quad + 49.07 - (12.0 \times 4.0) + F_5 = 0$ $\quad \therefore F_5 = - 1.07$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad + 24.30 - F_6 = 0$ $\quad \therefore F_6 = 24.30$ kN ←

Consider member FGH:

+ve ↑ $\Sigma F_y = 0$ $\quad + 78.93 - F_{11} = 0$ $\quad \therefore F_{11} = 78.93$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad - 37.30 + 8.0 + F_{12} = 0$ $\quad \therefore F_{12} = 29.30$ kN →

Consider joint F:

+ve ↑ $\Sigma F_y = 0$ There is an applied vertical load at joint F = 20 kN ↓

$F_{11} + F_9 = - 20.0$ $\quad \therefore F_9 = 58.93$ kN ↑

+ve → $\Sigma F_x = 0$ There is an applied horizontal load at joint F = 5 kN →

$+ F_{12} - F_{10} = + 5.0$ $\quad \therefore F_{10} = 24.30$ kN ←

Consider member DF:

+ve ↑ $\Sigma F_y = 0$ $\quad + 58.93 - 35.0 + F_7 = 0$ $\quad \therefore F_7 = 23.93$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad - 24.30 + F_8 = 0$ $\quad \therefore F_8 = 24.30$ kN →

The calculated values can be checked by considering the equilibrium at joint D.

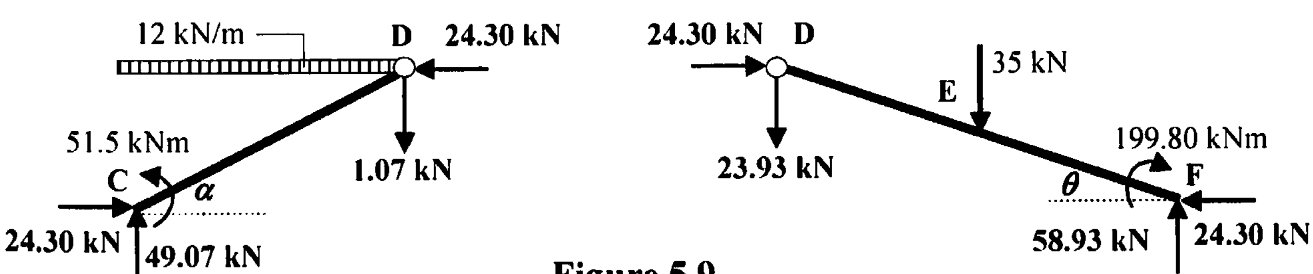


Figure 5.9

+ve → ΣF_x $\quad - 24.30 + 24.30 = 0$

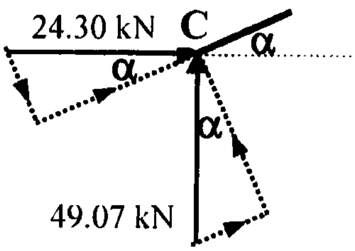
+ve ↑ ΣF_y $\quad - 1.07 - 23.93 = - 25.0$ kN (equal to the applied vertical load at D).

The axial force and shear force in member CD can be found from:

Axial load = +/– (Horizontal force × Cosα) +/– (Vertical force × Sinα)
Shear force = +/– (Horizontal force × Sinα) +/– (Vertical force × Cosα)
The signs are dependent on the directions of the respective forces.

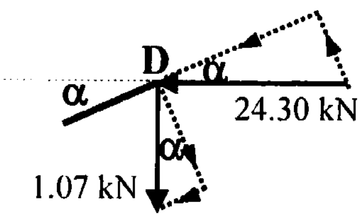
Similarly with θ for member DEF.

Member CD:



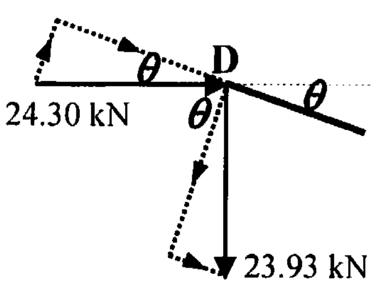
$\alpha = \tan^{-1}(2.0/4.0) = 26.565°$
Cos α = 0.894;   Sin α = 0.447

Assume axial compression to be positive.
At joint C
Axial force = + (24.30 × Cosα) + (49.07× Sinα) = + 43.66 kN
Shear force = – (24.30 × Sinα) + (49.07× Cosα) = + 33.01 kN



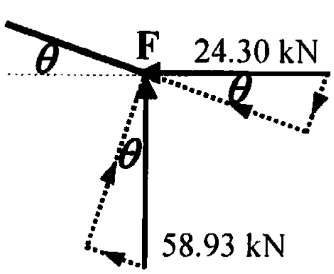
At joint D
Axial force = + (24.30 × Cosα) + (1.07× Sinα) = + 22.20 kN
Shear force = – (24.30 × Sinα) + (49.07× Cosα) = – 9.91 kN

Member DEF:



$\theta = \tan^{-1}(2.0/6.0) = 18.435°$
Cos θ = 0.947;   Sin θ = 0.316

Assume axial compression to be positive.
At joint D
Axial force = + (24.30 × Cosθ) + (23.93× Sinθ) = + 30.57 kN
Shear force = + (24.30 × Sinθ) – (23.93 × Cosθ) = + 14.98 kN



At joint F
Axial force = + (24.30 × Cosθ) + (58.93× Sinθ) = + 41.63 kN
Shear force = – (24.30 × Sinθ) + (58.93× Cosθ) = + 48.13 kN

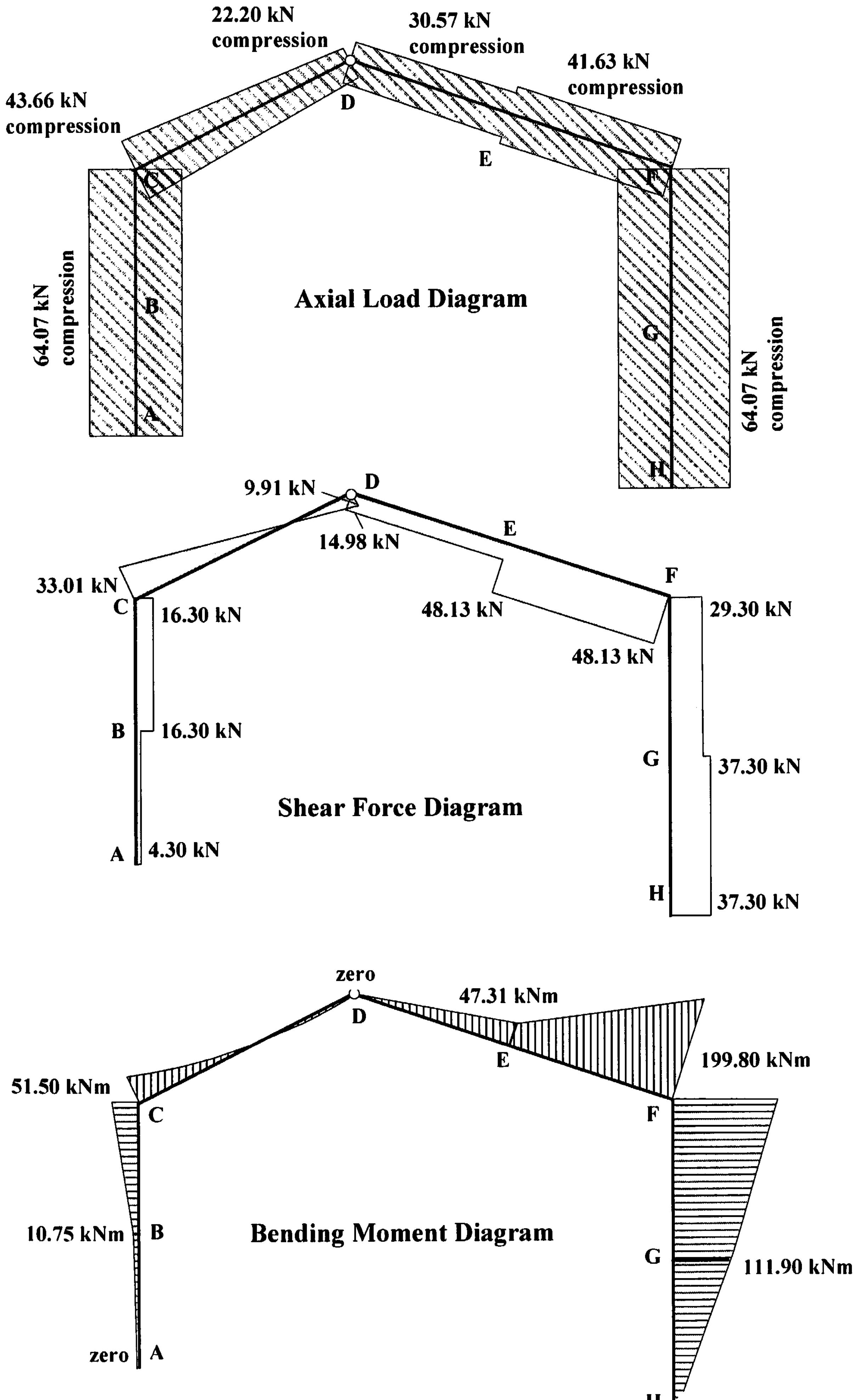
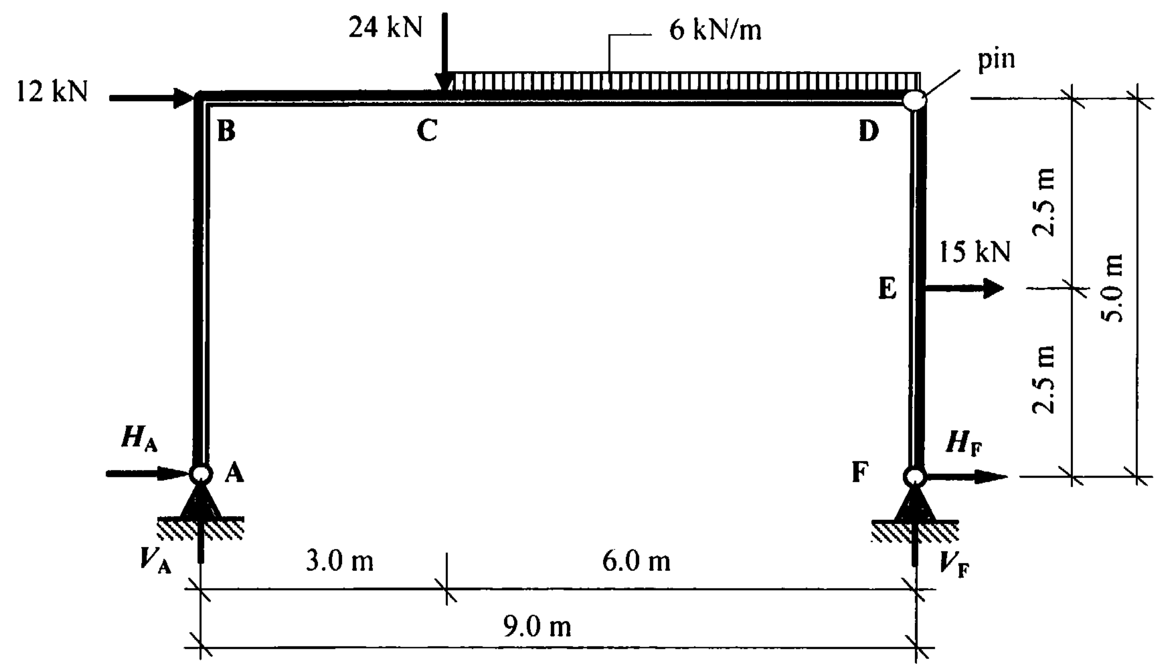

22.20 kN
compression
30.57 kN
compression
41.63 kN
compression
43.66 kN
compression
D
E
C
F
64.07 kN
compression
B
Axial Load Diagram
G
A
64.07 kN
compression
H
9.91 kN
D
14.98 kN
E
33.01 kN
F
C
16.30 kN
48.13 kN
29.30 kN
48.13 kN
B
16.30 kN
G
37.30 kN
Shear Force Diagram
A
4.30 kN
H
37.30 kN
zero
47.31 kNm
D
E
199.80 kNm
51.50 kNm
C
F
10.75 kNm
B
Bending Moment Diagram
G
111.90 kNm
zero
A
H
zero

Figure 5.10

Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.1 **Page No. 1**



Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

+ve ↑ $\Sigma F_y = 0$
$V_A - 24.0 - (6.0 \times 6.0) + V_F = 0$ Equation (1)

+ve → $\Sigma F_x = 0$
$H_A + 12.0 + 15.0 + H_F = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$
$(12.0 \times 5.0) + (24.0 \times 3.0) + (6.0 \times 6.0)(6.0) + (15.0 \times 2.5) - (V_F \times 9.0) = 0$
Equation (3)

+ve ↻ $\Sigma M_{pin} = 0$ (right-hand side)
$-(15.0 \times 2.5) - (H_F \times 5.0) = 0$ Equation (4)

From Equation (4): $-37.5 - 5.0H_F = 0$ **$H_F = -7.5$ kN** ←

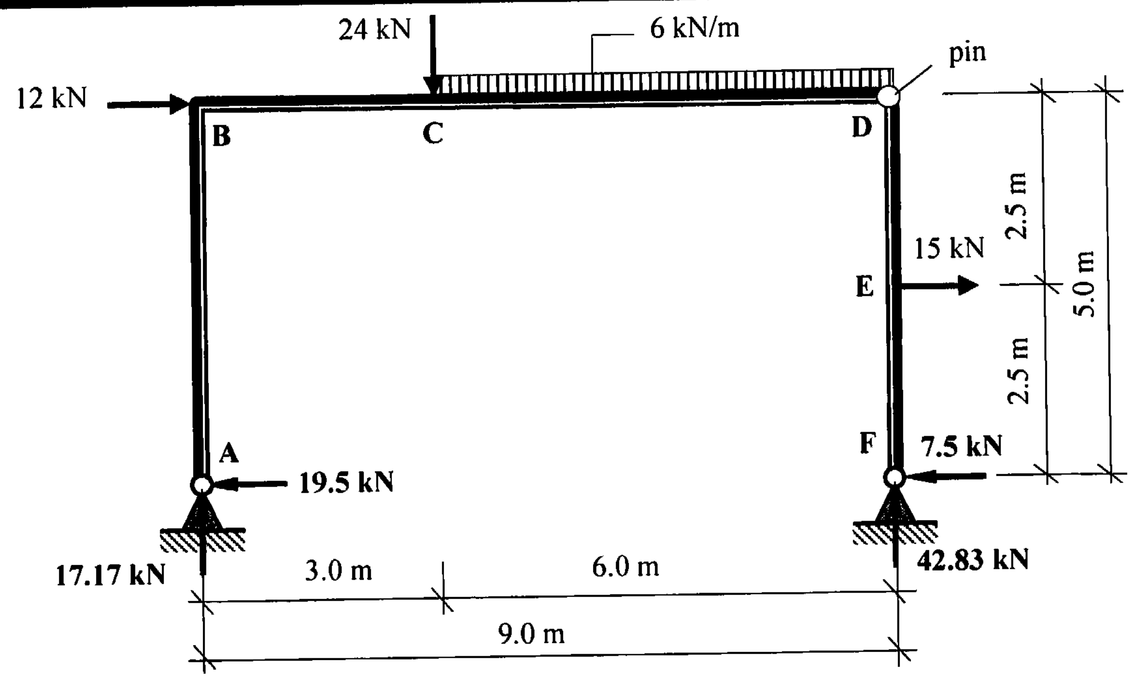
From Equation (2): $H_A + 27.0 - 7.5 = 0$ **$H_A = -19.5$ kN** ←

From Equation (3): $385.5 - 9.0V_F = 0$ **$V_F = +42.83$ kN** ↑

From Equation (1): $V_A - 60.0 + 42.83 = 0$ **$V_A = +17.17$ kN** ↑

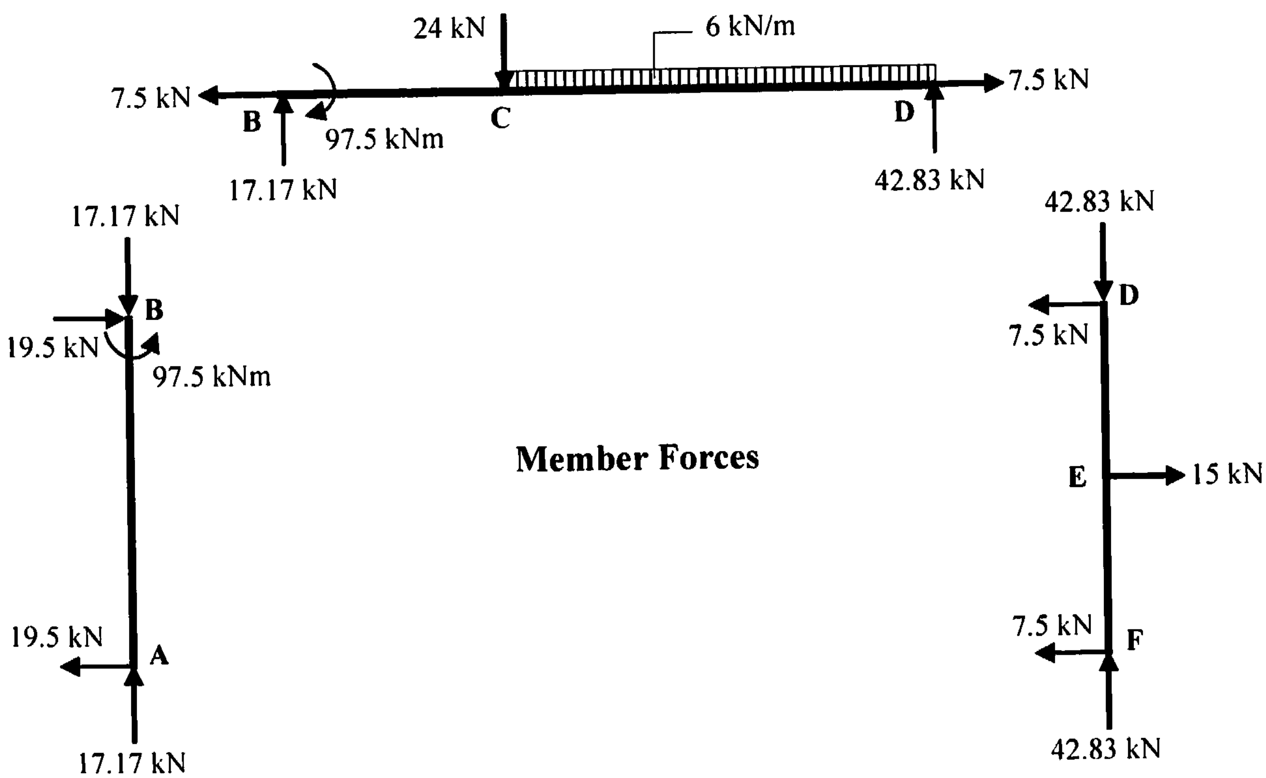
Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.1

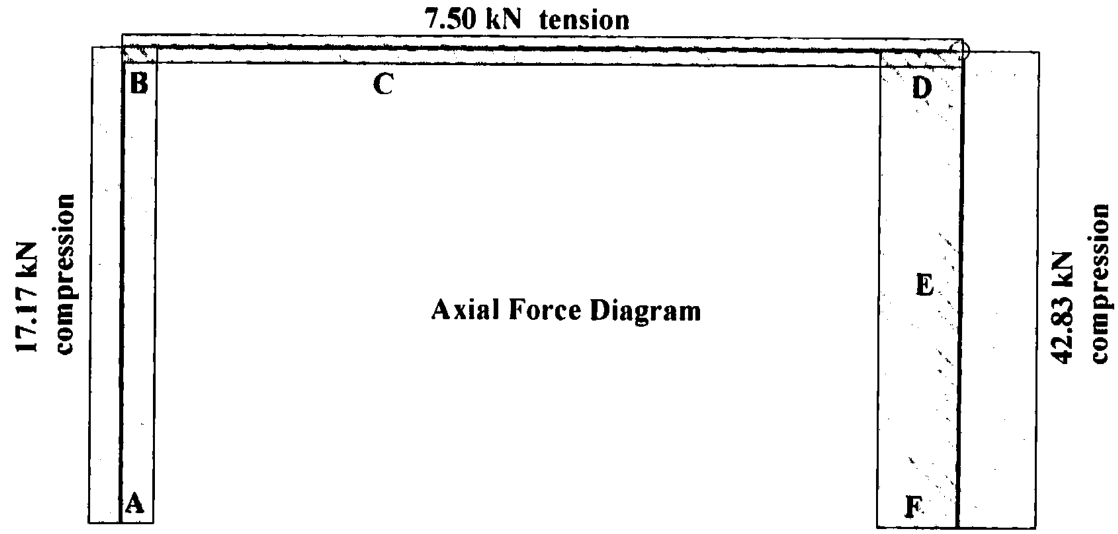


Assuming positive bending moments induce tension **inside** the frame:

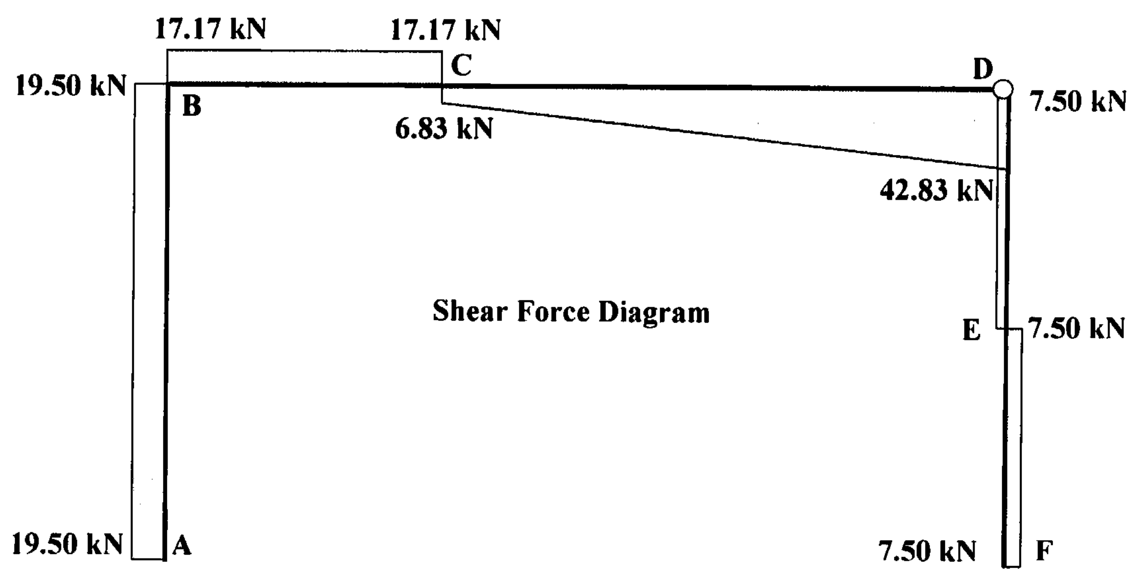
$M_B = +(19.5 \times 5.0) = +97.50$ kNm
$M_C = +(17.17 \times 3.0) + (19.5 \times 5.0) = +149.0$ kNm
$M_D =$ zero (pin)
$M_E = -(7.5 \times 2.5) = -18.75$ kNm



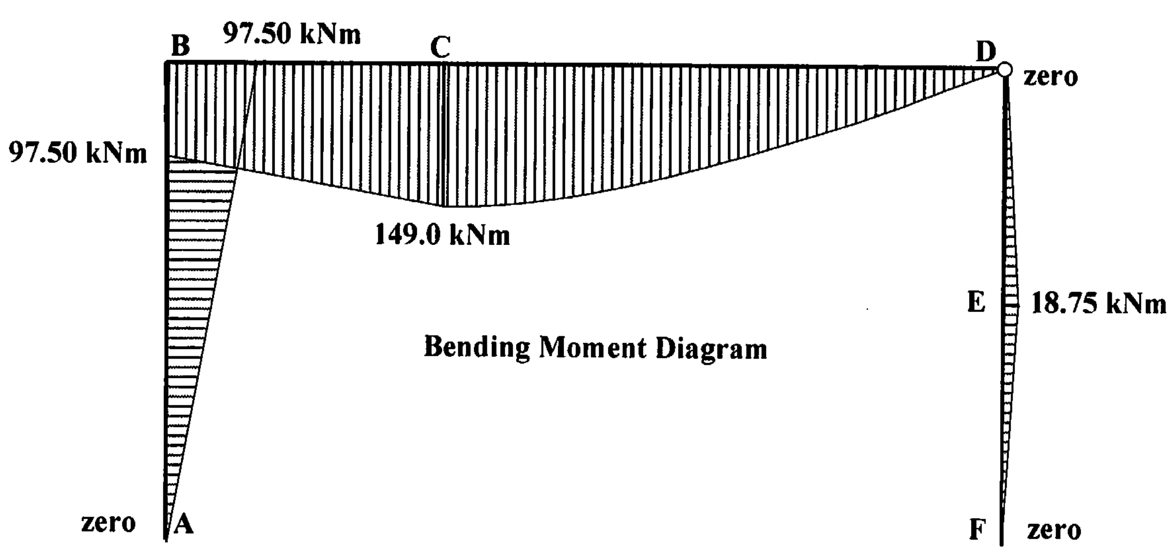
Member Forces



Axial Force Diagram



Shear Force Diagram

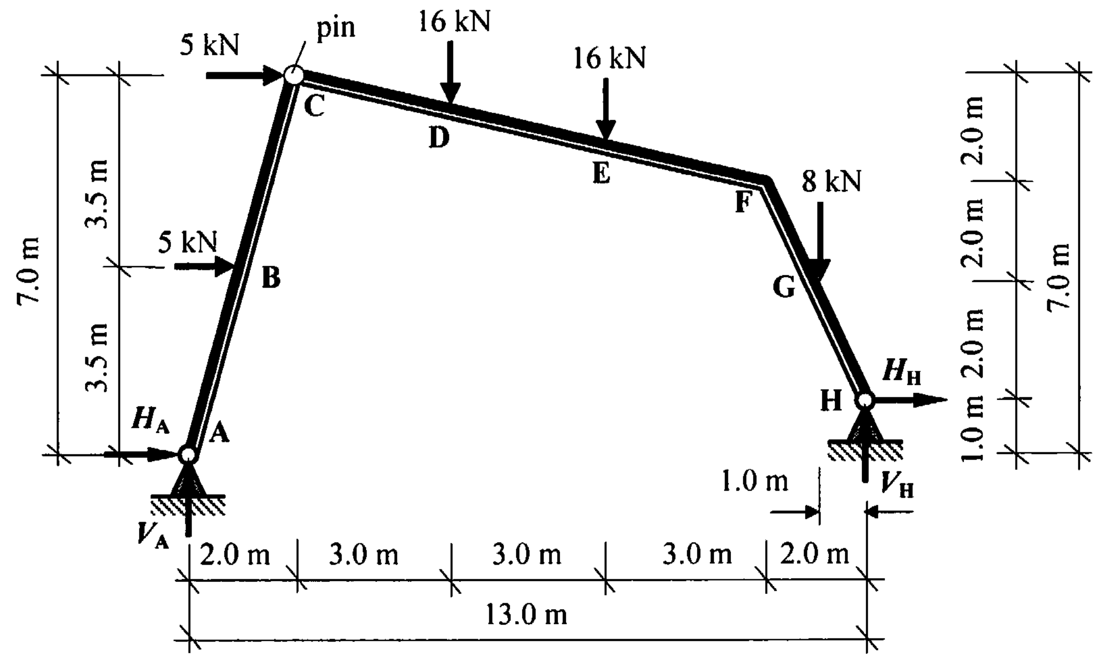


Bending Moment Diagram

Solution

Topic: Statically Determinate Rigid-Jointed Frames

Problem Number: 5.2 **Page No. 1**



Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

+ve ↑ $\Sigma F_y = 0$

$V_A - 16.0 - 16.0 - 8.0 + V_H = 0$ Equation (1)

+ve → $\Sigma F_x = 0$

$H_A + 5.0 + 5.0 + H_H = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$

$(5.0 \times 3.5) + (5.0 \times 7.0) + (16.0 \times 5.0) + (16.0 \times 8.0) + (8.0 \times 12.0) - (V_H \times 13.0)$
$+ (H_H \times 1.0) = 0$ Equation (3)

+ve ↻ $\Sigma M_{pin} = 0$

$+ (16.0 \times 3.0) + (16.0 \times 6.0) + (8.0 \times 10.0) - (V_H \times 11.0) - (H_H \times 6.0) = 0$ Equation (4)

From Equation (3): $+356.5 - 13.0V_H + H_H = 0$ Equation (3a)
From Equation (4): $+224.0 - 11.0V_H - 6.0H_H = 0$ Equation (3b)

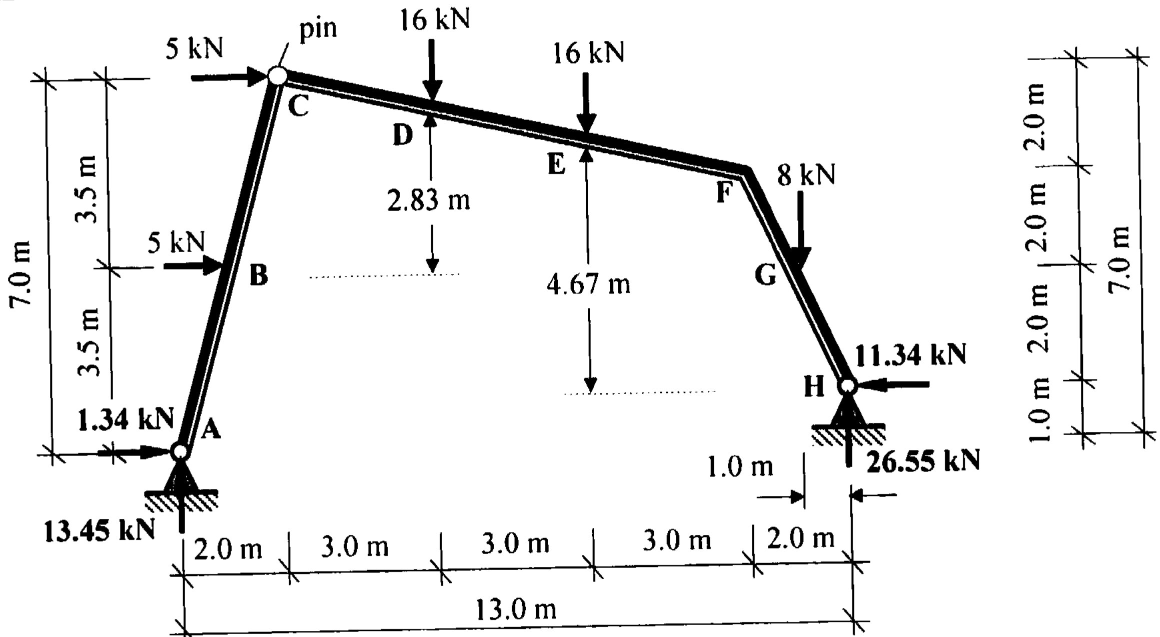
Solve equations 3(a) and 3(b) simultaneously: $\boldsymbol{V_H = +26.55}$ **kN** ↑ $\boldsymbol{H_H = -11.34}$ **kN** ←

From Equation (2): $H_A + 10.0 + H_H = 0$ $\boldsymbol{H_A = +1.34}$ **kN** →

From Equation (1): $V_A + 64.0 + V_H = 0$ $\boldsymbol{V_A = +13.45}$ **kN** ↑

Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.2



Assuming positive bending moments induce tension **inside** the frame:

$M_B = -(1.34 \times 3.5) + (13.45 \times 1.0) = +8.76$ kNm

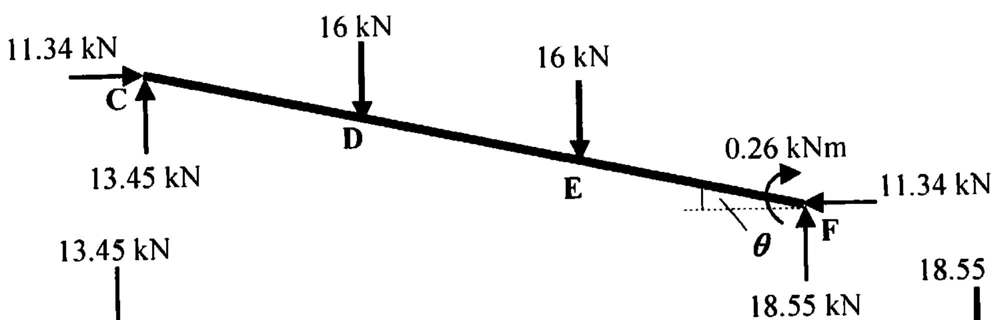
$M_C =$ zero (pin)

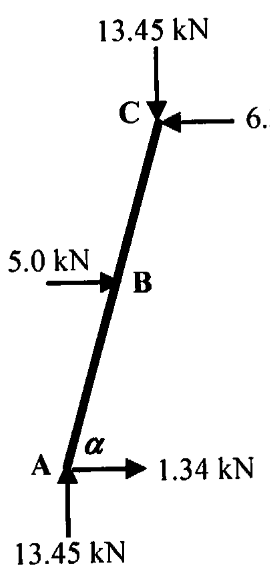
$M_D = +(13.45 \times 5.0) - (1.34 \times 6.33) - (5.0 \times 2.83) + (5.0 \times 0.67) = +47.97$ kNm

$M_E = +(26.55 \times 5.0) - (11.34 \times 4.67) - (8.0 \times 4.0) = +47.79$ kNm

$M_F = -(8.0 \times 1.0) - (11.34 \times 4.0) + (26.55 \times 2.0) = -0.26$ kNm

$M_G = -(11.34 \times 2.0) + (26.55 \times 1.0) = +3.87$ kNm



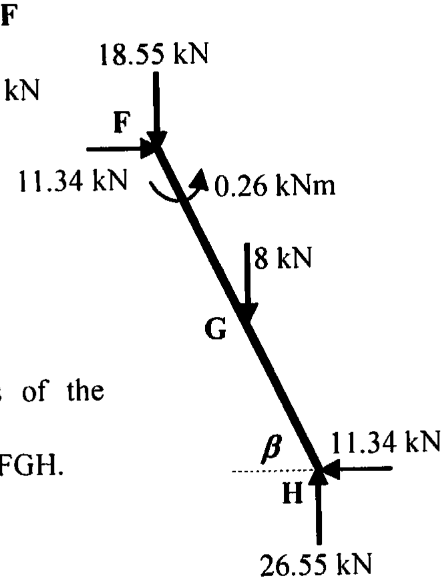


Note: For member ABC.

Axial load = +/– (Horizontal force × Cosα)
+/– (Vertical force × Sinα)

Shear force = +/– (Horizontal force × Sinα)
+/– (Vertical force × Cosα)

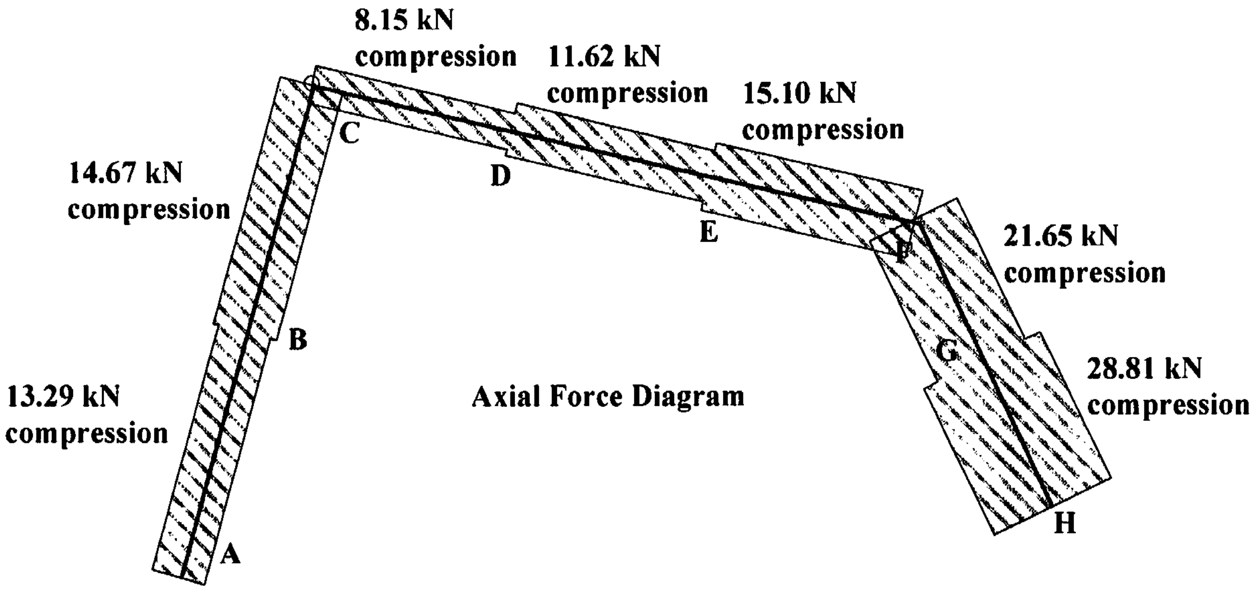
The signs are dependent on the directions of the respective forces.
Similarly with θ and β for member CDEF and FGH.



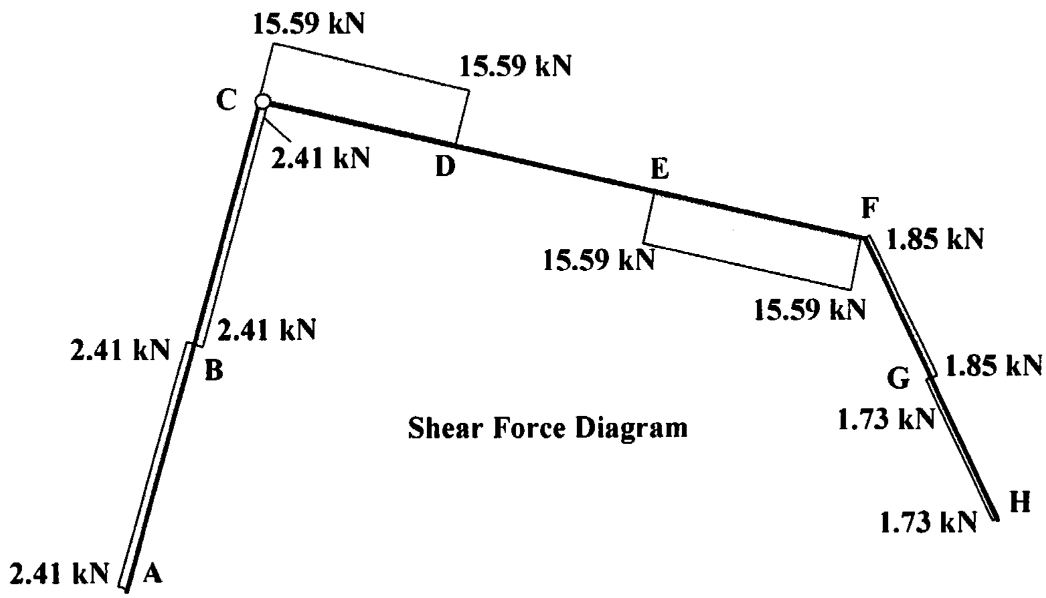
Member Forces

Solution

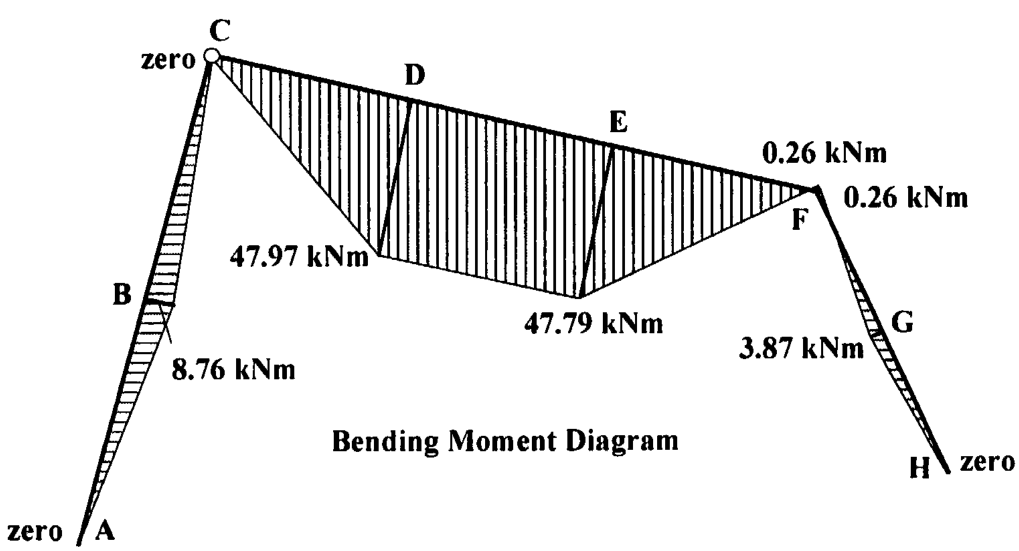
Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.2



Axial Force Diagram



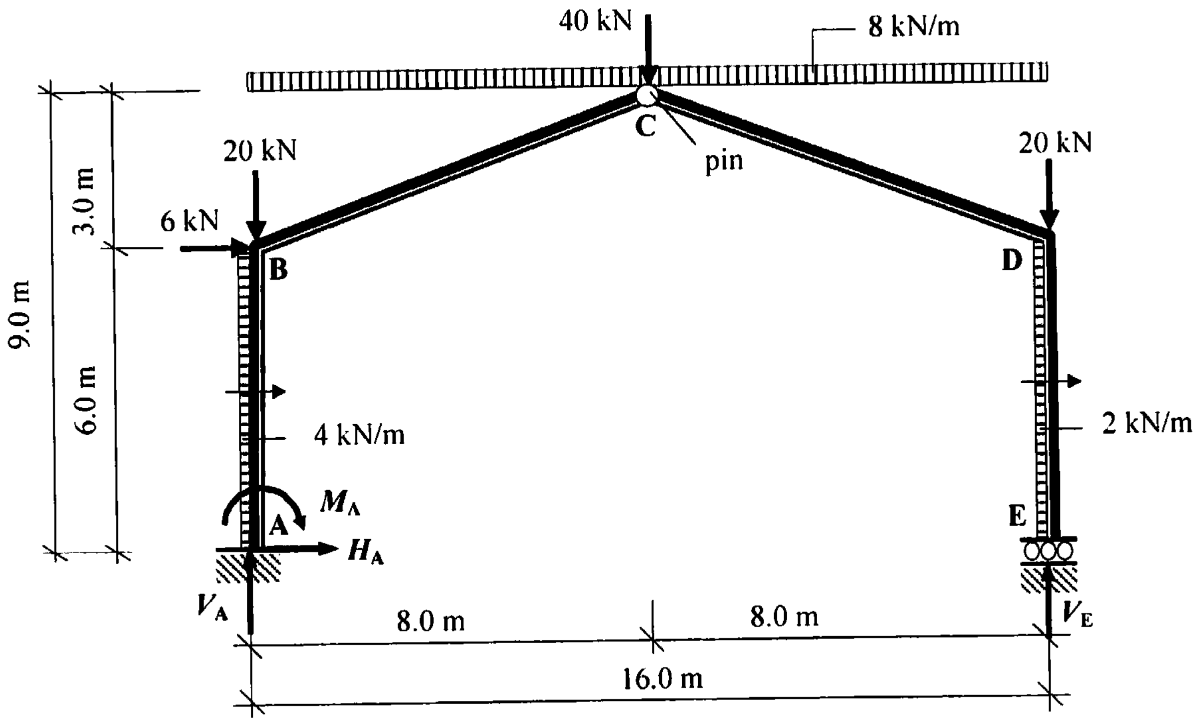
Shear Force Diagram



Bending Moment Diagram

Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.3



Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

+ve ↑ $\Sigma F_y = 0$

$V_A - 20.0 - (8.0 \times 16.0) - 40.0 - 20.0 + V_E = 0$ Equation (1)

+ve → $\Sigma F_x = 0$

$H_A + (4.0 \times 6.0) + 6.0 + (2.0 \times 6.0) = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$

$M_A + (4.0 \times 6.0)(3.0) + (6.0 \times 6.0) + (8.0 \times 16.0)(8.0) + (40.0 \times 8.0) + (20.0 \times 16.0)$
$+ (2.0 \times 6.0)(3.0) - (V_E \times 16.0) = 0$ Equation (3)

+ve ↻ $\Sigma M_{pin} = 0$

$+ (8.0 \times 8.0)(4.0) + (20.0 \times 8.0) - (2.0 \times 6.0)(6.0) - (V_E \times 8.0) = 0$ Equation (4)

From Equation (2): $H_A + 42.0 = 0$ **$H_A = -42.0$ kN** ←

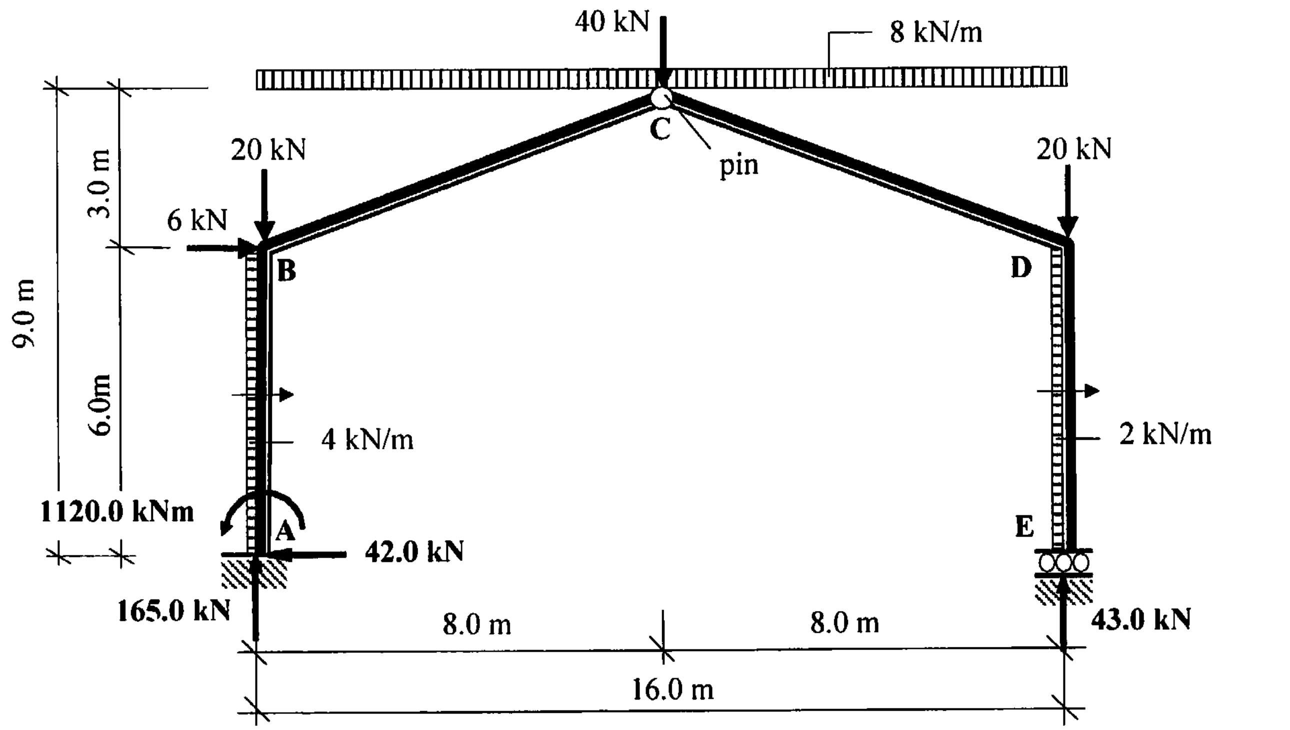
From Equation (4): $+344.0 - 8.0V_E = 0$ **$V_E = +43.0$ kN** ↑

From Equation (3): $M_A + 1808.0 - (43.0 \times 16.0) = 0$ **$M_A = -1120.0$ kNm** ↺

From Equation (1): $V_A - 208.0 + 43.0 = 0$ **$V_A = +165.0$ kN** ↑

Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.3



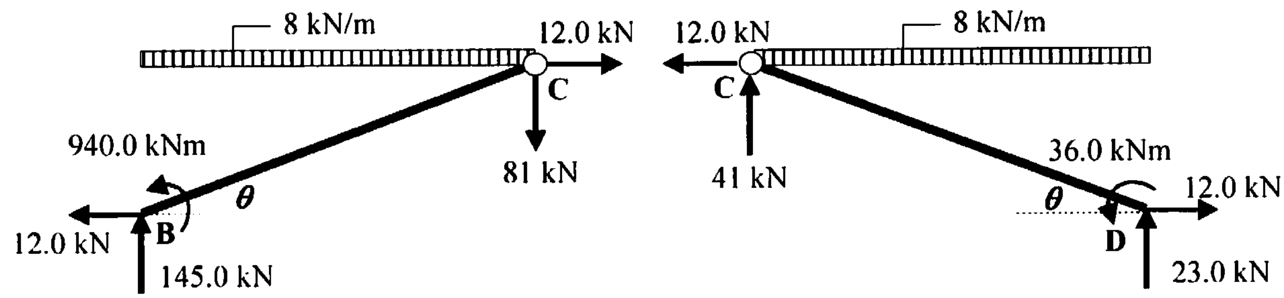
Assuming positive bending moments induce tension **inside** the frame:

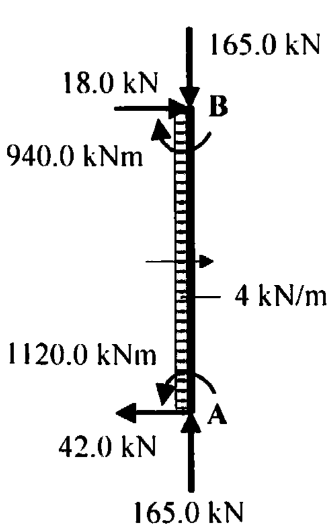
$M_A = -\ 1120.0$ kNm

$M_B = -\ 1120.0 - (4.0 \times 6.0)(3.0) + (42.0 \times 6.0) = -\ 940.0$ kNm

M_C = zero (pin)

$M_D = +\ (2.0 \times 6.0)(3.0) = +\ 36.0$ kNm



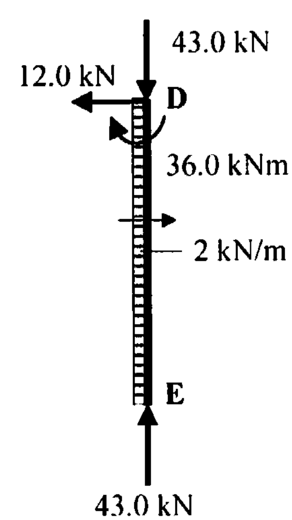


Note: For members BC and CD.

Axial load = +/– (Horizontal force × Cosθ)
+/– (Vertical force × Sinθ)

Shear force = +/– (Horizontal force × Sinθ)
+/– (Vertical force × Cosθ)

The signs are dependent on the directions of the respective forces.

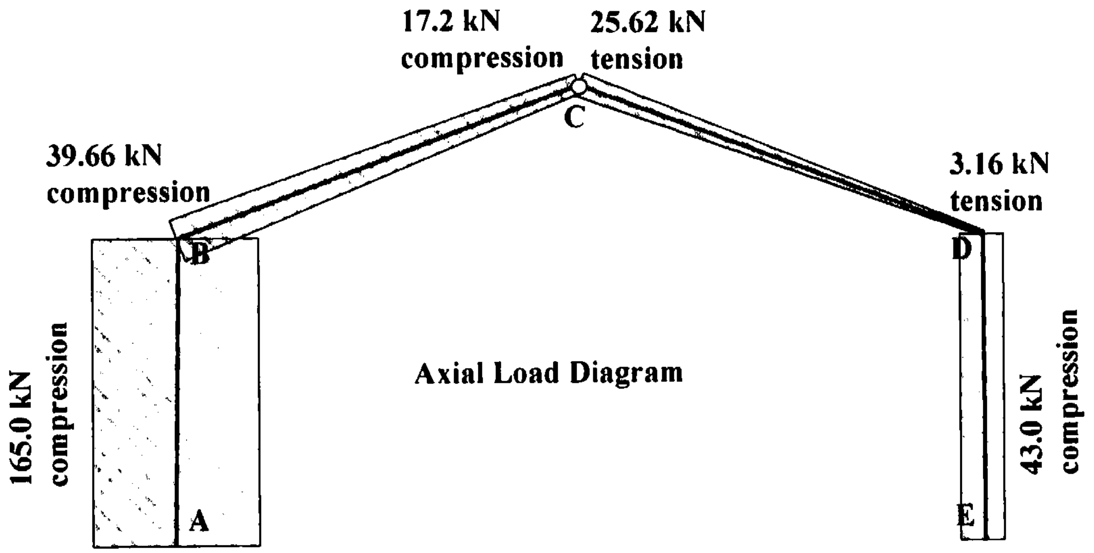


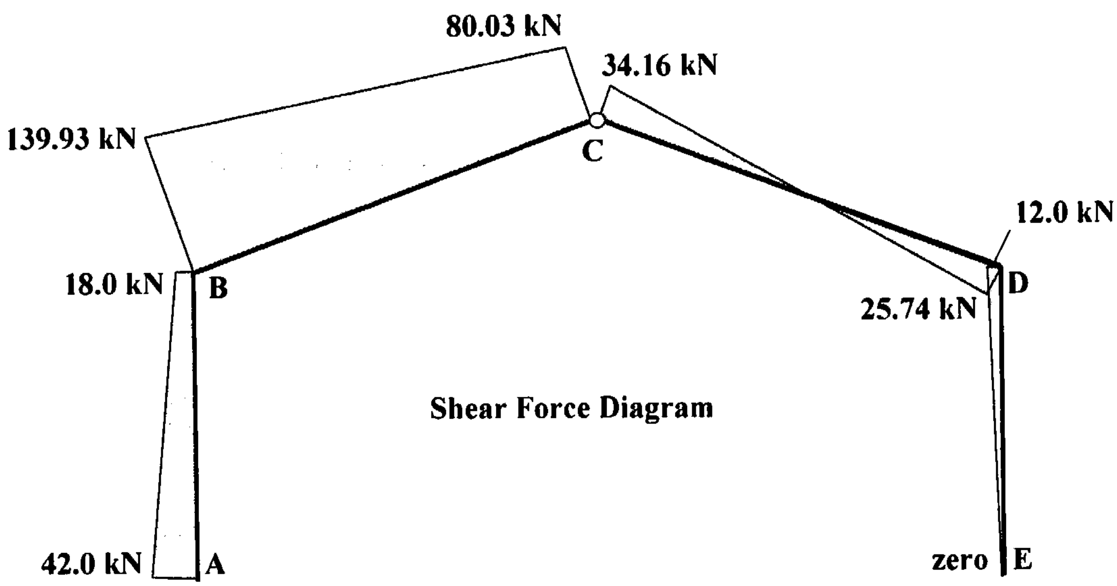
Member Forces

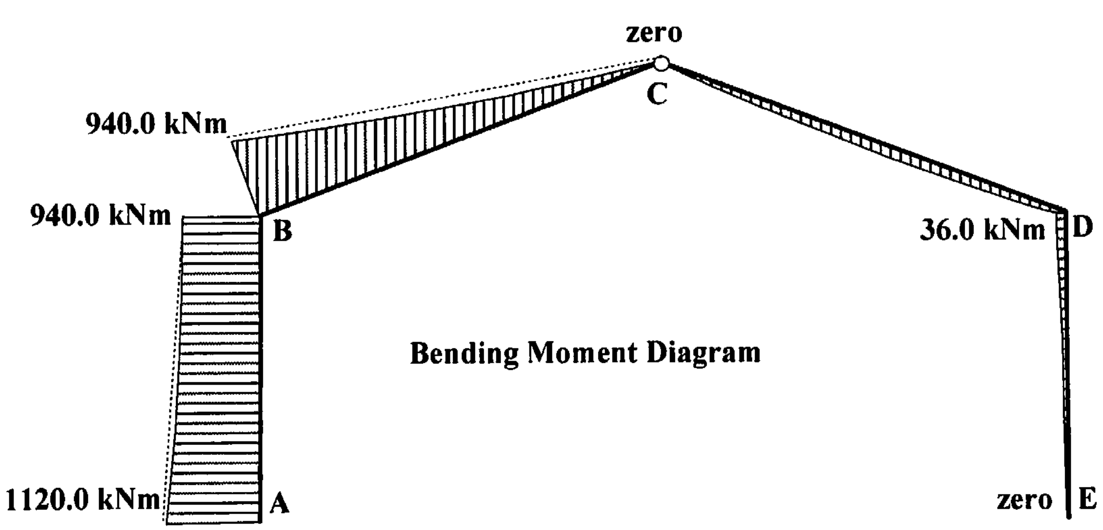
Solution

Topic: Statically Determinate Rigid-Jointed Frames

Problem Number: 5.3

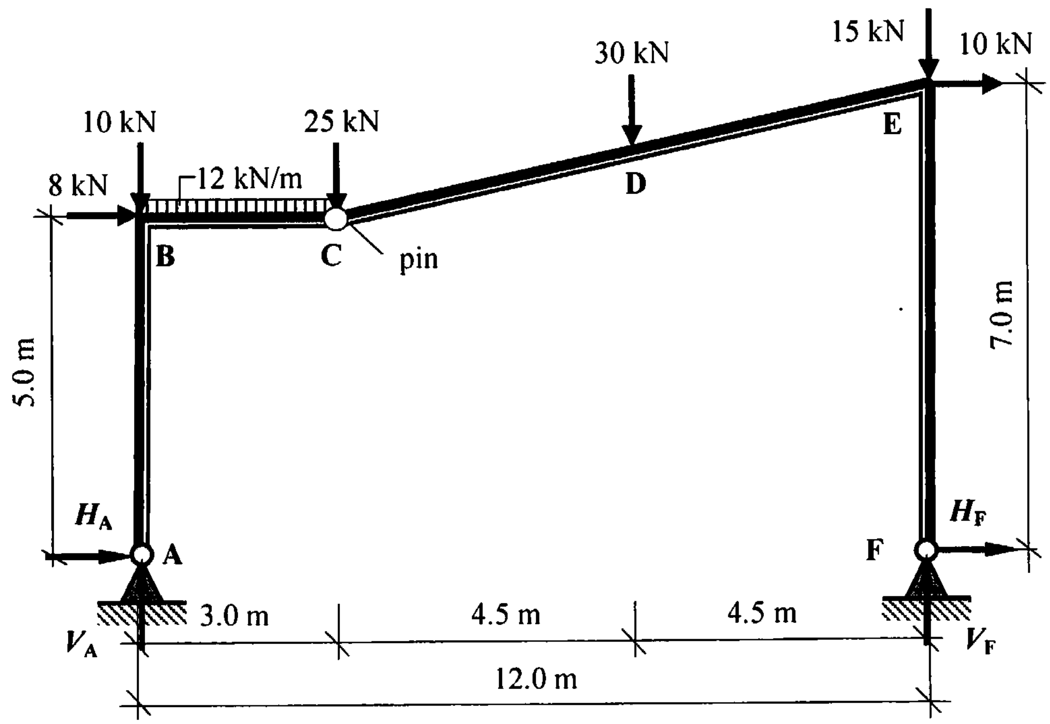






Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.4



Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

+ve ↑ $\Sigma F_y = 0$

$V_A - 10.0 - (12.0 \times 3.0) - 25.0 - 30.0 - 15.0 + V_F = 0$ Equation (1)

+ve → $\Sigma F_x = 0$

$H_A + 8.0 + 10.0 + H_F = 0$ Equation (2)

+ve ↻ $\Sigma M_A = 0$

$(8.0 \times 5.0) + (12.0 \times 3.0)(1.5) + (25.0 \times 3.0) + (30.0 \times 7.5) + (15.0 \times 12.0)$
$+ (10.0 \times 7.0) - (V_F \times 12.0) = 0$ Equation (3)

+ve ↻ $\Sigma M_{pin} = 0$

$+ (V_A \times 3.0) - (H_A \times 5.0) - (10.0 \times 3.0) - (12.0 \times 3.0)(1.5) = 0$ Equation (4)

From Equation (3): $2710.0 - 12.0V_F = 0$ **$V_F = + 53.67$ kN** ↑

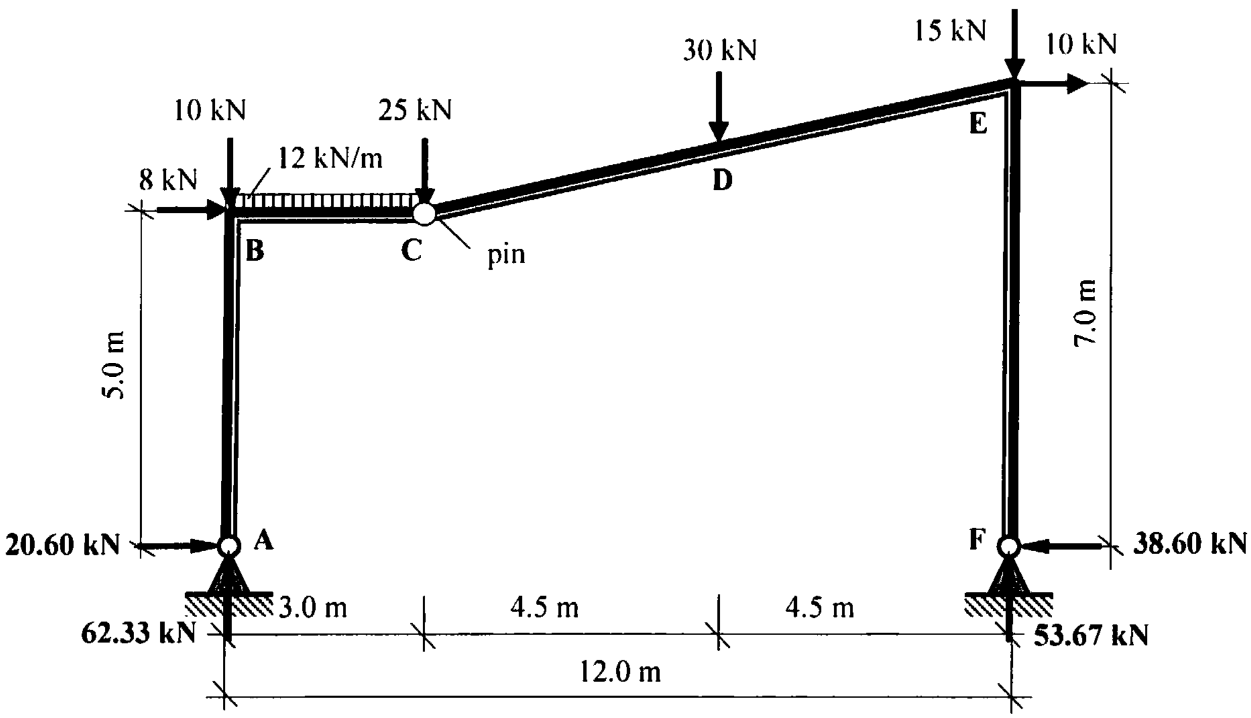
From Equation (1): $V_A - 116.0 + 53.67 = 0$ **$V_A = + 62.33$ kN** ↑

From Equation (4): $+ (62.33 \times 3.0) - 5.0H_A - 84.0 = 0$ **$H_A = + 20.60$ kN** →

From Equation (2): $+ 20.60 + 18.0 + H_F = 0$ **$H_F = - 38.60$ kN** ←

Solution

Topic: Statically Determinate Rigid-Jointed Frames
Problem Number: 5.4



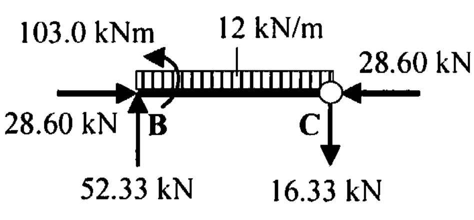
Assuming positive bending moments induce tension **inside** the frame:

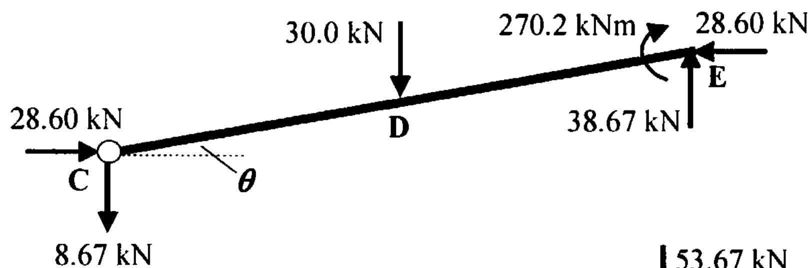
$M_B = -(20.60 \times 5.0) = -103.0$ kNm

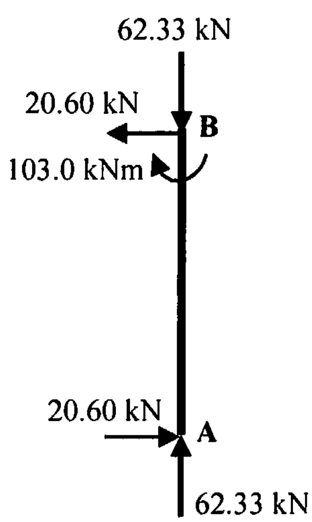
M_C = zero (pin)

$M_D = -(15.0 \times 4.5) - (10.0 \times 1.0) - (38.60 \times 6.0) + (53.67 \times 4.5) = -67.59$ kNm

$M_E = -(38.60 \times 7.0) = -270.2$ kNm





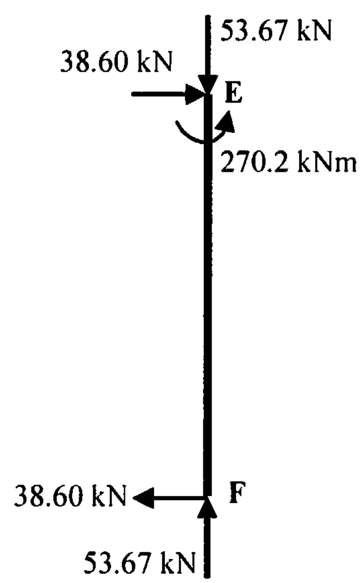


Note: For member CDE.

Axial load = +/– (Horizontal force × Cosθ)
+/– (Vertical force × Sinθ)

Shear force = +/– (Horizontal force × Sinθ)
+/– (Vertical force × Cosθ)

The signs are dependent on the directions of the respective forces.

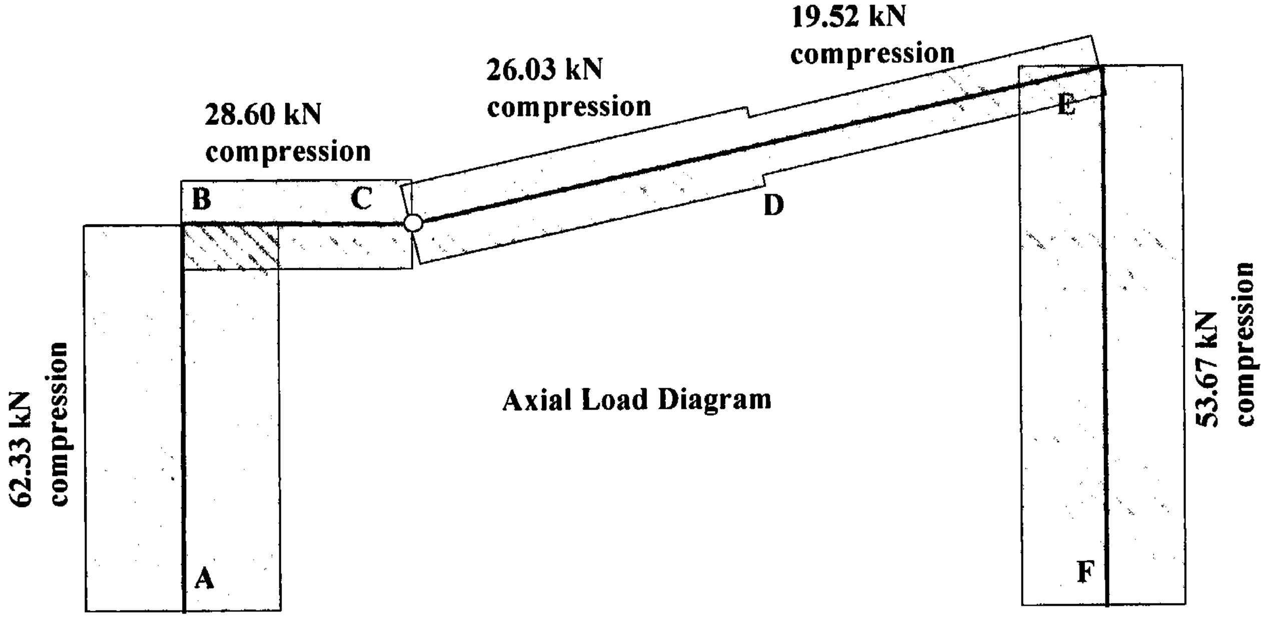


Member Forces

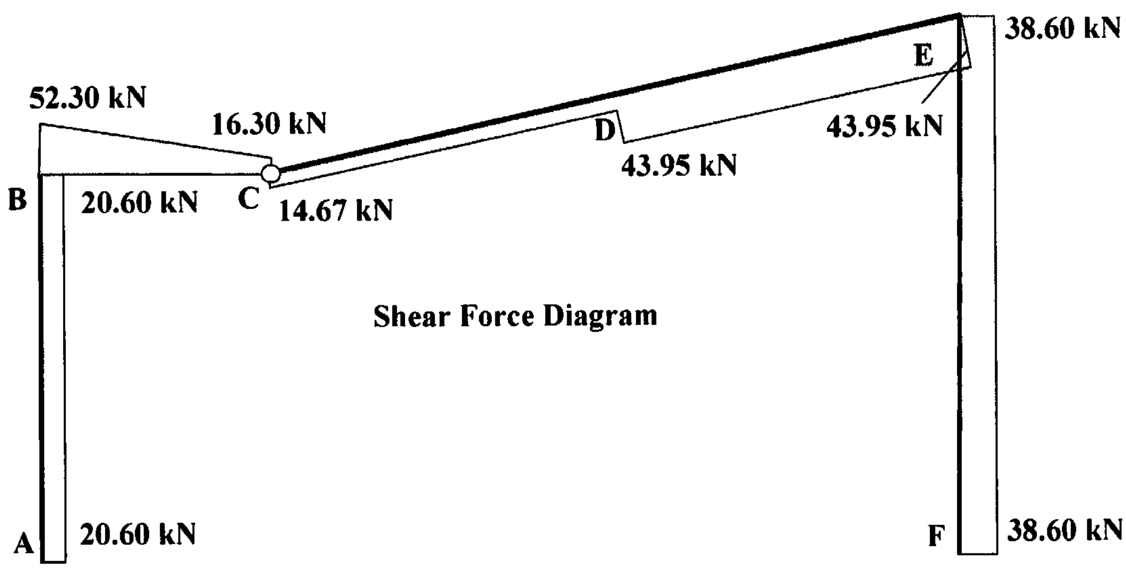
Solution

Topic: Statically Determinate Rigid-Jointed Frames

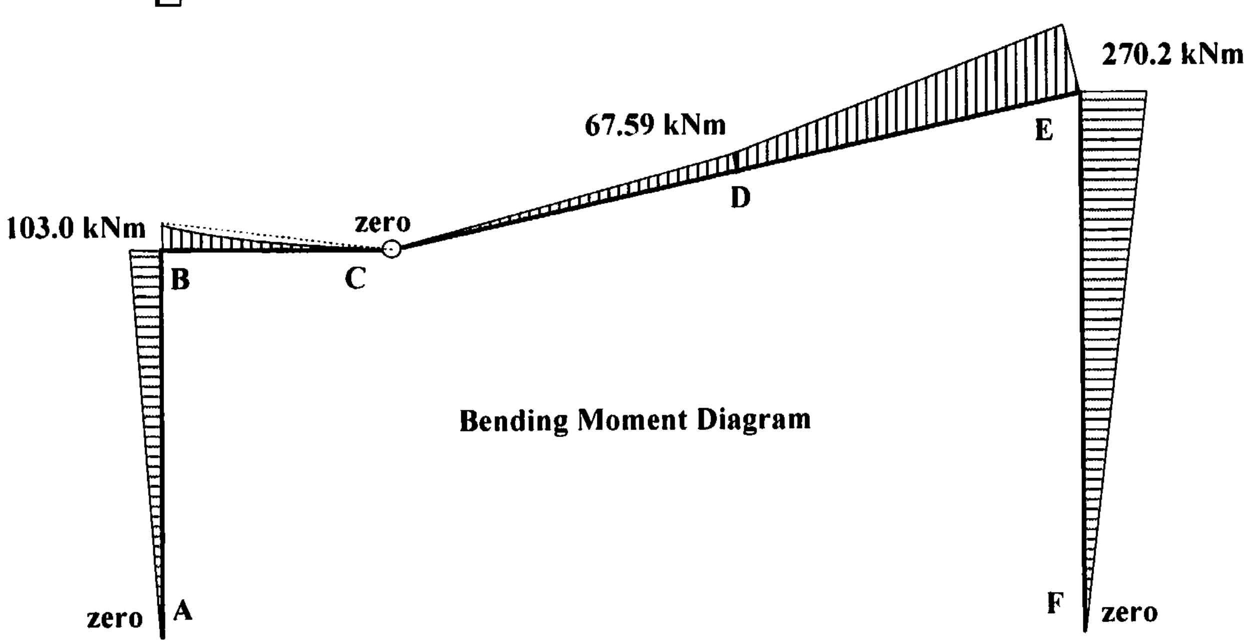
Problem Number: 5.4 **Page No. 3**



Axial Load Diagram



Shear Force Diagram



Bending Moment Diagram