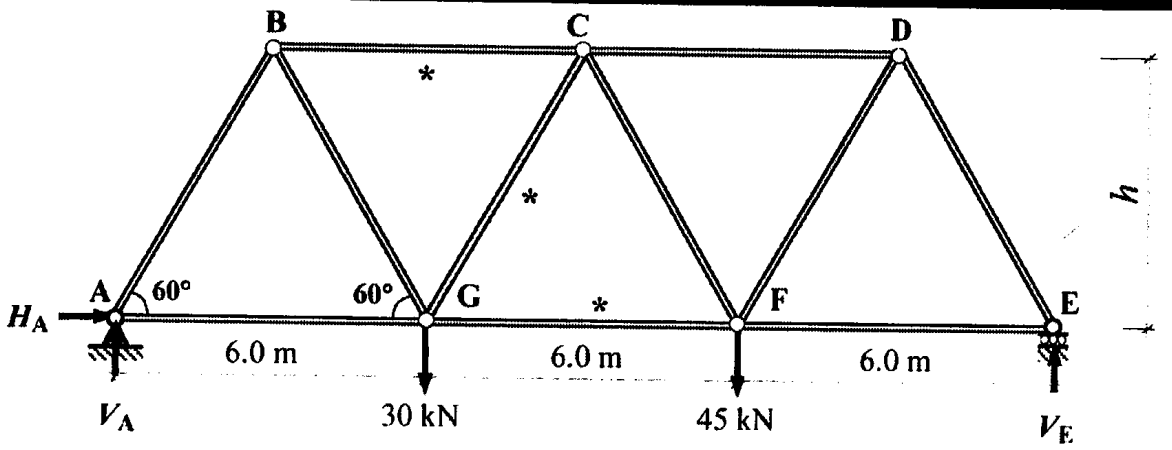
Solution

Topic: Pin-Jointed Frames – Method of Sections

Problem Number: 3.1



$h = (3.0 \times \text{Tan } 60°) = 5.196$ m

Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0$ $\quad + (30 \times 6.0) + (45.0 \times 12.0) - (V_E \times 18.0) = 0$ Equation (1)

$\therefore$ **$V_E = +40.0$ kN** ↑

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0$ $\quad + H_A = 0$ Equation (2)

$\therefore$ **H_A = zero**

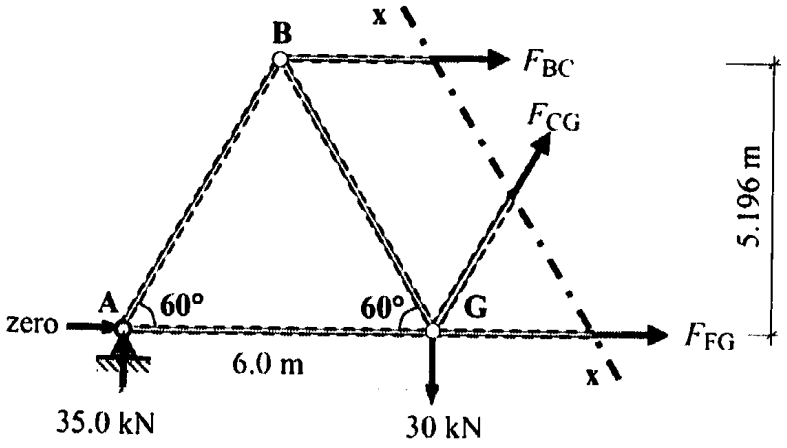
Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $\quad + V_A - 30.0 - 45.0 + V_E = 0$ Equation (3)

$V_A = 30.0 + 45.0 - 40.0$

$\therefore$ **$V_A = +35.0$ kN** ↑

Consider a section x–x through members BC, CG and FG:



Readers should consider the equilibrium of the right-hand-side of the section x-x and confirm the values for the unknown forces F_{BC}, F_{CG} and F_{FG}.

+ve ↻ $\Sigma M_G = 0$ $\quad + (35.0 \times 6.0) + (F_{BC} \times 5.196) = 0$ $\therefore$ **$F_{BC} = -40.42$ kN (Strut)** ⟷

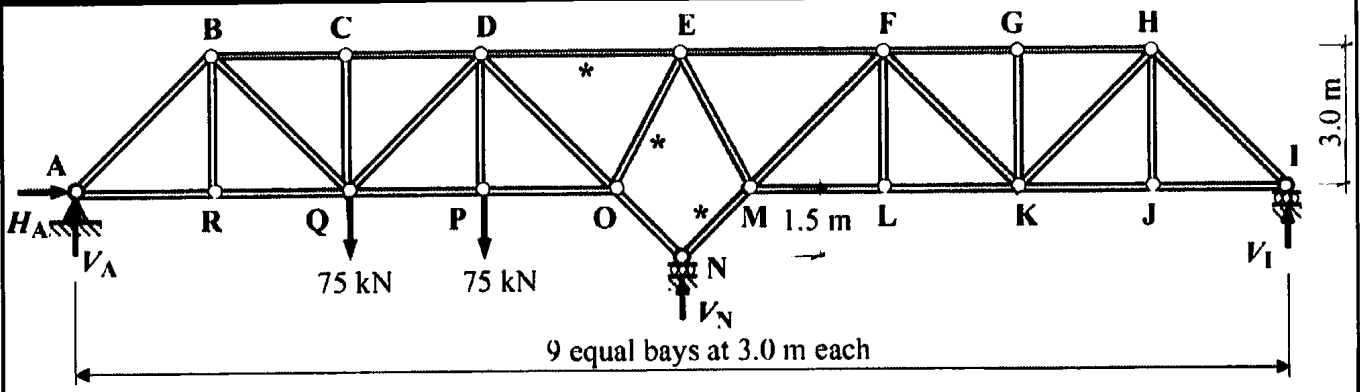
+ve ↑ $\Sigma F_y = 0$ $\quad + 35.0 - 30.0 + (F_{CG} \text{ Sin } 60°) - 0$ $\therefore$ **$F_{CG} = -5.77$ kN (Strut)** ⟷

+ve → $\Sigma F_x = 0$ $\quad - 40.42 - 5.77 \text{ Cos } 60° + F_{FG} = 0$ $\therefore$ **$F_{FG} = +43.31$ kN (Tie)** →←

Solution

Topic: Pin-Jointed Frames – Method of Sections

Problem Number: 3.4



This frame is similar to the frame given in Chapter 1: Figure 1.21 comprising two statically determinate frames.

There are four unknown reactions, however in addition to the three equations of static equilibrium, at support N the magnitude of the forces in members MN and NO are equal. (**Note:** the horizontal components must balance each other). This provides an additional equation which can be used to solve the problem.

Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve $\circlearrowleft$ $\Sigma M_I = 0$

$+ (27.0 \times V_A) - (75.0 \times 21.0) - (75.0 \times 18.0) + (V_N \times 13.5) = 0$ Equation (1)

$+ 27.0V_A - 2925.0 + 13.5V_N$ $\therefore V_A = + 108.33 - 0.5V_N$

Consider the horizontal equilibrium of the frame:

+ve $\rightarrow$ $\Sigma F_x = 0$ $+ H_A = 0$ Equation (2)

$\therefore$ $\boldsymbol{H_A = \text{zero}}$

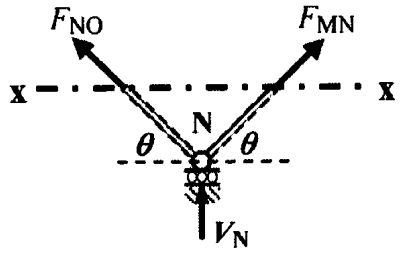
Consider the vertical equilibrium of the frame:

+ve $\uparrow$ $\Sigma F_y = 0$

$+ V_A - 75.0 - 75.0 + V_N + V_I = 0$ Equation (3)

$\therefore V_I = + 150.0 - V_A - V_N$

Consider section x–x at support N



$L_{NO} = L_{MN} = \sqrt{1.5^2 + 1.5^2} = 2.121$ m

$\text{Sin}\theta = (1.5/2.121) = 0.707$

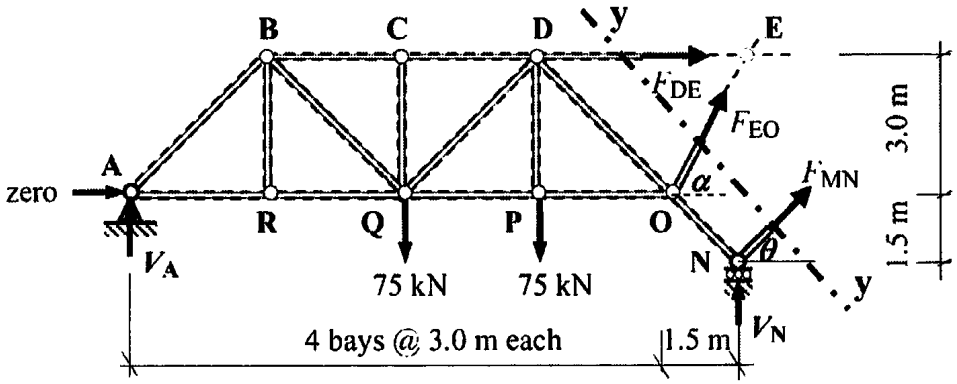
$\text{Cos}\theta = (1.5/2.121) = 0.707$

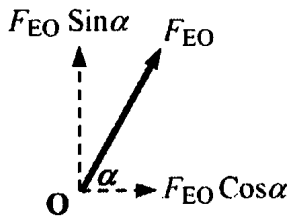
+ve $\uparrow$ $\Sigma F_y = 0$

$+ V_N + (F_{NO}\,\text{Sin}\theta) + (F_{MN}\,\text{Sin}\theta) = 0$ also $F_{NO} = F_{MN}$

$+ V_N + [2 \times (F_{MN} \times 0.707)] = 0$ $\therefore F_{MN} = - 0.707V_N$

Consider section y–y through members DE, EO and MN.





$L_{EO} = \sqrt{1.5^2 + 3.0^2} = 3.354$ m

$\text{Sin}\alpha = 3.0/3.354 = 0.894$ $\quad \text{Cos}\alpha = 1.5/3.354 = 0.447$

+ve $\circlearrowright$ $\Sigma M_E = 0$

$+ (13.5 \times V_A) - (75.0 \times 7.5) - (75.0 \times 4.5) - (F_{MN}\,\text{Cos}\theta \times 4.5) = 0$ Equation (1)

$+ 13.5V_A - 900.0 - 3.182F_{MN} = 0$ $\quad \therefore\ F_{MN} = + 4.243V_A - 282.84$

From section x–x: $F_{MN} = - 0.707V_N$

$- 0.707V_N = + 4.243V_A - 282.84$ $\quad \therefore\ V_A = - 0.167V_N + 66.66$

From Equation (1): $V_A = + 108.33 - 0.5V_N$

$- 0.167V_N + 66.66 = + 108.33 - 0.5V_N$ $\quad \therefore\ V_N = +$ **125.14 kN** ↑

$V_A = - (0.167 \times 125.14) + 66.66$ $\quad \therefore\ V_A =$ **+ 45.76 kN** ↑

From Equation (3): $V_1 = + 150.0 - V_A - V_N$

$V_1 = + 150.0 - 45.76 - 125.14$ $\quad \therefore\ V_1 =$ **– 20.9 kN** ↓

$F_{MN} = + (4.243 \times 45.76) - 282.84$ $\quad \therefore\ F_{MN} =$ **– 88.68 kN (Strut)**

+ve ↑ $\Sigma F_y = 0$

$+ V_A - 75.0 - 75.0 + V_N + (F_{MN}\,\text{Sin}\theta) + (F_{EO}\,\text{Sin}\alpha) = 0$

$F_{EO} = [- 45.76 + 75.0 + 75.0 - 125.14 - (- 88.68 \times 0.707)]/0.894 = 0$ ←→

$\therefore\ F_{EO} =$ **+ 46.75 kN (Tie)**

+ve → $\Sigma F_x = 0$

$+ H_A + F_{DE} + (F_{MN}\,\text{Cos}\theta) + (F_{EO}\,\text{Cos}\alpha) = 0$

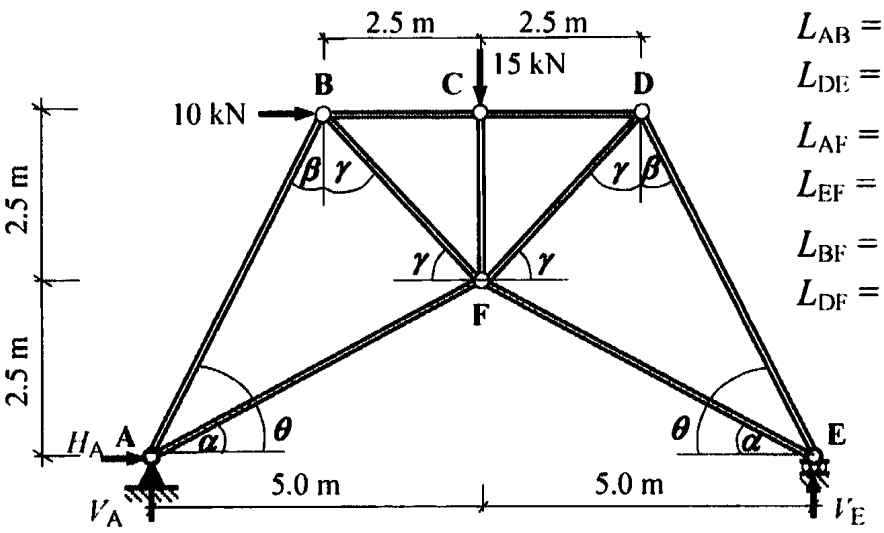
$F_{DE} = [\text{zero} - (- 88.68 \times 0.707) - (46.75 \times 0.447)] = 0$ ←→

$\therefore\ F_{DE} =$ **– 41.80 kN (Tie)**

Solution

Topic: Pin-Jointed Frames – Joint Resolution

Problem Number: 3.9



$L_{AB} = \sqrt{2.5^2 + 5.0^2} = 5.59$ m

$L_{DE} = 5.59$ m

$L_{AF} = \sqrt{5.0^2 + 2.5^2} = 5.59$ m

$L_{EF} = 5.59$ m

$L_{BF} = \sqrt{2.5^2 + 2.5^2} = 3.536$ m

$L_{DF} = 3.536$ m

Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve $\circlearrowright$ $\Sigma M_A = 0$ $\quad + (10.0 \times 5.0) + (15.0 \times 5.0) - (V_E \times 10.0) = 0$ Equation (1)

$\therefore$ **$V_E = +12.5$ kN** ↑

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0$ $\quad + H_A + 10.0 = 0$ Equation (2)

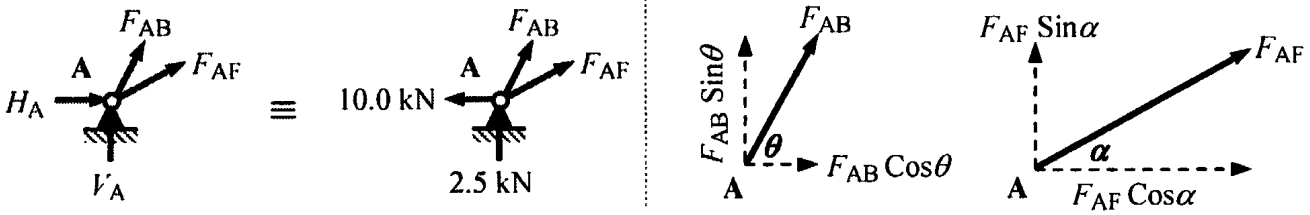
$\therefore$ **$H_A = -10.0$ kN** ←

Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $\quad + V_A - 15.0 + V_E = 0 \quad \therefore V_A = 15.0 - V_E$ Equation (3)

$V_A = 15.0 - 12.5 \quad \therefore$ **$V_A = +2.5$ kN** ↑

Consider joint A:



$\text{Sin}\theta = (5.0/5.59) = 0.894 \quad \text{Cos}\theta = (2.5/5.59) = 0.447$

$\text{Sin}\alpha = (2.5/5.59) = 0.447 \quad \text{Cos}\alpha = (5.0/5.59) = 0.894$

+ve → $\Sigma F_x = 0$ $\quad -10.0 + F_{AB}\text{Cos}\theta + F_{AF}\text{Cos}\alpha = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $\quad + 2.5 + F_{AB}\text{Sin}\theta + F_{AF}\text{Sin}\alpha = 0$ Equation (b)

From Equation (a):

$F_{AB} = [+10.0 - (F_{AF} \times 0.894)]/\ 0.447$ $\therefore\ \boldsymbol{F_{AB} = +22.371 - 2.0F_{AF}}$

Substitute for F_{AB} in Equation (b)

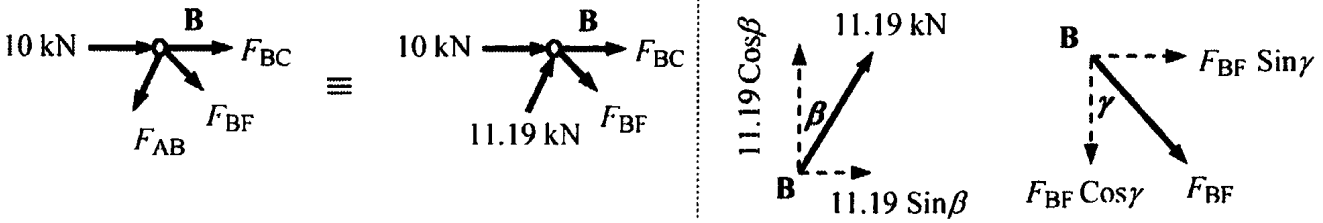
$+2.5 + (22.371 - 2.0F_{AF})\ \mathrm{Sin}\theta + F_{AF}\ \mathrm{Sin}\alpha = 0$

$+2.5 + [(22.371 \times 0.894) - (2.0F_{AF} \times 0.894) + (F_{AF} \times 0.447) = 0$

$+22.5 - 1.341F_{AF} = 0$ $\therefore\ \boldsymbol{F_{AF} = +16.78\ \mathrm{kN}}$ **(Tie)**

$F_{AB} = +22.371 - (2.0 \times 16.78)$ $\therefore\ \boldsymbol{F_{AB} = -11.19\ \mathrm{kN}}$ **(Strut)**

Consider joint B:



$\mathrm{Sin}\beta = (2.5/5.59) = 0.447$ $\mathrm{Cos}\beta = (5.0/5.59) = 0.894$

$\mathrm{Sin}\gamma = (2.5/3.536) = 0.707$ $\mathrm{Cos}\gamma = (2.5/3.536) = 0.707$

+ve → $\Sigma F_x = 0$ $+ 10.0 + 11.19\ \mathrm{Sin}\beta + F_{BF}\ \mathrm{Sin}\gamma + F_{BC} = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $+ 11.19\ \mathrm{Cos}\beta - F_{BF}\ \mathrm{Cos}\ \gamma = 0$ Equation (b)

From Equation (b):

$F_{BF} = +(11.19\ \mathrm{Cos}\beta/\mathrm{Cos}\gamma) = +[(11.19 \times 0.894)/0.707)]$

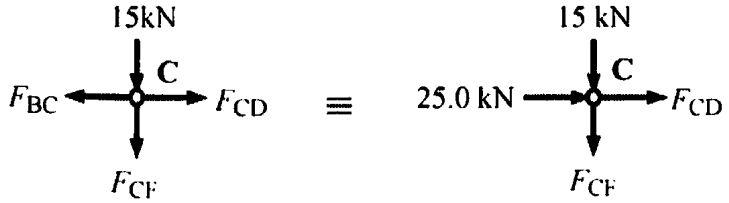
$\therefore\ \boldsymbol{F_{BF} = +14.15\ \mathrm{kN}}$ **(Tie)**

From Equation (a):

$F_{BC} = -[10.0 + (11.19 \times 0.447) + (14.15 \times 0.707)]$

$\therefore\ \boldsymbol{F_{BC} = -25.0\ \mathrm{kN}}$ **(Strut)**

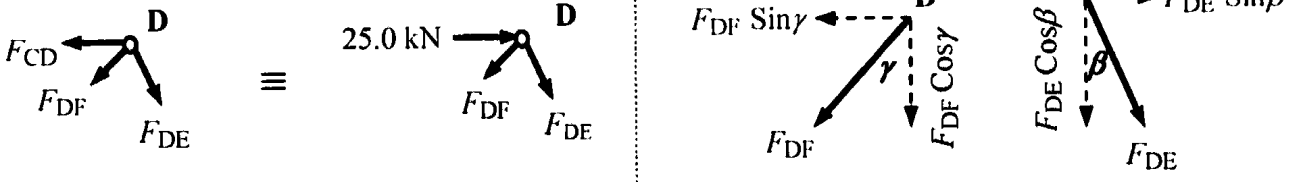
Consider joint C:



+ve → $\Sigma F_x = 0$ $+ 25.0 + F_{CD} = 0$ $\therefore\ \boldsymbol{F_{CD} = -25.0\ \mathrm{kN}}$ **(Strut)**

+ve ↑ $\Sigma F_y = 0$ $- 15.0 - F_{CF} = 0$ $\therefore\ \boldsymbol{F_{CF} = -15.0\ \mathrm{kN}}$ **(Strut)**

Consider joint D:



Sinβ = (2.5/5.59) = 0.447 Cosβ = (5.0/5.59) = 0.894

Sinγ = (2.5/3.536) = 0.707 Cosγ = (2.5/3.536) = 0.707

+ve → $\Sigma F_x = 0$ $+ 25.0 - F_{DF}\,\text{Sin}\gamma + F_{DE}\,\text{Sin}\beta = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $- F_{DF}\,\text{Cos}\gamma - F_{DE}\,\text{Cos}\beta = 0$ Equation (b)

From Equation (a):

$F_{DE} = [-25.0 + (F_{DF} \times 0.707)]/\ 0.447$ $\therefore\ F_{DE} = -55.928 + 1.582F_{DF}$

Substitute for F_{DE} in Equation (b)

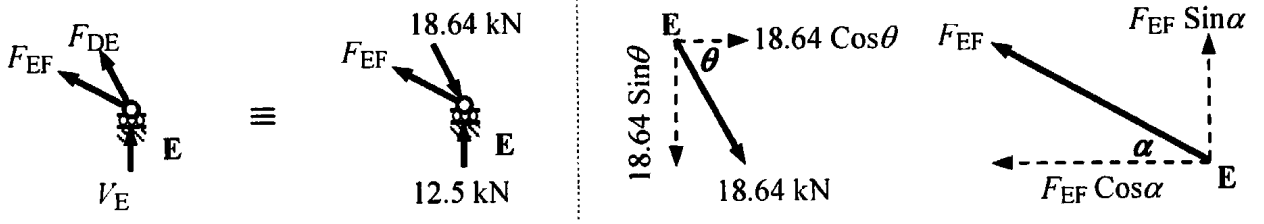
$- F_{DF}\,\text{Cos}\gamma - F_{DE}\,\text{Cos}\beta = 0$

$-(F_{DF} \times 0.707) - [(-55.928 + 1.582F_{DF}) \times 0.894] = 0$

$+ 50.0 - 2.121F_{DF} = 0$ $\therefore$ **F_{DF} = + 23.57 kN (Tie)** ⟵⟶

$F_{DE} = -55.928 + (1.582 \times 23.57)$ $\therefore$ **F_{DE} = – 18.64 kN (Strut)** ⟶⟵

Consider joint E:



Sinθ = (5.0/5.59) = 0.894 Cosθ = (2.5/5.59) = 0.447

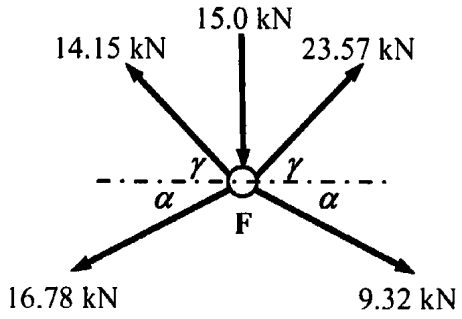
Sinα = (2.5/5.59) = 0.447 Cosα = (5.0/5.59) = 0.894

+ve → $\Sigma F_x = 0$ $+ (18.64 \times \text{Cos}\theta) - F_{EF}\,\text{Cos}\alpha = 0$

$F_{EF} = +\,(18.64 \times 0.447)/0.894$ $\therefore$ **F_{EF} = + 9.32 kN (Tie)** ⟵⟶

The values obtained above can be checked by confirming the horizontal and vertical equilibrium at joint F as follows:

Joint F:



$\text{Sin}\gamma = (2.5/3.536) = 0.707$ $\quad \text{Cos}\gamma = (5.0/3.536) = 0.707$

$\text{Sin}\alpha = (2.5/5.59) = 0.447$ $\quad \text{Cos}\alpha = (5.0/5.59) = 0.894$

+ve $\rightarrow \Sigma F_x$

$= -16.78\ \text{Cos}\alpha - 14.15\ \text{Cos}\gamma + 9.32\ \text{Cos}\alpha + 23.57\ \text{Cos}\gamma$

$= -(16.78 \times 0.894) - (14.15 \times 0.707) + (9.32 \times 0.894) + (23.57 \times 0.707)$

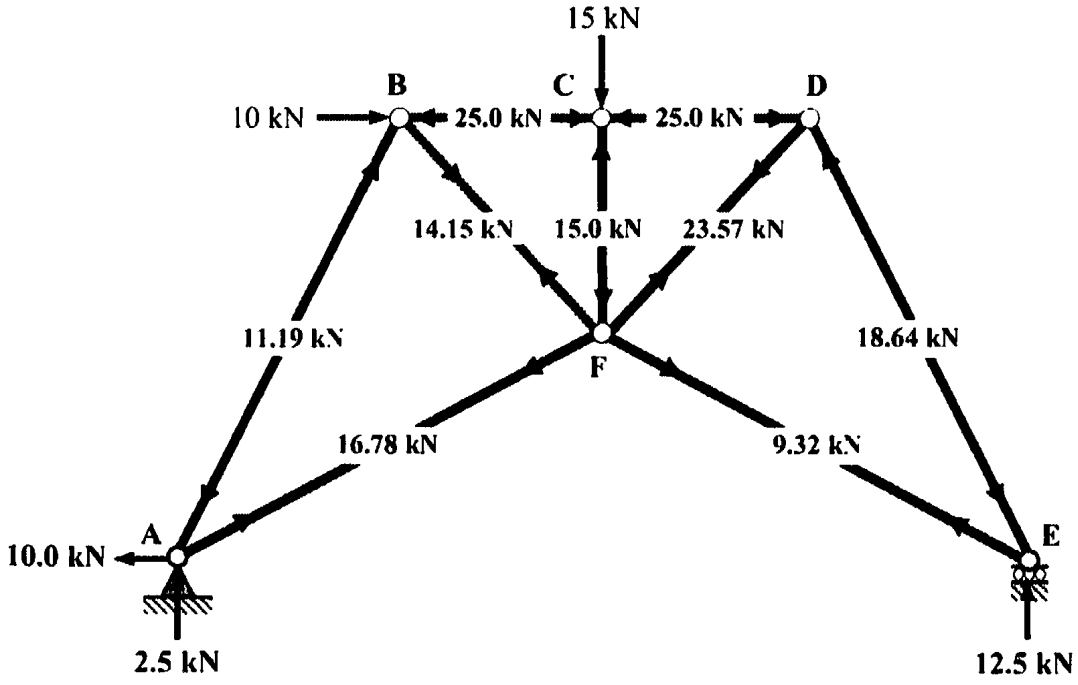
= zero

+ve $\uparrow \Sigma F_y = 0$

$= -16.78\ \text{Sin}\alpha + 14.15\ \text{Sin}\gamma - 9.32\ \text{Sin}\alpha + 23.57\ \text{Sin}\gamma - 15.0$

$= -(16.78 \times 0.447) + (14.15 \times 0.707) - (9.32 \times 0.447) + (23.57 \times 0.707) - 15.0$

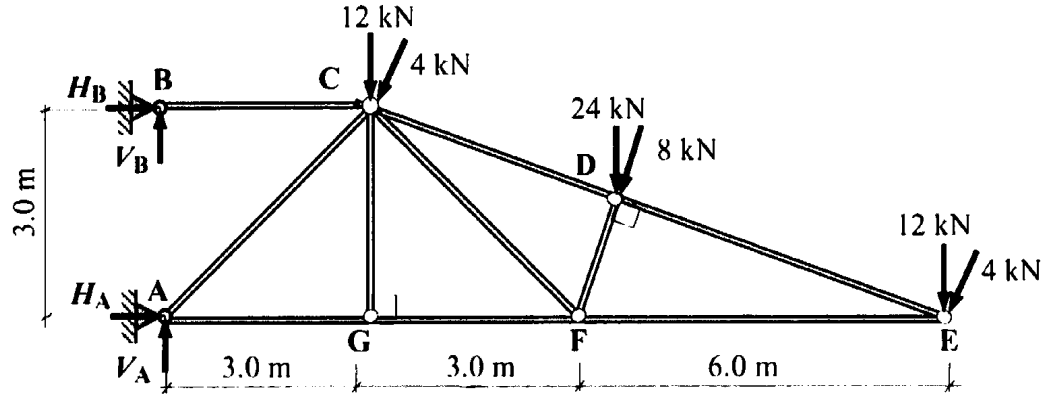
= zero

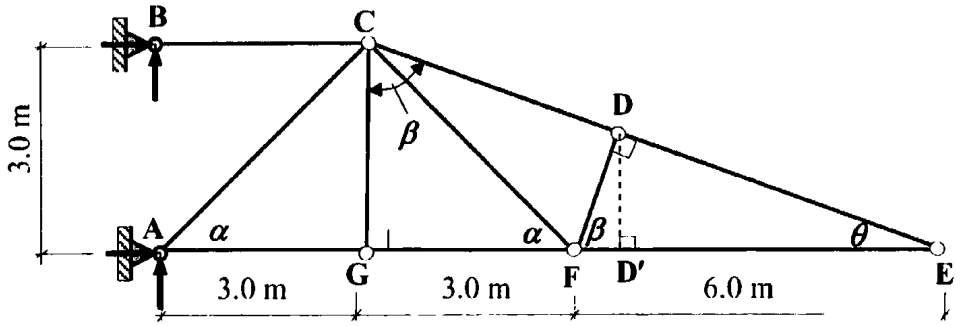


Solution

Topic: Pin-Jointed Frames – Joint Resolution

Problem Number: 3.10





$L_{CE} = \sqrt{3.0^2 + 9.0^2} = 9.487$ m; $L_{AC} = L_{CF} = \sqrt{3.0^2 + 3.0^2} = 4.243$ m

Consider triangle CEG:

$\text{Sin}\theta = (3.0/9.487) = 0.316$; $\text{Cos}\theta = (9.0/9.487) = 0.949$

$\text{Sin}\beta = (9.0/9.487) = 0.949$; $\text{Cos}\beta = (3.0/9.487) = 0.316$

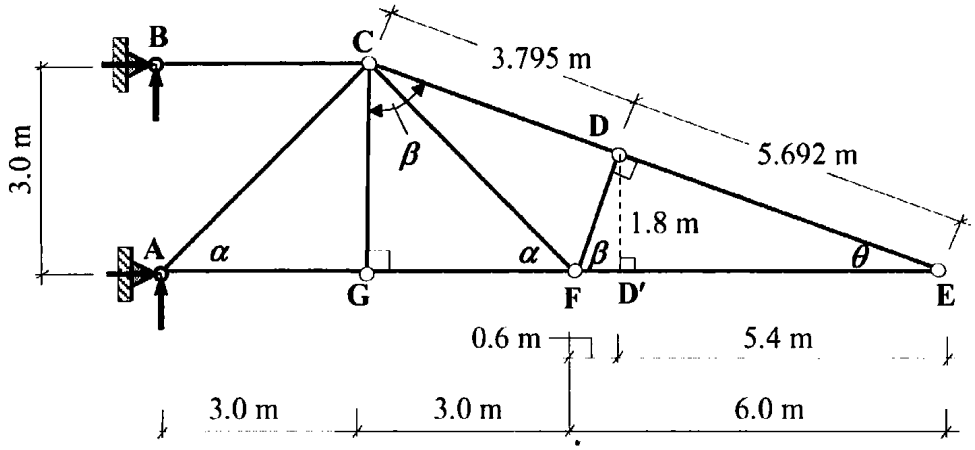
Consider triangle DEF:

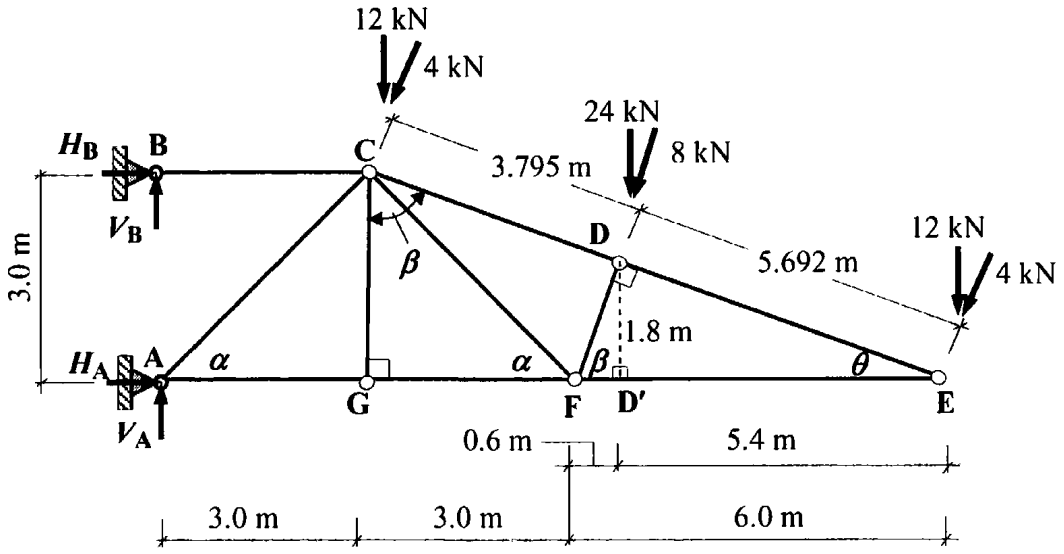
$\text{Sin}\beta = (L_{DE}/L_{EF})$ $\therefore$ $L_{DE} = L_{EF}\,\text{Sin}\beta = (6.0 \times 0.949) = 5.692$ m

Consider triangle DED′:

$\text{Sin}\theta = (L_{DD'}/L_{DE})$ $\therefore$ $L_{DD} = L_{DE}\,\text{Sin}\theta = (5.692 \times 0.316) = 1.8$ m

$\text{Cos}\theta = (L_{ED'}/L_{DE})$ $\therefore$ $L_{ED} = L_{DE}\,\text{Cos}\theta = (5.692 \times 0.949) = 5.4$ m





Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve) $\Sigma M_E = 0$ It is convenient to consider joint E in this case

$+ (12.0 \times V_A) + (12.0 \times V_B) + (3.0 \times H_B) - (12.0 \times 9.0) - (4.0 \times 9.487) - (24.0 \times 5.4) - (8.0 \times 5.692) = 0$

$\therefore + 12.0V_A + 12.0V_B + 3.0H_B = 321.1$ Equation (1)

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0$

$+ H_A + H_B - (4.0 \cos\beta) - (8.0 \cos\beta) - (4.0 \cos\beta) = 0$

$\therefore + H_A + H_B - 1.264 - 2.528 - 1.264 = 0$

$\therefore + H_A + H_B = 5.06$ Equation (2)

Consider the vertical equilibrium of the frame:

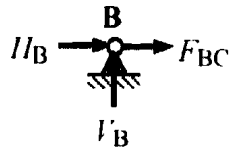
+ve ↑ $\Sigma F_y = 0$

$+ V_A + V_B - 12.0 - (4.0 \sin\beta) - 24.0 - (8.0 \sin\beta) - 12.0 - (4.0 \sin\beta) = 0$

$+ V_A + V_B - 12.0 - 3.796 - 24.0 - 7.592 - 12.0 - 3.796 = 0$

$\therefore + V_A + V_B = 63.18$ Equation (3)

Consider joint B:



+ve ↑ $\Sigma F_y = 0$ $V_B = 0$ **V_B = zero**

From Equation (3): $+ V_A + V_B = 63.18$ $\therefore$ **V_A = + 63.18 kN ↑**

From Equation (1): $+ 12.0V_A + 12.0V_B + 3.0H_B = 321.1$

$+ (12.0 \times 63.18) + (3.0H_B) = 321.1$ $\therefore$ **H_B = – 145.69 kN ←**

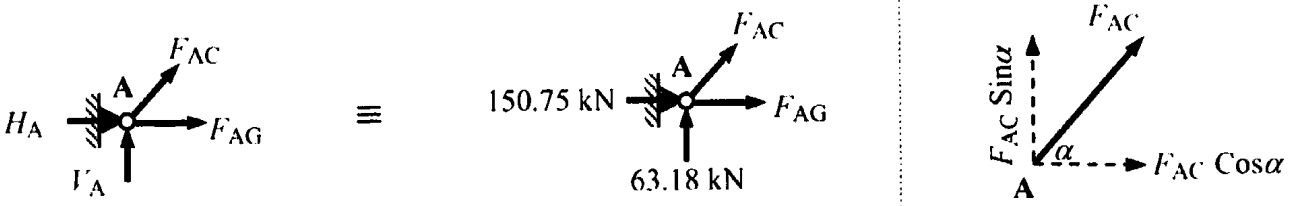
From Equation (2): $+ H_A + H_B = 5.06$

$+ H_A - 145.69 = 5.06$ $\therefore$ **H_A = + 150.75 kN →**

+ve → $\Sigma F_x = 0$

$+ H_B + F_{BC} = 0$ $\therefore$ **F_{BC} = + 145.69 kN (Tie) →←**

Consider joint A:



Sinα = (3.0/4.243) = 0.707 Cosα = (3.0/4.243) = 0.707

+ve → $\Sigma F_x = 0$ $+ 150.75 + F_{AC}\,\text{Cos}\alpha + F_{AG} = 0$ Equation (a)

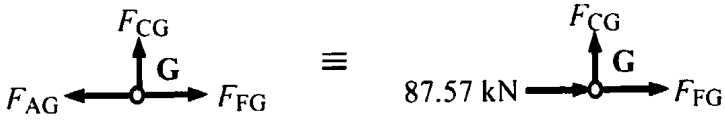
+ve ↑ $\Sigma F_y = 0$ $+ 63.18 + F_{AC}\,\text{Sin}\alpha = 0$ Equaton (b)

From Equation (b): $F_{AC} = -(63.18/0.707) = 0$ $\therefore$ **$F_{AC} = -89.36$ kN (Strut)**

From Equation (a): $F_{AG} = -150.75 - (-89.36 \times 0.707)$

$\therefore$ **$F_{AG} = -87.57$ kN (Strut)**

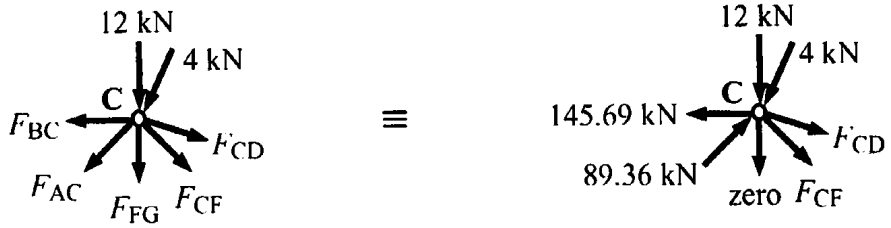
Consider joint G:

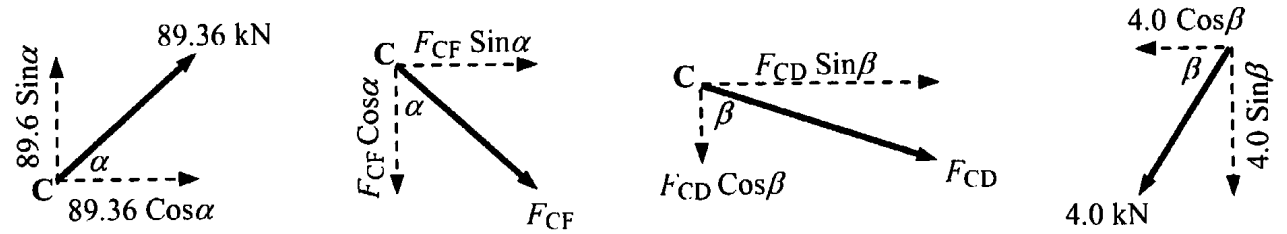


+ve → $\Sigma F_x = 0$ $+ 87.57 + F_{FG} = 0$ $\therefore$ **$F_{FG} = -87.57$ kN (Strut)**

+ve ↑ $\Sigma F_y = 0$ $+ F_{CG} = 0$ $\therefore$ **F_{CG} = zero member**

Consider joint C:





Sinα = (3.0/4.243) = 0.707 Cosα = (3.0/4.243) = 0.707

Sinβ = (9.0/9.487) = 0.949 Cosβ = (3.0/9.487) = 0.316

+ve $\rightarrow \Sigma F_x$

$-145.69 + 89.36\,\text{Cos}\alpha - 4.0\,\text{Cos}\beta + F_{CF}\,\text{Sin}\alpha + F_{CD}\,\text{Sin}\beta = 0$

$-145.69 + (89.36 \times 0.707) - (4.0 \times 0.316) + (F_{CF} \times 0.707) + (F_{CD} \times 0.949) = 0$

$-83.776 + 0.707F_{CF} + 0.949F_{CD} = 0$ Equation (a)

+ve $\uparrow \Sigma F_y = 0$

$-12.0 + 89.36\,\text{Sin}\alpha - 4.0\,\text{Sin}\beta - F_{CF}\,\text{Cos}\alpha - F_{CD}\,\text{Cos}\beta = 0$

$-12.0 + (89.36 \times 0.707) - (4.0 \times 0.949) - (F_{CF} \times 0.707) - (F_{CD} \times 0.316) = 0$

$+47.382 - 0.707F_{CF} - 0.316F_{CD} = 0$ Equation (b)

From Equation (a):

$F_{CF} = (+83.776 - 0.949F_{CD})/0.707$ $\therefore F_{CF} = +118.5 - 1.342F_{CD}$

Substitute for F_{CF} in Equation (b)

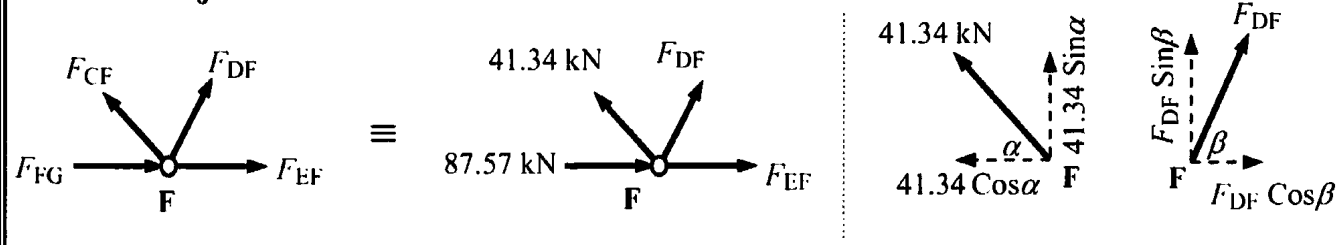
$+47.382 - 0.707F_{CF} - 0.316F_{CD} = 0$

$+47.382 - [0.707 \times (118.5 - 1.342F_{CD})] - 0.316F_{CD} = 0$

$+36.4 + 0.633F_{CD} = 0$ $\therefore$ **$F_{CD} = +57.50$ kN (Tie)**

$\therefore F_{CF} = +118.5 - (1.342 \times 57.5)$ $\therefore$ **$F_{CF} = +41.34$ kN (Tie)**

Consider joint F:



$\text{Sin}\alpha = (3.0/4.243) = 0.707$ $\text{Cos}\alpha = (3.0/4.243) = 0.707$

$\text{Sin}\beta = (9.0/9.487) = 0.949$ $\text{Cos}\beta = (3.0/9.487) = 0.316$

+ve $\rightarrow \Sigma F_x = 0$ $+87.57 - 41.34\,\text{Cos}\alpha + F_{DF}\,\text{Cos}\beta + F_{EF} = 0$ Equation (a)

+ve $\uparrow \Sigma F_y = 0$ $+41.34\,\text{Sin}\alpha + F_{DF}\,\text{Sin}\beta = 0$ Equation (b)

From Equation (b):

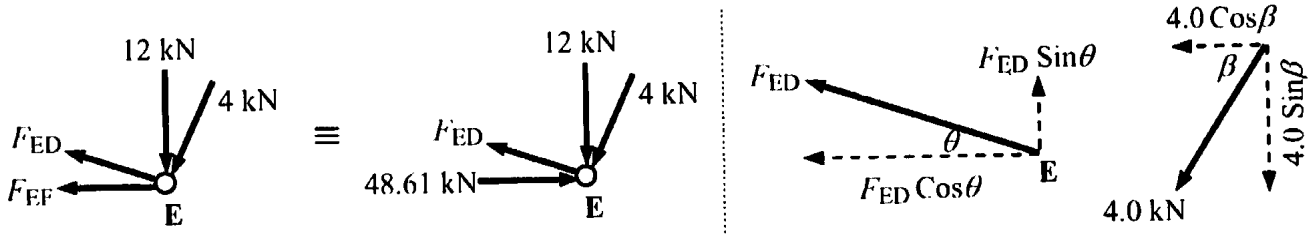
$F_{DF} = -(41.34\,\text{Sin}\alpha/\text{Sin}\beta) = -[(41.34 \times 0.707)/0.949)]$

$\therefore$ **$F_{DF} = -30.8$ kN (Strut)**

From Equation (a):

$F_{EF} = -87.57 + (41.34 \times 0.707) - (-30.8 \times 0.316)$ $\therefore$ **$F_{EF} = -48.61$ kN (Strut)**

Consider joint E:



$\text{Sin}\theta = (3.0/9.487) = 0.316$ $\quad$ $\text{Cos}\theta = (9.0/9.487) = 0.949$

$\text{Sin}\beta = (9.0/9.487) = 0.949$ $\quad$ $\text{Cos}\beta = (3.0/9.487) = 0.316$

+ve $\rightarrow \Sigma F_x$

$+ 48.61 - 4.0\ \text{Cos}\beta - F_{ED}\ \text{Cos}\theta = 0$

$F_{ED} = [48.61 - (4.0 \times 0.316)]/0.949$

Equation (a)

$\therefore$ **F_{ED} = + 49.9 kN (Tie)**

or

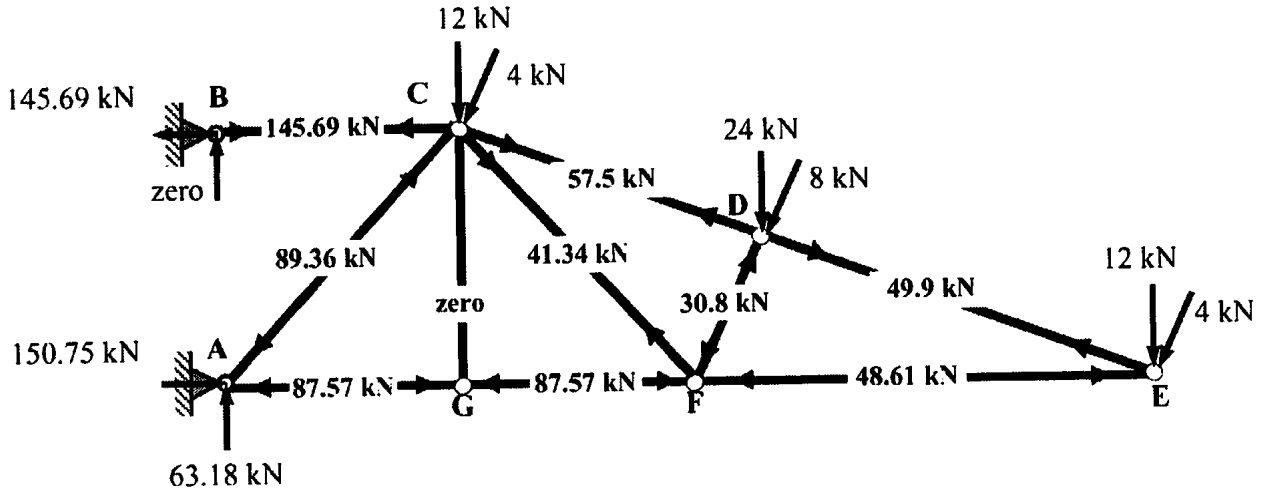
+ve $\uparrow \Sigma F_y = 0$

$- 12.0 - 4.0\ \text{Sin}\beta + F_{ED}\ \text{Sin}\theta = 0$

$F_{ED} = [12.0 + (4.0 \times 0.949)]/0.316$

Equation (b)

$\therefore$ **F_{ED} = + 49.9 kN (Tie)**



The reader should consider the equilibrium of joint D to confirm the calculated values are correct by checking that: +ve $\rightarrow \Sigma F_x = 0$ and +ve $\uparrow \Sigma F_y = 0$