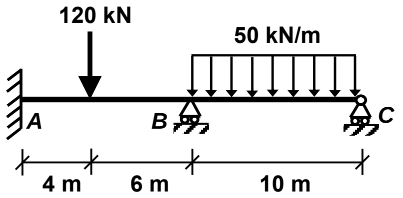
Example:



EI **constant**

The two degrees of freedom are θ_B and θ_C. Using $K_{AB} = EI_{AB} / L_{AB} = EI/10 = K$ and $K_{BC} = EI_{BC} / L_{BC} = EI/10 = K$, we can write the general slope-deflection formula as

$$M_{NF} = 2K_{NF}\left(2\theta_N + \theta_F - 3\psi_{NF}\right) + FEM_{NF}$$

As

$$FEM_{AB} = -172.8 \text{ kNm} \qquad FEM_{BC} = -416.7 \text{ kNm}$$
$$FEM_{BA} = +115.2 \text{ kNm} \qquad FEM_{CB} = +416.7 \text{ kNm}$$

member end moments expressed in terms of the degrees of freedom are

$$M_{AB} = 2K\left(\theta_B\right) - 172.8 \qquad M_{BC} = 2K\left(2\theta_B + \theta_C\right) - 416.7$$
$$M_{BA} = 2K\left(2\theta_B\right) + 115.2 \qquad M_{CB} = 2K\left(2\theta_C + \theta_B\right) + 416.7$$

From the governing equilibrium equations at joints *B* and *C*, that is,

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} = 0$$

we find the following expressions

$$8K\theta_B + 2EK\theta_C = 301.5$$
$$2K\theta_B + 4EK\theta_C = -416.7$$

which can be rewritten, for convenience, as

$$K\theta_B = 37.7 - 0.25\left(K\theta_C\right)$$
$$K\theta_C = -104.2 - 0.5\left(K\theta_B\right)$$

We can use the numerical iterative solution to find the pair of θ_B and θ_C satisfying the governing equilibrium equations simultaneously. Tabulating the results for each iteration step we have:

Step	**$K\theta_B$**	**$K\theta_C$**
Initial	0	0
1	37.7	0
2	37.7	-123.1
3	68.5	-123.1
4	68.5	-138.4
5	72.3	-138.4
6	72.3	-140.4
7	72.8	-140.4
8	72.8	-140.6
9	72.8	-140.6

Physical interpretation of the iterative solution can be seen by calculating the member-end moments for each iteration step. Using the following slope-deflection relationships

$$M_{AB} = 2K\theta_B - 172.8 \qquad M_{BC} = 4K\theta_B + 2K\theta_C - 416.7$$
$$M_{BA} = 4K\theta_B + 115.2 \qquad M_{CB} = 4K\theta_C + 2K\theta_B + 416.7$$

we can calculate the member-end moments at each iteration step. Recall that the equilibrium equations we have to satisfy are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} = 0$$

The results for each step are given below:

Step	$K\theta_B$	$K\theta_C$	M_{AB}	M_{BA}	M_{BC}	M_{CB}
Initial	0	0	-172.8	115.2	-416.7	416.7
1	37.7	0	-97.4	266	-265.9	492.1
2	37.7	-123.1	-97.4	266.0	-512.0	-0.1
3	68.5	-123.1	-35.9	389.1	-389.0	61.4
4	68.5	-138.4	-35.9	389.1	-419.7	-0.1
5	72.3	-138.4	-28.2	404.4	-404.3	7.6
6	72.3	-140.4	-28.2	404.4	-408.2	-0.1
7	72.8	-140.4	-27.2	406.4	-406.3	0.9
8	72.8	-140.6	-27.2	406.4	-406.7	-0.1
9	72.8	-140.6	-27.1	406.6	-406.5	0.0

An examination of the joint rotations and moments listed in the last table reveals the following:

- Initially, neither equilibrium equation is satisfied.
- From the initial step to step #1, joint *B* is allowed to rotate to while joint *C* remains clamped. In the process, the moment equilibrium at joint *B* is satisfied but the moment equilibrium at *C* remains unsatisfied.
- From step #1 to step #2, joint *B* is clamped in a rotated position while joint *C* is permitted to rotate. As a result, the moment equilibrium at joint *C* is satisfied but the moment equilibrium at joint *B* is not satisfied.
- From step #2 to step #3, joint *C* is clamped in its rotated position while joint *B* is again permitted to rotate. This action satisfies the moment equilibrium at joint *B* but the moment equilibrium at joint *C* is, again, not satisfied.

The pattern continues: at each step, one of the equilibrium conditions is satisfied and the other one is not. In going to the next step, the joint where equilibrium is not satisfied is allowed to rotate; this produces equilibrium at this joint but disturbs the equilibrium at the other joint. The process continues until both joints are in equilibrium, at which point the correct moments are obtained.

This technique of finding the set of moments that satisfy all equilibrium equations simultaneously is called, aptly, the ***moment distribution method***.