Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
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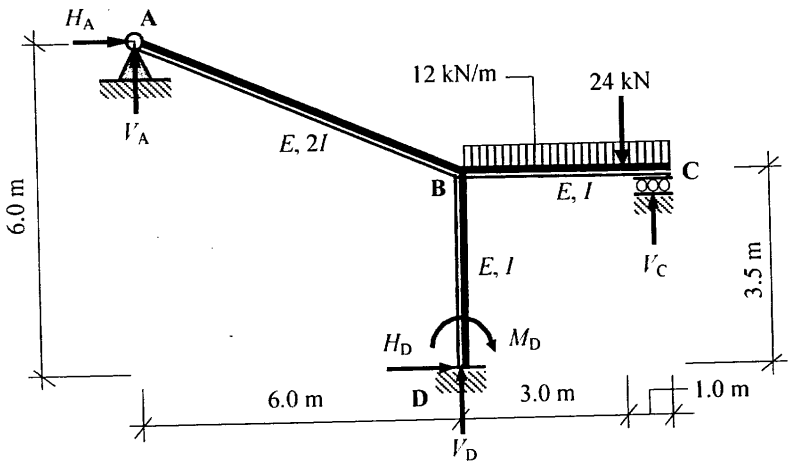
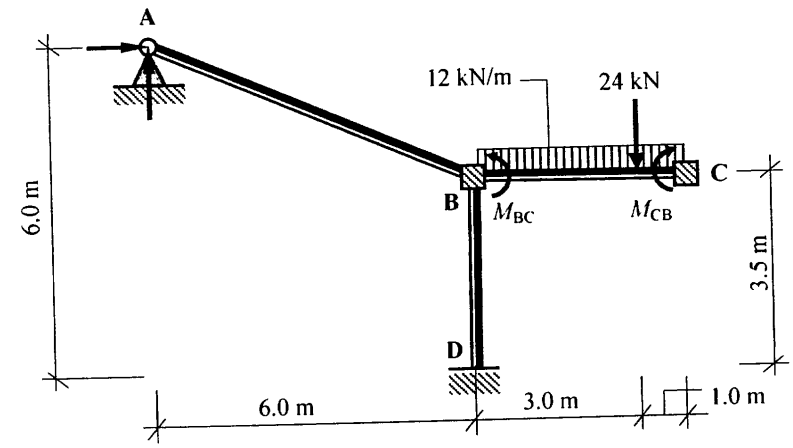


Fixed-end Moments:



Member BC*

$$M_{BC} = -\frac{Pab^2}{L^2} - \frac{wL^2}{12} = -\frac{24.0\times3\times1^2}{4^2} - \frac{12.0\times4^2}{12} = -20.5 \text{ kNm}$$

$$M_{CB} = +\frac{Pa^2b}{L^2} + \frac{wL^2}{12} = +\frac{24.0\times3^2\times1}{4^2} + \frac{12.0\times4^2}{12} = +29.5 \text{ kNm}$$

* Since support C is a roller, the fixed-end moments are ($M_{BC} - 0.5M_{CB}$) at B and zero at C.
($M_{BC} - 0.5M_{CB}$) = [– 20.5 – (0.5 × 29.5)] = – 35.25 kNm.

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Length of member AB = $\sqrt{(6.0^2 + 2.5^2)} = 6.5$ m

Distribution Factors : Joint B

$$k_{BA} = \frac{3}{4} \times \left(\frac{2I}{6.5}\right) = 0.23I$$

$$k_{BC} = \frac{3}{4} \times \left(\frac{I}{4.0}\right) = 0.19I$$

$$k_{BD} = \left(\frac{I}{3.5}\right) = 0.29I$$

$$k_{total} = \mathbf{0.71}I$$

$$DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.23}{0.71} = \mathbf{0.32}$$

$$DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.19}{0.71} = \mathbf{0.27}$$

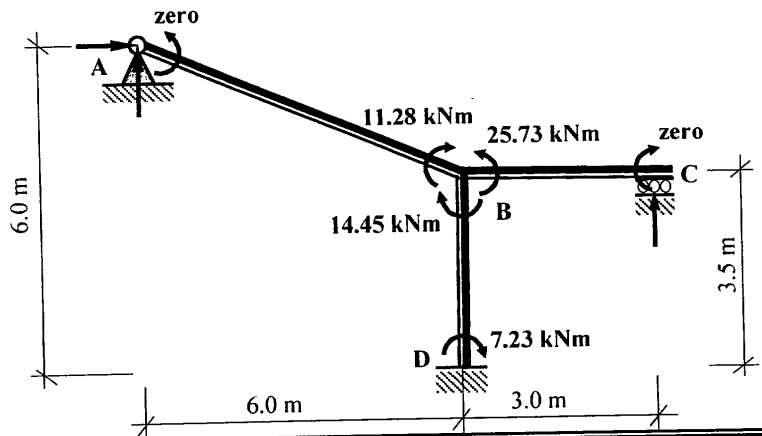
$$DF_{BD} = \frac{k_{BD}}{k_{Total}} = \frac{0.29}{0.71} = \mathbf{0.41}$$

Moment Distribution Table:

Joint	A	B			C	D
	AB	BA	BD	BC	CB	DB
Distribution Factors	**1.0**	**0.32**	**0.41**	**0.27**	**1.0**	**0**
Fixed-end Moments				− 35.25		
Balance		+ 11.28	+ 14.45	+ 9.52		
Carry-over						+ 7.23
Total	**0**	**+ 11.28**	**+ 14.45**	**− 25.73**	**0**	**+ 7.23**

Note: the sum of the moments at joint B = zero

Continuity Moments:



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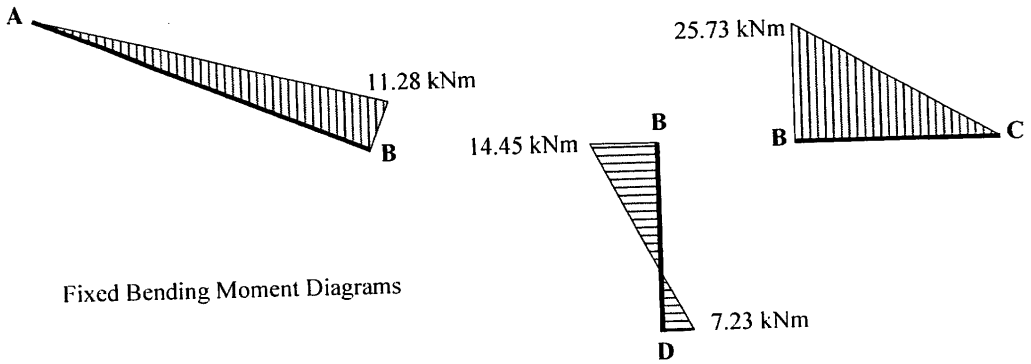
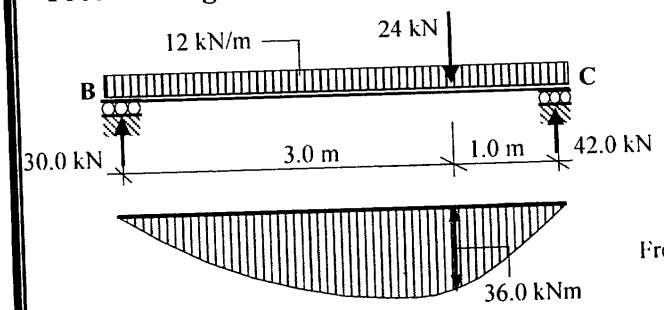


Fixed Bending Moment Diagrams

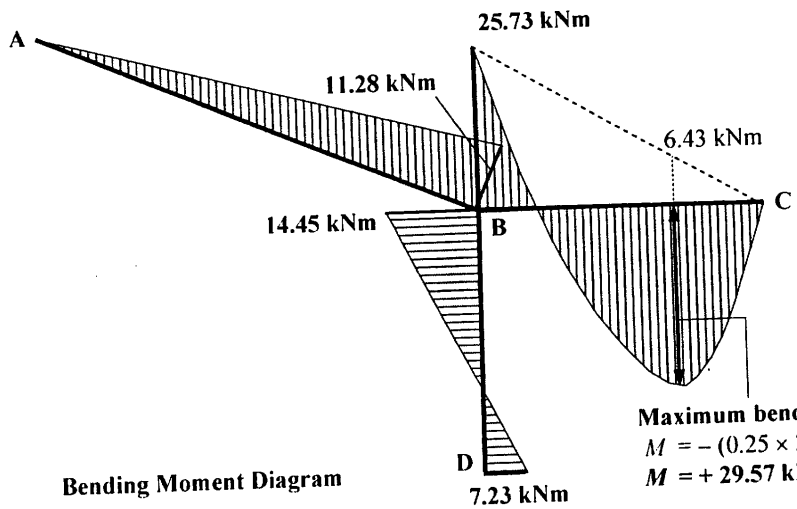
Free bending moments:



Note:
In this problem, the point of zero shear in member BC occurs under the point load.

Free Bending Moment Diagram

Member BC: $M_{free} = +[(42.0 \times 1.0) - (12.0 \times 1.0 \times 0.5)] = +36.0$ kNm



Bending Moment Diagram

Maximum bending moment:
$M = -(0.25 \times 25.73) + 36.0$
$M = +29.57$ kNm

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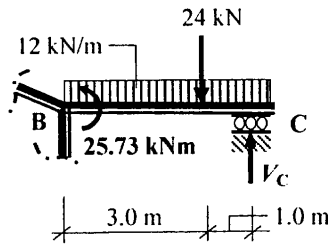
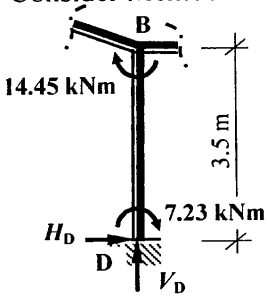
Consider Member BC:



+ve $\circlearrowleft \Sigma M_B = 0$

$-25.73 + (12.0 \times 4.0 \times 2.0) + (24.0 \times 3.0) - (V_C \times 4.0) = 0 \quad \therefore V_C = +\mathbf{35.57\ kN}$ ↑

Consider Member BD:



+ve $\circlearrowleft \Sigma M_B = 0$

$+ 14.45 + 7.23 - (H_D \times 3.5) = 0 \quad \therefore H_D = +\mathbf{6.19\ kN}$ →

For the complete frame:

+ve → $\Sigma F_x = 0$

$+ H_A + H_D = 0 \quad \therefore H_A = -\mathbf{6.19\ kN}$ ←

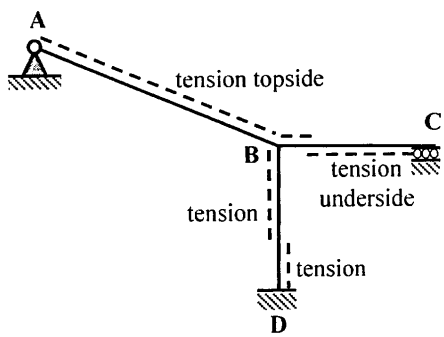
+ve $\circlearrowleft \Sigma M_A = 0$

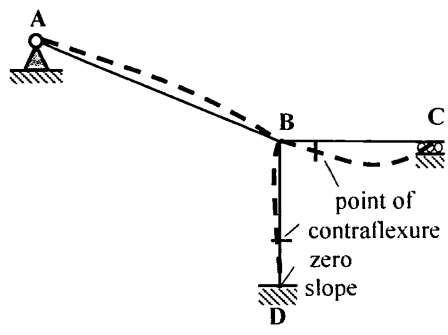
$+ 7.23 + (12.0 \times 4.0 \times 8.0) + (24.0 \times 9.0) - (35.57 \times 10.0) - (6.19 \times 6.0) - (V_D \times 6.0) = 0$

$\therefore V_D = +\mathbf{35.73\ kN}$ ↑

+ve ↑ $\Sigma F_y = 0$

$35.73 - (12.0 \times 4.0) - 24.0 + 35.57 + V_A = 0 \quad \therefore V_A = +\mathbf{0.7\ kN}$ ↑





Deflected Shape