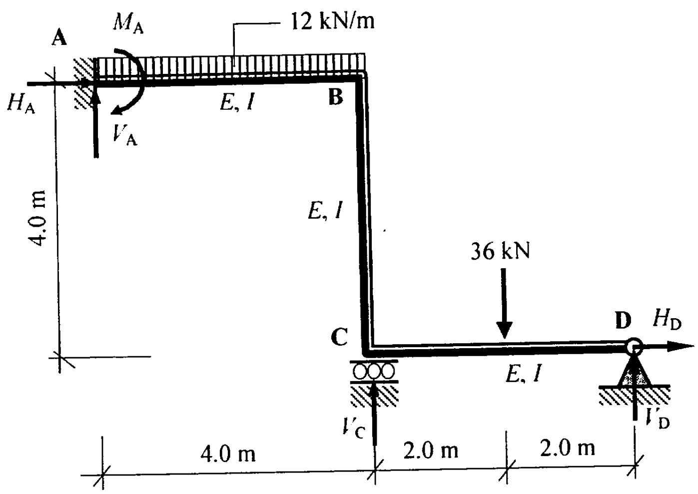
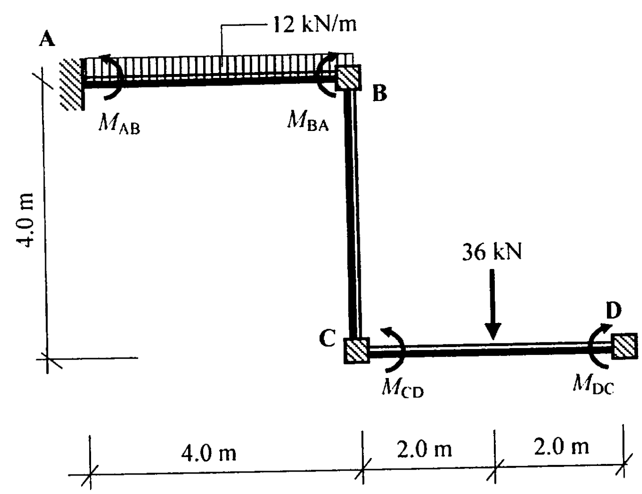
Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.9 **Page No. 1**



Fixed-end Moments:



Member AB

$$M_{AB} = -\frac{wL^2}{12} = -\frac{12.0 \times 4^2}{12} = -16.0 \text{ kNm}$$

$$M_{BA} = +\frac{wL^2}{12} = +\frac{12.0 \times 4^2}{12} = +16.0 \text{ kNm}$$

Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.9 **Page No. 2**

Member CD*

$$M_{CD} = -\frac{PL}{8} = -\frac{36.0 \times 4}{8} = -18.0 \text{ kNm}$$

$$M_{DC} = +\frac{PL}{8} = +\frac{36.0 \times 4}{8} = +18.0 \text{ kNm}$$

* Since support D is pinned, the fixed-end moments are $(M_{CD} - 0.5M_{DC})$ at C and zero at D.

$(M_{CD} - 0.5M_{DC}) = [-18.0 - (0.5 \times 18.0)] = -27.0$ kNm.

Distribution Factors : Joint B

$$k_{BA} = \left(\frac{I}{4.0}\right) = 0.25I \qquad DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.25}{0.5} = \mathbf{0.5}$$

$$\mathbf{k_{total} = 0.51\mathit{I}}$$

$$k_{BC} = \left(\frac{I}{4.0}\right) = 0.25I \qquad DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.25}{0.5} = \mathbf{0.5}$$

Distribution Factors : Joint C

$$k_{CB} = \left(\frac{I}{4.0}\right) = 0.25I \qquad DF_{CB} = \frac{k_{CB}}{k_{Total}} = \frac{0.25}{0.44} = \mathbf{0.57}$$

$$\mathbf{k_{total} = 0.44\mathit{I}}$$

$$k_{CD} = \frac{3}{4} \times \left(\frac{I}{4.0}\right) = 0.19I \qquad DF_{CD} = \frac{k_{CD}}{k_{Total}} = \frac{0.19}{0.44} = \mathbf{0.43}$$

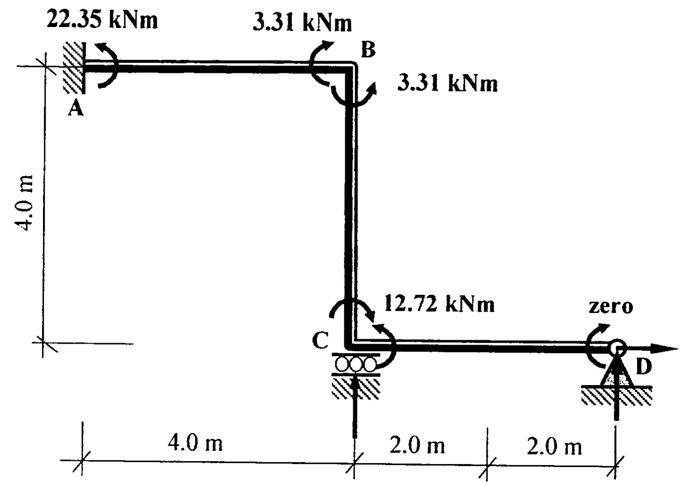
Moment Distribution Table:

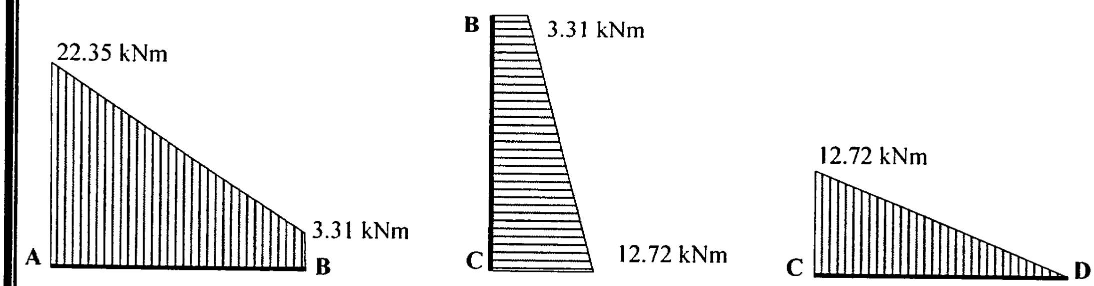
Joint	**A**	**B**		**C**		**D**
	AB	**BA**	**BC**	**CB**	**CD**	**DC**
Distribution Factors	**0**	**0.5**	**0.5**	**0.57**	**0.43**	**1.0**
Fixed-end Moments	– 16.0	+ 16.0			– 27.0	
Balance		– 8.0	– 8.0	+ 15.39	+ 11.61	
Carry-over	– 4.0		+ 7.7	– 4.0		
Balance		– 3.85	– 3.85	+ 2.28	+ 1.72	
Carry-over	– 1.79		+ 1.14	– 1.93		
Balance		– 0.57	– 0.57	+ 1.1	+ 0.83	
Carry-over	– 0.29		+ 0.55	– 0.29		
Balance		– 0.27	– 0.27	+ 0.17	+ 0.12	
Carry-over	– 0.13					
Total	**– 22.35**	**+3.31**	**– 3.31**	**+ 12.72**	**– 12.72**	

Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.9 **Page No. 3**

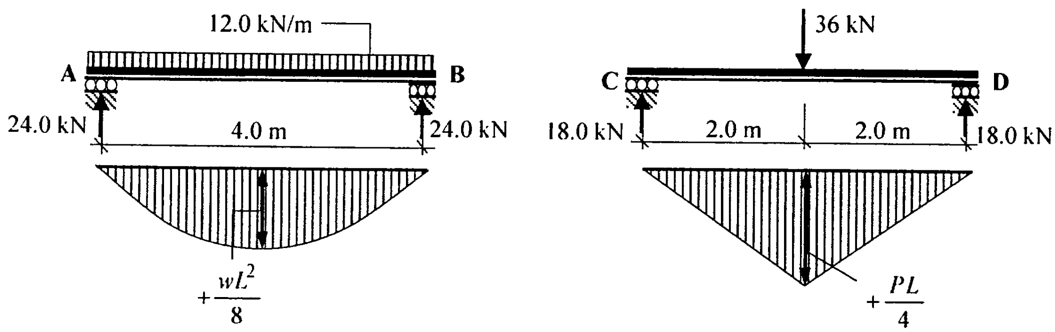
Continuity Moments:





Fixed Bending Moment Diagrams

Free bending moments:

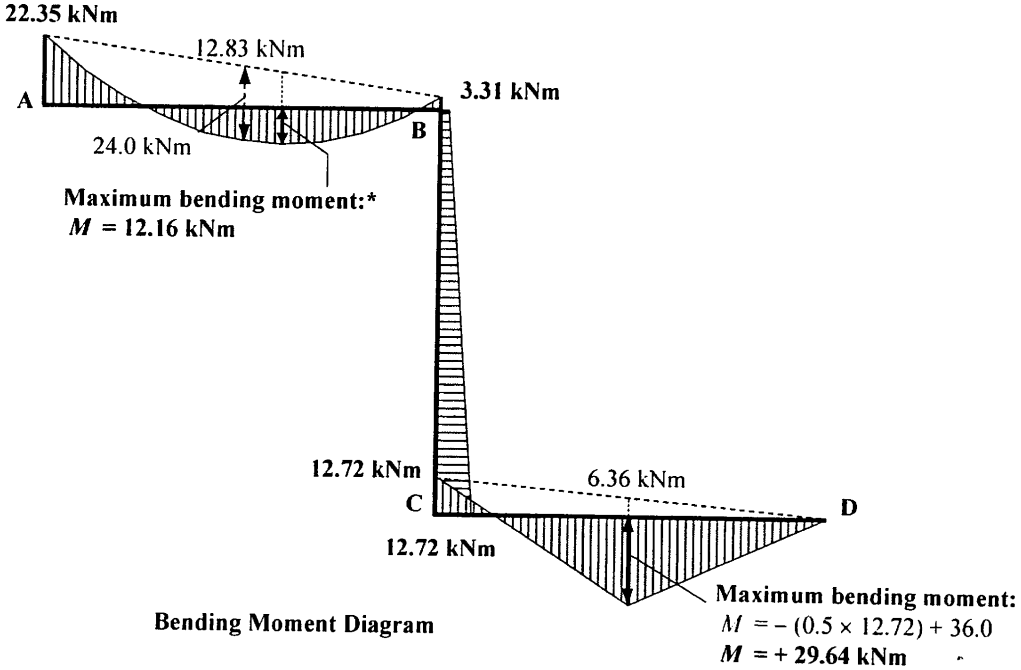


Free Bending Moment Diagrams

Member AB: $M_{free} = (12.0 \times 4^2)/8 = 24.0$ kNm
Member CD: $M_{free} = (36.0 \times 4)/4 = 36.0$ kNm

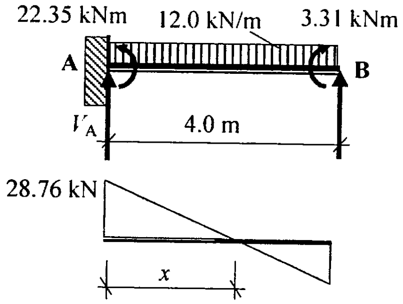
Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.9 **Page No. 4**



Bending Moment Diagram

- **The maximum value along the length of member AB can be found by identifying the point of zero shear as follows:**



Shear Force Diagram

+ve $\sum M_B = 0$

$$-22.35 - (12.0 \times 4.0 \times 2.0) + 3.31 + (V_A \times 4.0) = 0 \qquad \therefore\ V_A = +\,\mathbf{28.76\ kN}\ \uparrow$$

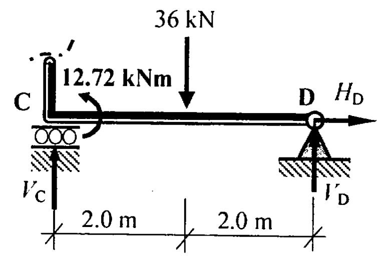
$$x = (28.76/12.0) = 2.4\ \text{m}$$

$$M_{\text{maximum}} = (0.5 \times 2.4 \times 28.76) - 22.35 = \mathbf{12.16\ kNm}$$

Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.9 **Page No. 4**

Consider Member CD:



+ve ↻ $\Sigma M_C = 0$

$-12.72 + (36.0 \times 2.0) - (V_D \times 4.0) = 0$ $\quad\therefore$ **$V_D = +14.82$ kN ↑**

For the complete frame:

+ve ↑ $\Sigma F_y = 0$

$28.76 - (12.0 \times 4.0) - 36.0 + 14.82 + V_C = 0$ $\quad\therefore$ **$V_C = +40.42$ kN ↑**

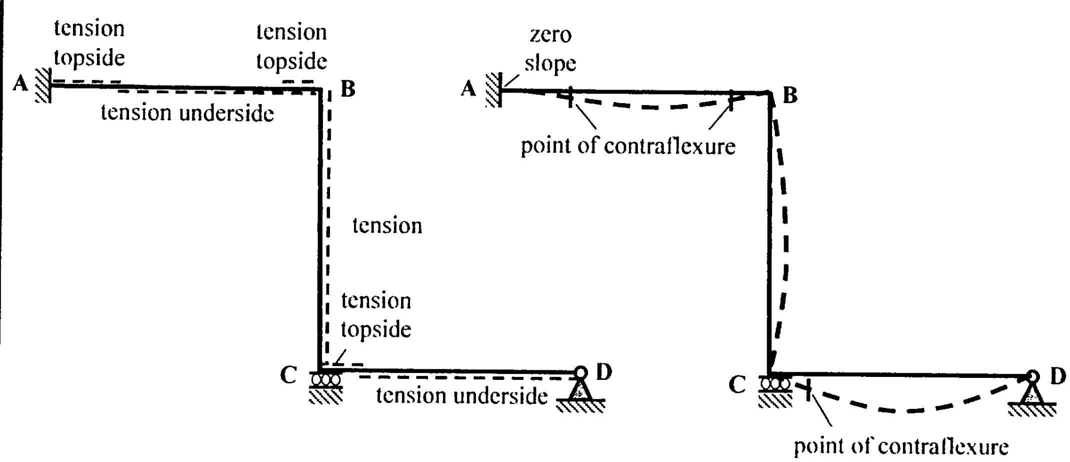
+ve ↻ $\Sigma M_A = 0$

$-22.35 + (12.0 \times 4.0 \times 2.0) + (36.0 \times 6.0) - (40.42 \times 4.0) - (14.82 \times 8.0) - (H_D \times 4.0) = 0$

$\therefore$ **$H_D = +2.35$ kN →**

+ve → $\Sigma F_x = 0$

$+H_A + H_D = 0$ $\quad\therefore$ **$H_A = -2.35$ kN ←**



Deflected Shape