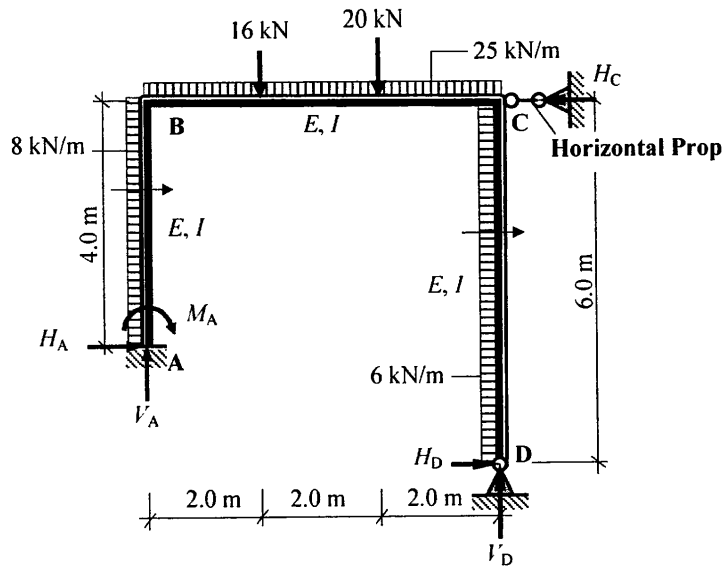
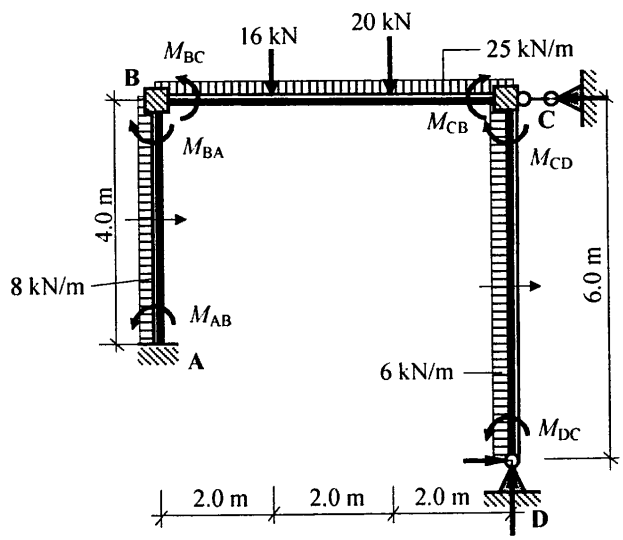
# Solution

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

**Problem Number: 5.12** **Page No. 1**




**Fixed-end Moments:**




Member AB

$$M_{AB} = -\frac{wL^2}{12} = -\frac{8.0 \times 4^2}{12} = -10.67 \text{ kNm}$$

$$M_{BA} = +\frac{wL^2}{12} = +\frac{8.0 \times 4^2}{12} = +10.67 \text{ kNm}$$

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Member BC

$$M_{BC} = -\frac{wL^2}{12} - \frac{P_1ab^2}{L^2} - \frac{P_2ab^2}{L^2}$$

$$= -\left[\left(\frac{25.0\times 6^2}{12}\right)+\left(\frac{16.0\times 2.0\times 4.0^2}{6^2}\right)+\left(\frac{20.0\times 4.0\times 2.0^2}{6^2}\right)\right] = -98.1 \text{ kNm}$$

$$M_{CB} = +\frac{wL^2}{12} + \frac{P_1a^2b}{L^2} + \frac{P_2a^2b}{L^2}$$

$$= +\left[\left(\frac{25.0\times 6^2}{12}\right)+\left(\frac{16.0\times 2.0^2\times 4.0}{6^2}\right)+\left(\frac{20.0\times 4.0^2\times 2.0}{6^2}\right)\right] = +99.9 \text{ kNm}$$

Member CD *

$$M_{CD} = +\frac{wL^2}{12} = +\frac{6.0\times 6^2}{12} = +18.0 \text{ kNm}$$

$$M_{DC} = -\frac{wL^2}{12} = \frac{6.0\times 6^2}{12} = -18.0 \text{ kNm}$$

* Since support D is pinned, the fixed-end moments are ($M_{CD} - 0.5M_{DC}$) at C and zero at D.
($M_{CD} - 0.5M_{DC}$) = [+ 18.0 + (0.5 × 18.0) ] = + 27.0 kNm.

**Distribution Factors : Joint B**

$$k_{BA} = \left(\frac{I}{4.0}\right) = 0.25I \qquad DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.25}{0.42} = \mathbf{0.6}$$

$$\mathbf{k_{total} = 0.42I}$$

$$k_{BC} = \left(\frac{I}{6.0}\right) = 0.17I \qquad DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.17}{0.42} = \mathbf{0.4}$$

**Distribution Factors : Joint C**

$$k_{CB} = \left(\frac{I}{6.0}\right) = 0.17I \qquad DF_{CB} = \frac{k_{CB}}{k_{Total}} = \frac{0.17}{0.3} = \mathbf{0.57}$$

$$\mathbf{k_{total} = 0.3I}$$

$$k_{CD} = \frac{3}{4}\times\left(\frac{I}{6.0}\right) = 0.13I \qquad DF_{CD} = \frac{k_{CD}}{k_{Total}} = \frac{0.13}{0.3} = \mathbf{0.43}$$

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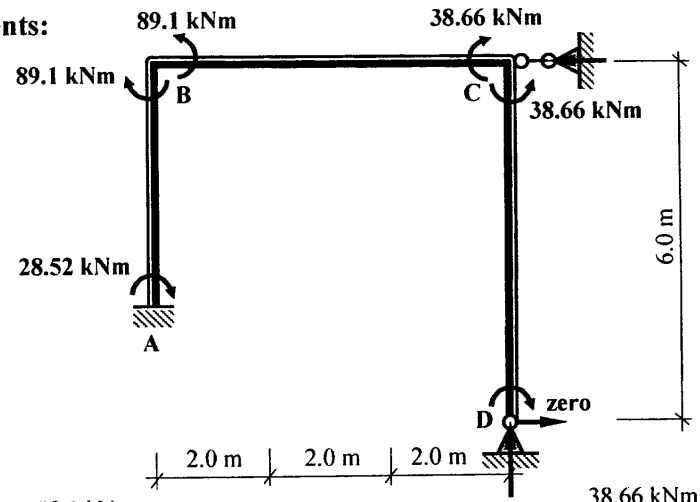
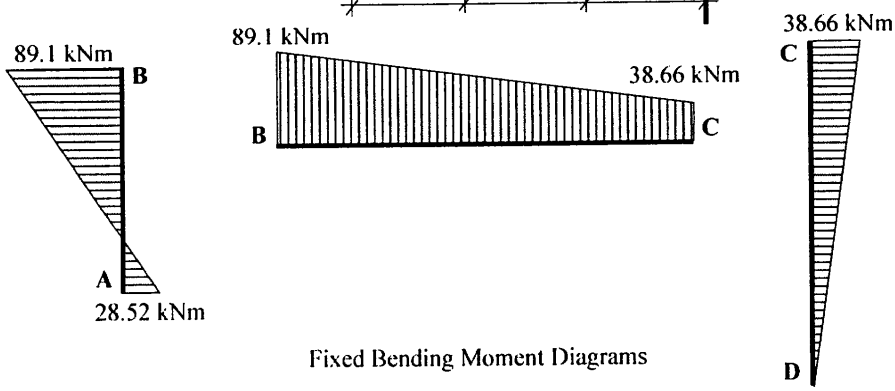
**Moment Distribution Table:**

| Joint | A | B | | C | | D |
|---|---|---|---|---|---|---|
| | AB | BA | BC | CB | CD | DC |
| Distribution Factors | 0 | 0.6 | 0.4 | 0.57 | 0.43 | 1.0 |
| Fixed-end Moments | –10.67 | + 10.67 | – 98.1 | + 99.9 | + 27.0 | |
| Balance | | + 52.46 | + 34.97 | – 72.3 | – 54.6 | |
| Carry-over | + 26.23 | | – 36.2 | + 17.49 | | |
| Balance | | + 21.72 | + 14.48 | – 9.97 | – 7.52 | |
| Carry-over | + 10.86 | | – 4.99 | + 7.24 | | |
| Balance | | + 3.0 | + 1.99 | – 4.13 | – 3.11 | |
| Carry-over | + 1.5 | | – 2.07 | + 1.0 | | |
| Balance | | + 1.2 | + 0.87 | – 0.57 | – 0.43 | |
| Carry-over | + 0.6 | | | | | |
| Total | + 28.52 | + 89.1 | – 89.1 | + 38.66 | – 38.66 | 0 |

**Continuity Moments:**




Fixed Bending Moment Diagrams

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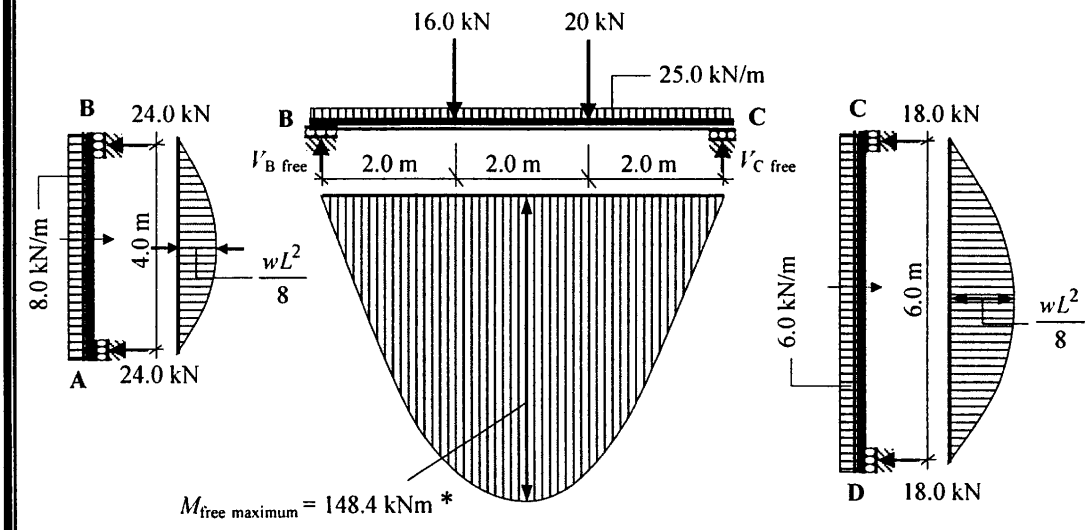
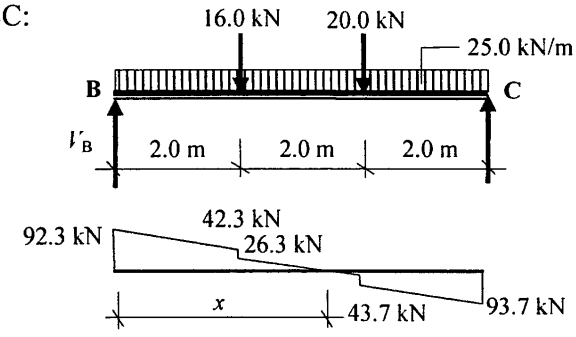
**Free bending moments:**




Free Bending Moment Diagrams

Member AB: $M_{free} = (8.0 \times 4^2)/8 = 16.0$ kNm

* Member BC:




+ve $\circlearrowright \Sigma M_C = 0$

$-(16.0 \times 4.0) - (20.0 \times 2.0) - (25.0 \times 6.0 \times 3.0 + (V_B \times 6.0) = 0 \qquad V_B = +92.3$ kN

Position of zero shear $x = [2.0 + (26.3 / 25.0)] = 3.05$ m

$M_{\text{maximum free bending moment}} = [0.5 \times (92.3 + 42.3) \times 2.0] + (0.5 \times 1.05 \times 26.3)$
$= 148.4$ kNm

Member DC: $M_{free} = (6.0 \times 6^2)/8 = 27.0$ kNm

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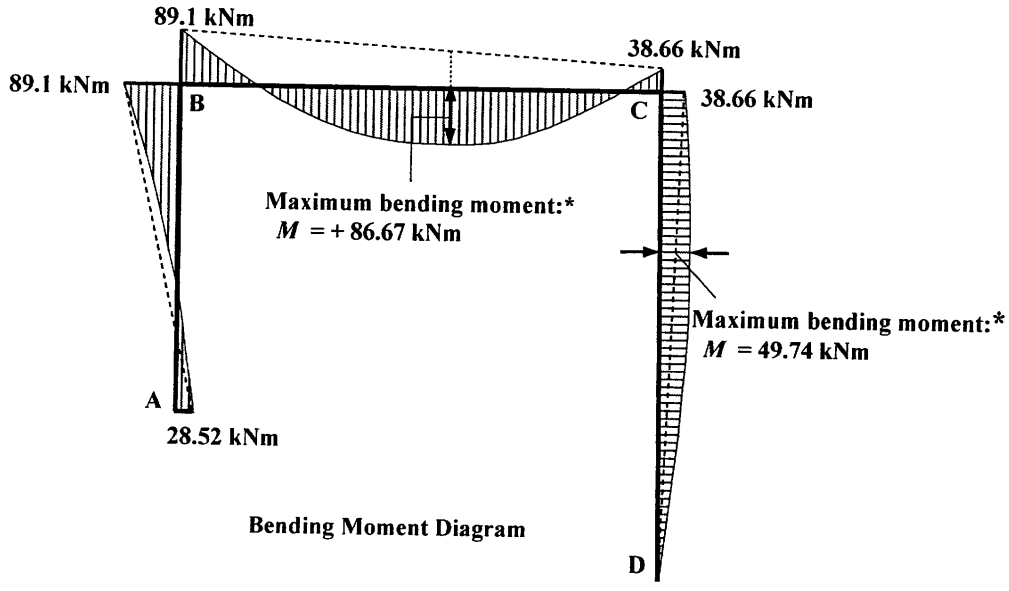
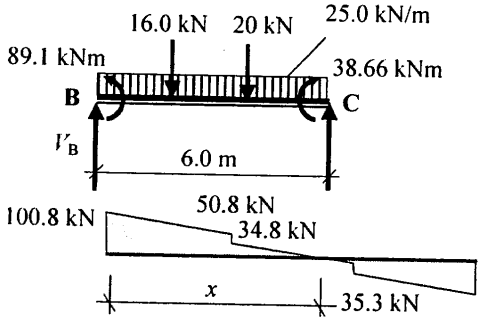



Bending Moment Diagram

* **The maximum value along the length of members BC and DC can be found by identifying the point of zero shear as follows:**

Member BC:




Shear Force Diagram

+ve $\circlearrowright \Sigma M_C = 0$

$-89.1 - (16.0 \times 4.0) - (20.0 \times 2.0)$
$- (25.0 \times 6.0 \times 3.0) + 38.66 + (V_B \times 6.0) = 0$

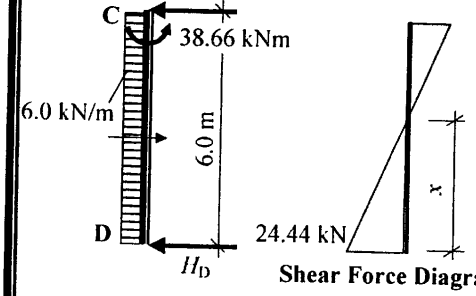
$V_B = +100.8$ kN

$x = 2.0 + (34.8/25.0) = 3.39$ m

$M_{maximum} = [0.5 \times (100.8 + 50.8) \times 2.0]$
$+ (0.5 \times 1.39 \times 34.8) - 89.1$

$M_{maximum} = \mathbf{86.67\ kNm}$

Member CD:




Shear Force Diagram

+ve $\circlearrowright \Sigma M_C = 0$

$-38.66 - (6.0 \times 6.0 \times 3.0) + (H_D \times 6.0) = 0$

$H_D = \mathbf{+24.44\ kN} \leftarrow$

$x = (24.4/6.0) = 4.07$ m

$M_{maximum} = (0.5 \times 4.07 \times 24.44) = \mathbf{49.74\ kNm}$

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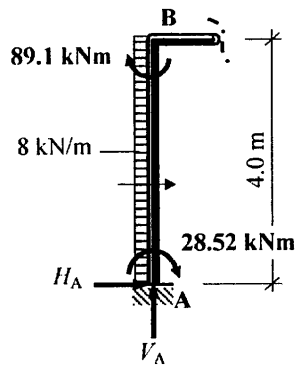
**Consider Member AB:**




Consider Member AB:

+ve $\circlearrowright \Sigma M_B = 0$

$+ 89.1 + 28.52 - (8.0 \times 4.0 \times 2.0) - (H_A \times 4.0) = 0$

$\therefore H_A = +13.41$ kN $\rightarrow$

For the complete frame:

+ve $\rightarrow \Sigma F_x = 0$

$13.41 + (8.0 \times 4.0) + (6.0 \times 6.0) - 24.44 - H_C = 0$ $\therefore H_C = +56.97$ kN $\leftarrow$

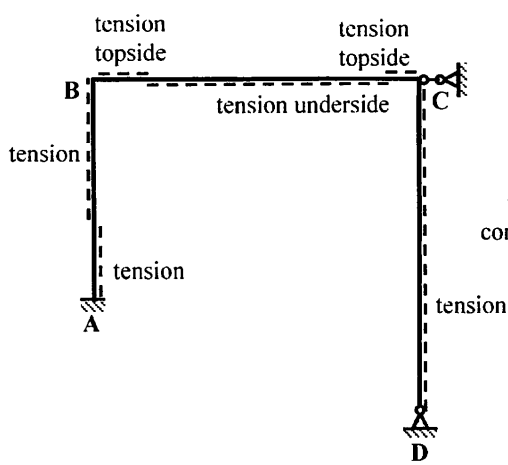
+ve $\circlearrowright \Sigma M_A = 0$

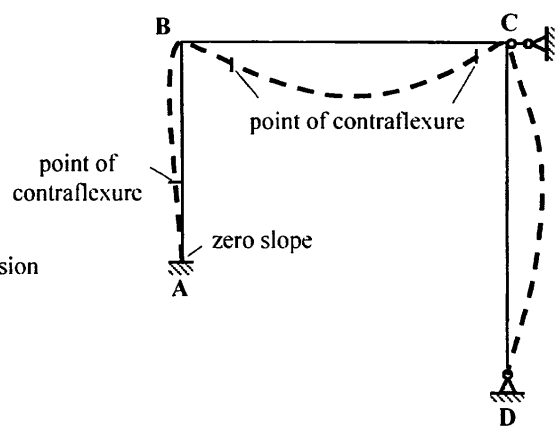
$+ 28.52 + (8.0 \times 4.0 \times 2.0) + (25.0 \times 6.0 \times 3.0) + (16.0 \times 2.0) + (20.0 \times 4.0)$
$- (56.97 \times 4.0) + (6.0 \times 6.0 \times 1.0) + (24.44 \times 2.0) - (V_D \times 6.0) = 0$

$\therefore V_D = +85.25$ kN $\uparrow$

+ve $\uparrow \Sigma F_y = 0$

$V_A - (25.0 \times 6.0) - 16.0 - 20.0 + 85.25 = 0$ $\therefore V_A = +100.75$ kN $\uparrow$




**Deflected Shape**