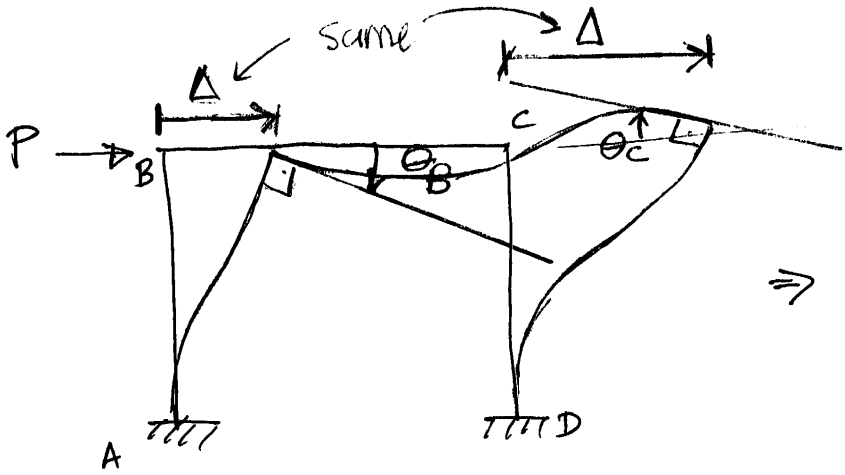
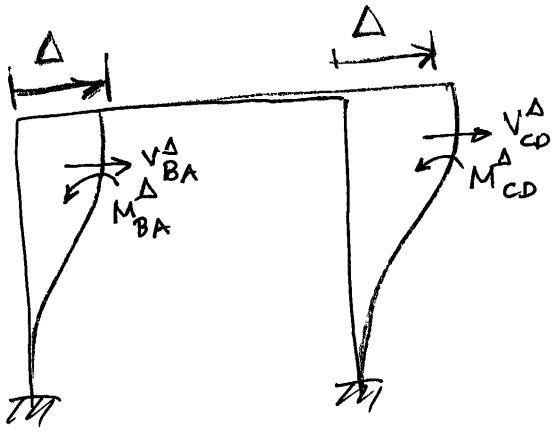
ex:



assuming axial deformations are negligible

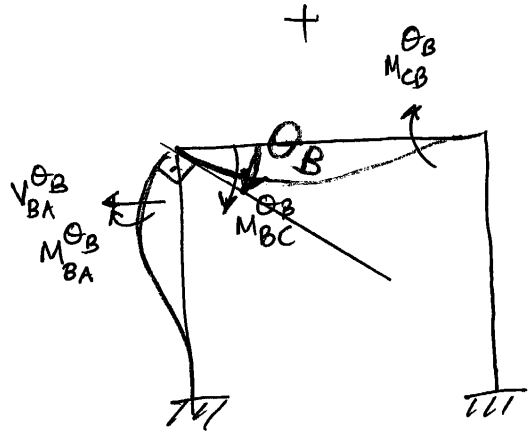
⇒ 3 DOF : Δ, θ_B, θ_C

=

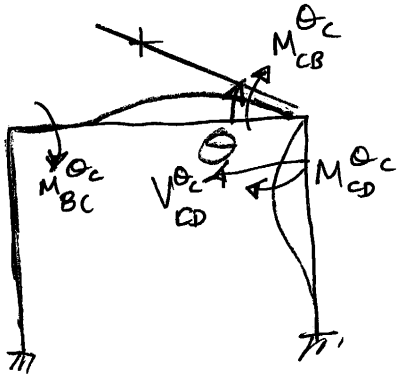


Δ only

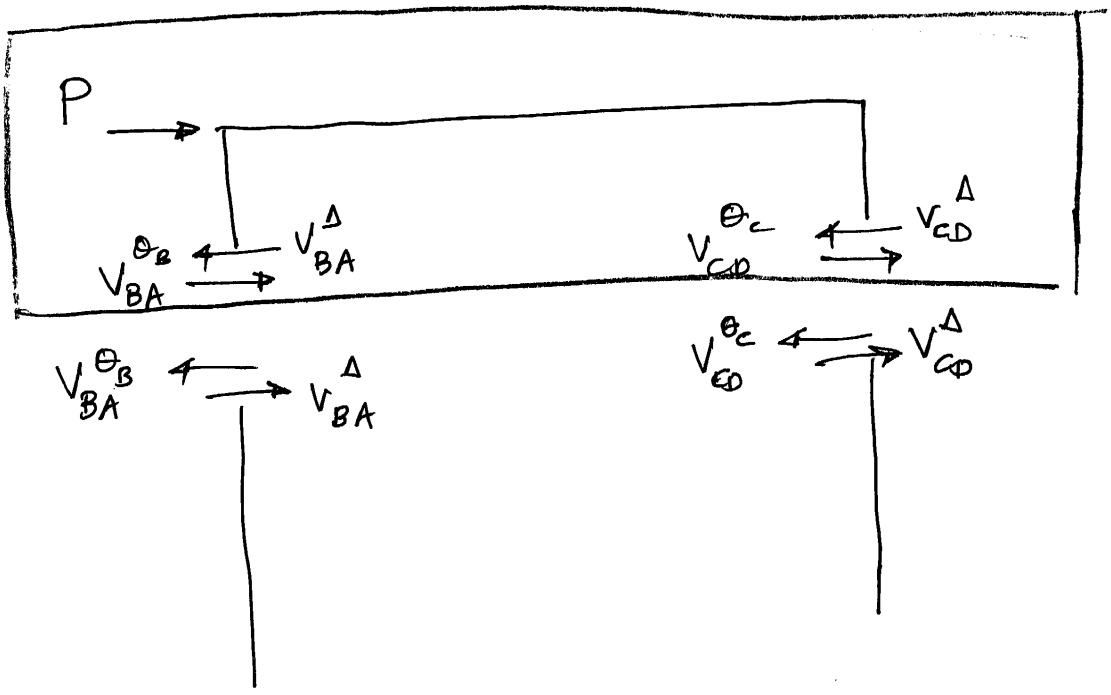
+



θ_B only



θ_C only



FBD associated w/ 1st degree of freedom

$$\Sigma F_{horiz} = 0$$

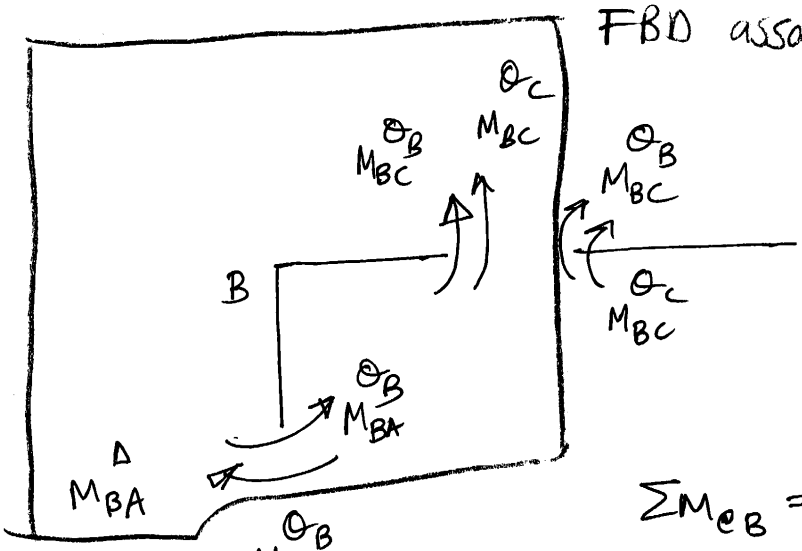
$$-V_{BA}^{\Delta} + V_{BA}^{\theta_B} - V_{CD}^{\Delta} + V_{CD}^{\theta_C} + P = 0$$

$$\Rightarrow \quad V_{BA}^{\Delta} + V_{CD}^{\Delta} - V_{BA}^{\theta_B} - V_{CD}^{\theta_C} = P$$

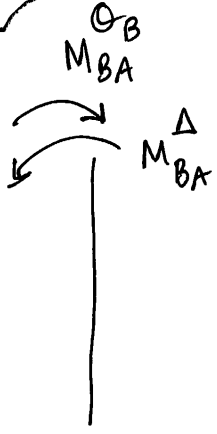
$$\frac{12EI}{L_{BA}^3}\Delta + \frac{12EI}{L_{CD}^3}\Delta - \frac{6EI}{L_{BA}^2}\theta_B - \frac{6EI}{L_{CD}^2}\theta_C = P \qquad \text{(1)}$$

1st eqn

FBD associated w/ 2nd DOF



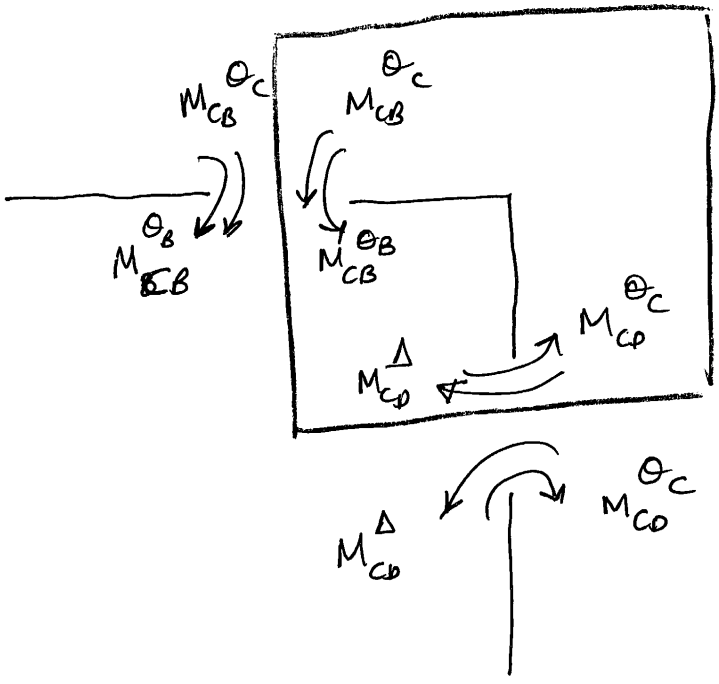
$\Sigma M_{@B} = 0$



$$\Rightarrow M_{BA}^{\theta_B} + M_{BC}^{\theta_B} + M_{BC}^{\theta_C} - M_{BA}^{\Delta} = 0 \quad \text{2nd eqn}$$

$$\frac{4EI}{L_{BA}}\theta_B + \frac{4EI}{L_{BC}}\theta_B + \frac{2EI}{L_{BC}}\theta_C - \frac{6EI}{L_{BA}^2}\Delta = 0 \quad (2)$$

FBD associated w/ 3rd DOF



$\Sigma M_{@C} = 0$

$$\Rightarrow M_{CB}^{\theta_B} + M_{CB}^{\theta_C} + M_{CD}^{\theta_C} - M_{CD}^{\Delta} = 0$$

$$\frac{2EI}{L_{CB}}\theta_B + \frac{4EI}{L_{CB}}\theta_C + \frac{4EI}{L_{CD}}\theta_C - \frac{6EI}{L_{CD}^2}\Delta = 0 \quad (3)$$

eqn. (1):
$$\left(\frac{12EI}{L_{BA}^3}+\frac{12EI}{L_{CD}^3}\right)\Delta-\frac{6EI}{L_{BA}^2}\theta_B-\frac{6EI}{L_{CD}^2}\theta_C=P$$

eqn. (2):
$$-\frac{6EI}{L_{BA}^2}\Delta+\left(\frac{4EI}{L_{BA}}+\frac{4EI}{L_{BC}}\right)\theta_B+\frac{2EI}{L_{BC}}\theta_C=0$$

eqn. (3):
$$-\frac{6EI}{L_{CD}^2}\Delta+\frac{2EI}{L_{CB}}\theta_B+\left(\frac{4EI}{L_{CB}}+\frac{4EI}{L_{CD}}\right)\theta_C=0$$

in matrix form

$$\begin{bmatrix} \left(\frac{12EI}{L_{BA}^3}+\frac{12EI}{L_{CD}^3}\right) & -\frac{6EI}{L_{BA}^2} & -\frac{6EI}{L_{CD}^2} \\ -\frac{6EI}{L_{BA}^2} & \left(\frac{4EI}{L_{BA}}+\frac{4EI}{L_{BC}}\right) & \frac{2EI}{L_{BC}} \\ -\frac{6EI}{L_{CD}^2} & \frac{2EI}{L_{CB}} & \left(\frac{4EI}{L_{CB}}+\frac{4EI}{L_{CD}}\right) \end{bmatrix} \begin{Bmatrix} \Delta \\ \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \\ 0 \end{Bmatrix}$$

$$[K]\qquad \{D\}=\{F\}$$

$$\{D\}=[K]^{-1}\{F\}$$