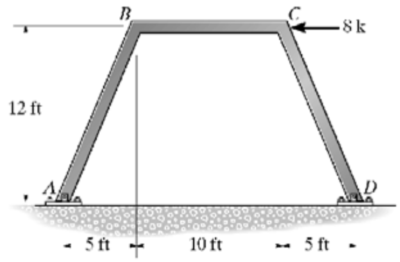
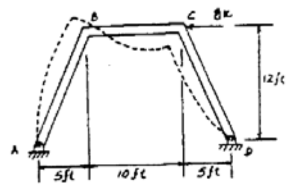
Slope-deflection examples with sloping members.

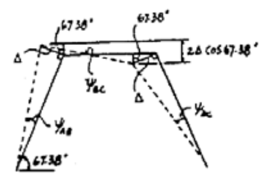
Note that in the first example, the pinned support conditions for the columns were taken advantage of and the “short-hand” slope-deflection formula is used in calculating the M_{BA} and M_{CD}.

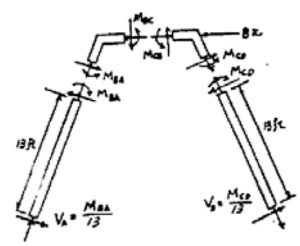
Example 1

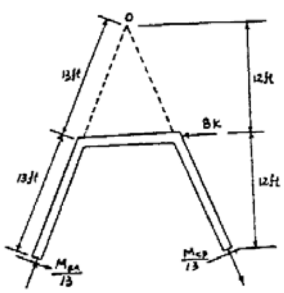
Determine the moment at each joint of the batter-column frame. The supports at *A* and *D* are pins. *EI* is constant.











$$(\text{FEM})_{BA} = (\text{FEM})_{BC} = (\text{FEM})_{CB} = (\text{FEM})_{CD} = 0$$

$$\psi_{AB} = \psi_{DC} = \frac{\Delta}{13} \qquad \psi_{BC} = \frac{2\Delta \cos 67.38°}{10}$$

$$\psi_{AB} = \psi_{DC} = \psi_{BC}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{13}\right)(\theta_B + \psi_{AB}) + 0$$

$$M_{BA} = 0.2308EI(\theta_B + \psi_{AB}) \qquad (1)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BC} = 2E\left(\frac{I}{10}\right)(2\theta_B + \theta_C - 3\,\psi_{AB}) + 0$$

$$M_{BC} = 0.2EI(2\theta_B + \theta_C - 3\,\psi_{AB}) + 0 \qquad (2)$$

$$M_{CB} = 2E\left(\frac{I}{10}\right)(2\theta_C + \theta_B - 3\,\psi_{AB}) + 0$$

$$M_{CB} = 0.2EI(2\theta_C + \theta_B - 3\,\psi_{AB}) + 0 \qquad (3)$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = 3E\left(\frac{I}{13}\right)(\theta_C + \psi_{AB}) + 0$$

$$M_{CD} = 0.2308EI(\theta_C + \psi_{AB}) \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

$$M_{CD} + M_{CB} = 0 \qquad (6)$$

$$\circlearrowleft + \Sigma M_O = 0: \quad \frac{M_{BA}}{13}(26) + \frac{M_{CD}}{13}(26) - 8(12) = 0$$

$$2M_{BA} + 2M_{CD} - 96 = 0 \qquad (7)$$

Solving these equations :

$$\theta_B = \theta_C = \frac{32}{EI}$$

$$\psi_{AB} = \frac{72}{EI}$$

$M_{BA} = 24$ k·ft **Ans**

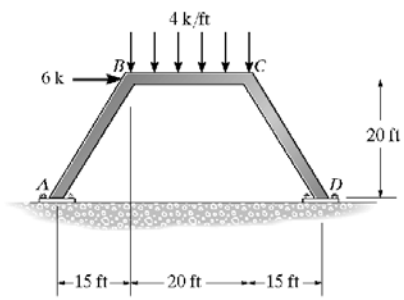
$M_{BC} = -24$ k·ft **Ans**

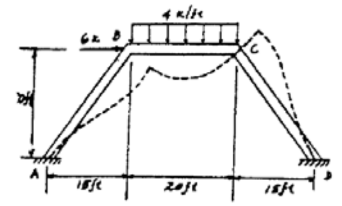
$M_{CB} = -24$ k·ft **Ans**

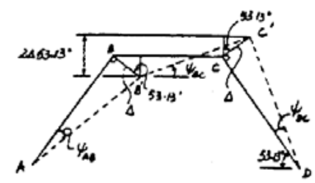
$M_{CD} = 24$ k·ft **Ans**

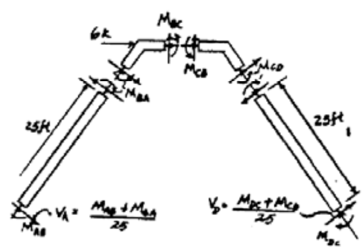
Example 2

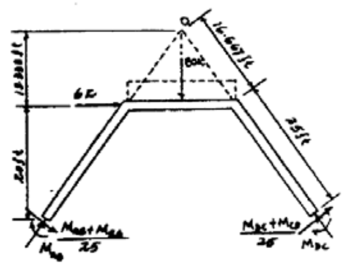
For the battered-column frame, determine the moments at each joint and at the fixed supports *A* and *D*. *EI* is constant.











$$\psi_{AB} = \frac{\Delta}{25}, \qquad \psi_{DC} = \frac{\Delta}{25}$$

$$\psi_{BC} = \frac{2\Delta \cos 53.13^\circ}{20} = 0.06\Delta$$

$$\psi_{AB} = \psi_{DC} = 0.6667\psi_{BC} \qquad (1)$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

$$(\text{FEM})_{BC} = \frac{-4(20)^2}{12} = -133.33 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{4(20)^2}{12} = 133.33 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{25}(0 + \theta_B - 3\psi_{AB}) + 0 \qquad (2)$$

$$M_{BA} = \frac{2EI}{25}(2\theta_B + 0 - 3\psi_{AB}) + 0 \qquad (3)$$

$$M_{BC} = \frac{2EI}{20}(2\theta_B + \theta_C + 3\psi_{BC}) - 133.3 \qquad (4)$$

$$M_{CB} = \frac{2EI}{20}(2\theta_C + \theta_B + 3\psi_{BC}) + 133.3 \qquad (5)$$

$$M_{CD} = \frac{2EI}{25}(2\theta_C + 0 - 3\psi_{DC}) + 0 \qquad (6)$$

$$M_{DC} = \frac{2EI}{25}(0 + \theta_C - 3\psi_{DC}) + 0 \qquad (7)$$

Equilibrium :

$$M_{BA} + M_{BC} = 0 \qquad (8)$$

$$M_{CB} + M_{CD} = 0 \qquad (9)$$

$$\circlearrowleft + \Sigma M_O = 0: \quad \left(\frac{M_{AB} + M_{BA}}{25}\right)(41.667) + \left(\frac{M_{DC} + M_{CD}}{25}\right)(41.667) - M_{AB} - M_{DC} + 6(13.333) = 0$$

$$0.6667M_{AB} + 1.6667M_{BA} + 1.6667M_{CD} + 0.6667M_{DC} = -80 \qquad (10)$$

Solving Eqs. 1 – 10,

$$\theta_B = \frac{487.0}{EI}$$

$$\theta_C = \frac{-538.7}{EI}$$

$$\psi_{AB} = \psi_{CD} = \frac{56.65}{EI}$$

$$\psi_{BC} = \frac{84.98}{EI}$$

$M_{AB} = 25.4$ k·ft **Ans**

$M_{BA} = 64.3$ k·ft **Ans**

$M_{BC} = -64.3$ k·ft **Ans**

$M_{CB} = 99.8$ k·ft **Ans**

$M_{CD} = -99.8$ k·ft **Ans**