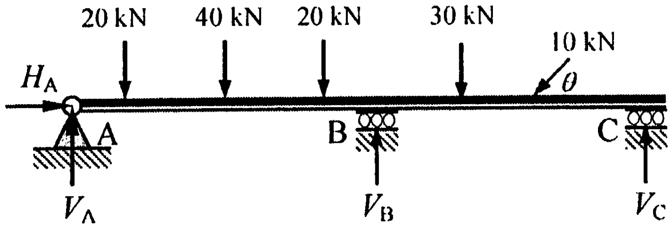
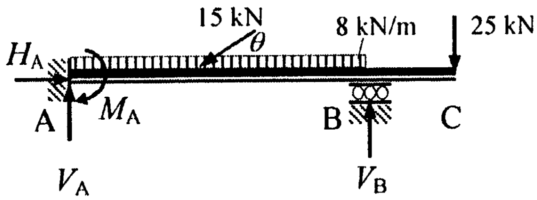
4.6 *Statically Indeterminate Beams*

In many instances multi-span beams are used in design, and consequently it is necessary to consider the effects of the continuity on the support reactions and member forces. Such structures are ***indeterminate*** (see Chapter 1) and there are more unknown variables than can be solved using only the three equations of equilibrium. A few examples of such beams are shown in Figure 4.57 (a) to (d).



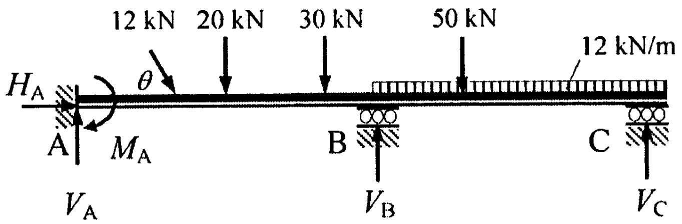
4 Unknown reactions:
1 horizontal
3 vertical

Figure 4.57(a)



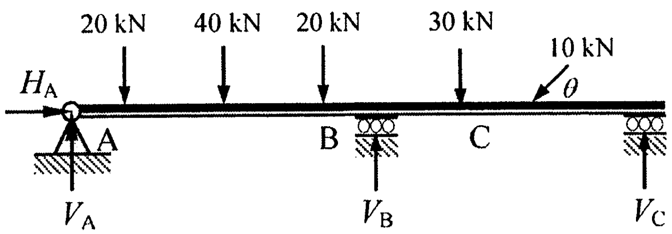
4 Unknown reactions:
1 horizontal
2 vertical
1 moment

Figure 4.57(b)



5 Unknown reactions:
1 horizontal
3 vertical
1 moment

Figure 4.57(c)



4 Unknown reactions:
1 horizontal
3 vertical

Figure 4.57(d)

A number of analysis methods are available for determining the support reactions, and member forces in indeterminate beams. In the case of singly-redundant beams the '*unit-load method*' can be conveniently used to analyse the structure. In multi-redundant structures the method of '*moment distribution*' is a particularly useful hand-method of analysis. These methods are considered in Sections 4.6.1 and 4.6.2 respectively.

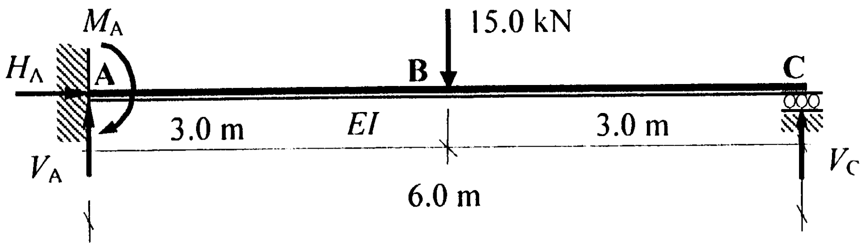
4.6.1 Unit Load Method for Singly–Redundant Beams

Using the method of analysis illustrated in Section 4.5 and considering the compatibility of displacements, member forces in singly–redundant beams can be determined as shown in Examples 4.17 and 4.18 and in Problems 4.24 to 4.27.

4.6.2 Example 4.17: Singly–Redundant Beam 1

A propped cantilever ABC is fixed at A, supported on a roller at C and carries a mid-span point load of 15 kN as shown in Figure 4.58,

(i) determine the value of the support reactions and
(ii) sketch the shear force and bending moment diagram.



E and *I* are constant.

Figure 4.58

The degree-of-indeterminacy $I_D = [(3m + r)] - 3n = [(3 \times 1) + 4] - (3 \times 2) = 1$

Assume that the reaction at C is the redundant reaction and consider the original beam to be the superposition of two beams as indicated in Figures 4.59(a) and (b). The beam in Figure 4.59(b) can be represented as shown in Figure 4.60. (**Note:** H_A = zero)

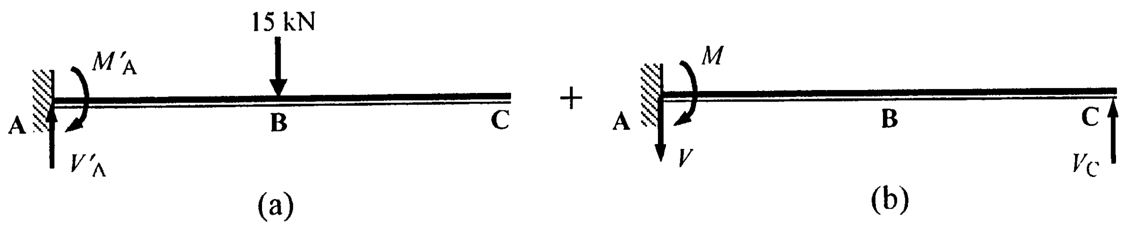


Figure 4.59

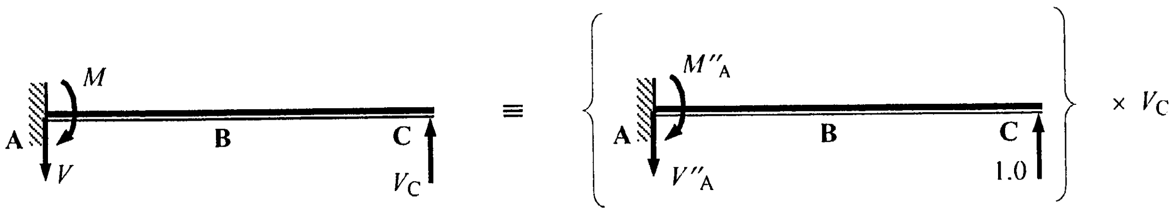


Figure 4.60

To maintain compatibility at the roller support, i.e. no resultant vertical displacement, the deformation of point C in Figure 4.59(a) must be equal and opposite to that in Figure 4.59(b) as shown in Figure 4.61.

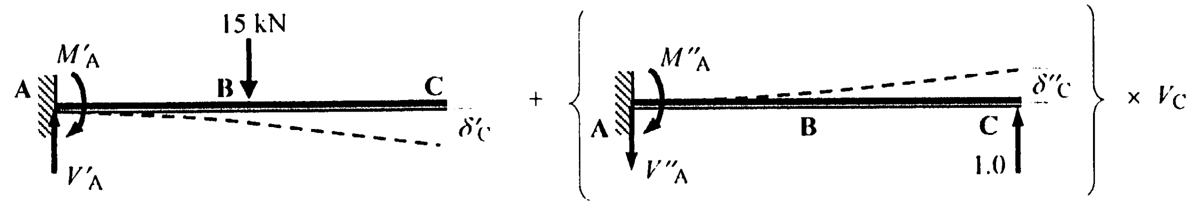


Figure 4.61

(δ' due to the applied load) + (δ'' due to the unit load) × $V_C = 0$

i.e. $\int_0^L \frac{Mm}{EI}dx + \left\{ \int_0^L \frac{mm}{EI}dx \right\} \times V_C = 0 \qquad \therefore \quad V_C = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx$

The product integrals can be evaluated as before in Section 4.5, e.g. using the coefficients in Table 4.1.

Solution:
The bending moment diagrams for the applied loads and a unit point load at B are shown in Figure 4.62.

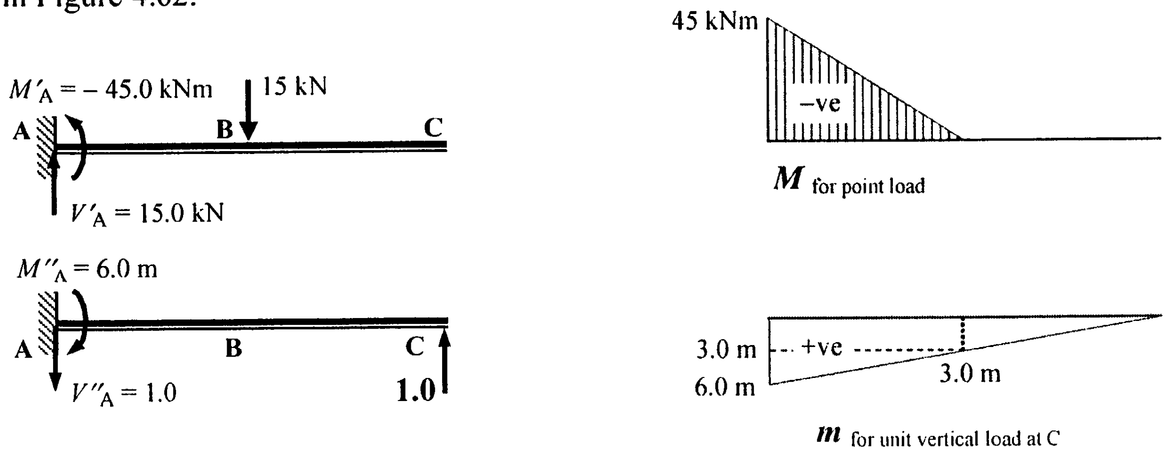


Figure 4.62

Using the coefficients given in Table 4.1:

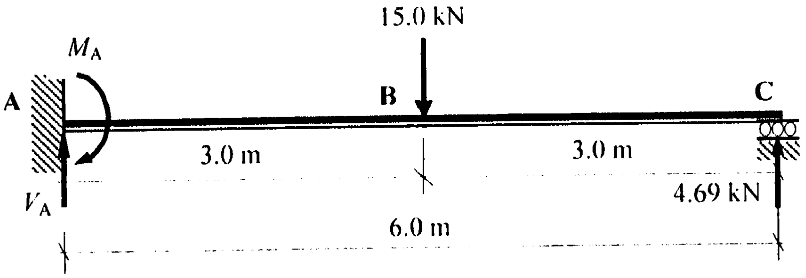
$$\delta'_{C,\text{ point load}} = \int_0^L \frac{Mm}{EI}dx$$

$$\delta'_{C,\text{ point load}} = [(\mathbf{0.5} \times 45.0 \times 3.0 \times 3.0) + (\mathbf{0.333} \times 45.0 \times 3.0 \times 3.0)]/EI = -\,337.5/EI$$

$$\delta''_{C,\text{ unit load}} = \int_0^L \frac{m^2}{EI}dx$$

$$\delta''_{C,\text{ unit load}} = (\mathbf{0.333} \times 6.0 \times 6.0 \times 6.0)/EI = +\,71.93/EI$$

$$V_C = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -\,(-\,337.5/EI)/(71.93/EI) = 4.69 \text{ kN}$$



$M_A = M'_A + (M''_A \times V_C) = -45.0 + (6.0 \times 4.69)$ $\quad\therefore$ $\mathbf{M_A = -16.86\ kNm}$

$M_B = M'_B + (M''_B \times V_C) = \text{zero} + (3.0 \times 4.69)$ $\quad\therefore$ $\mathbf{M_B = +14.07\ kNm}$

$V_A = V'_A + (V''_A \times V_C) = +15.0 - (1.0 \times 4.69)$ $\quad\therefore$ $\mathbf{V_A = +10.31\ kN}$

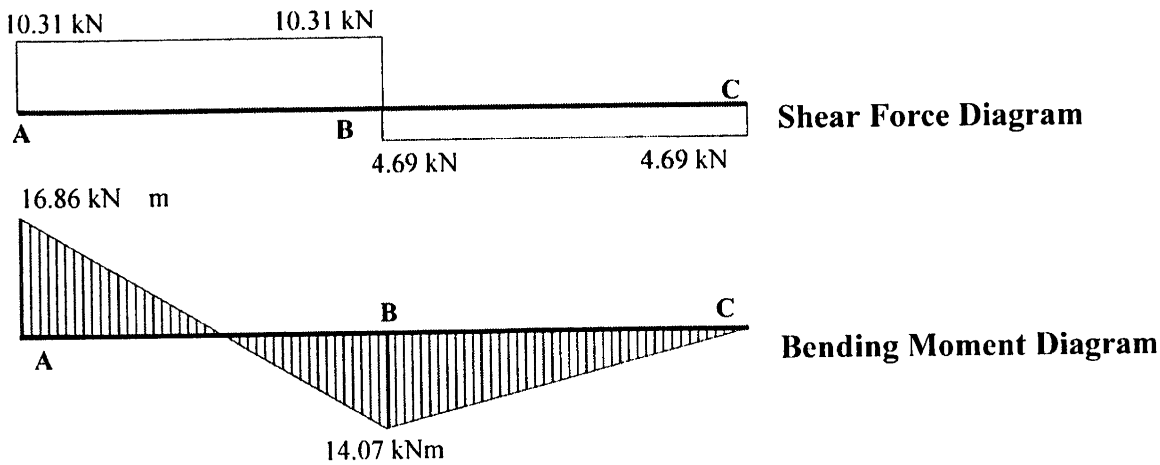


Figure 4.63

4.6.3 Example 4.18: Singly–Redundant Beam 2

A non-uniform, two-span beam ABCD is simply supported at A, B and D as shown in Figure 4.64. The beam carries a uniformly distributed load on span AB and a point at the mid-span point of BCD. Using the data given:

(i) determine the value of the support reactions,
(ii) sketch the shear force and bending moment diagrams.

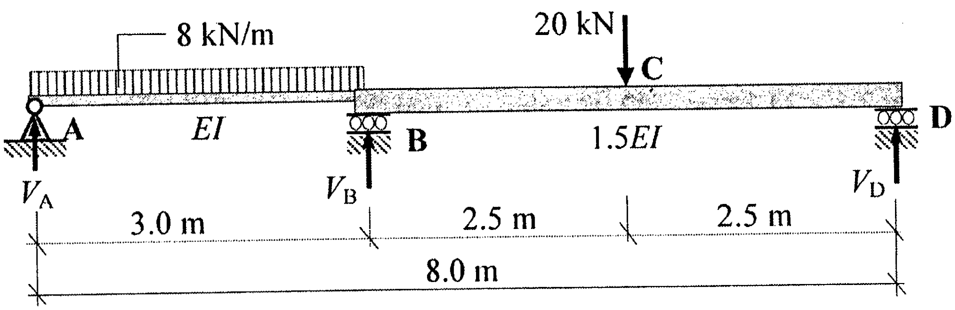


Figure 4.64

Solution:

Assume that the reaction at B is the redundant reaction. The bending moment diagrams for the applied loads and a unit point load at B are shown in Figure 4.65.

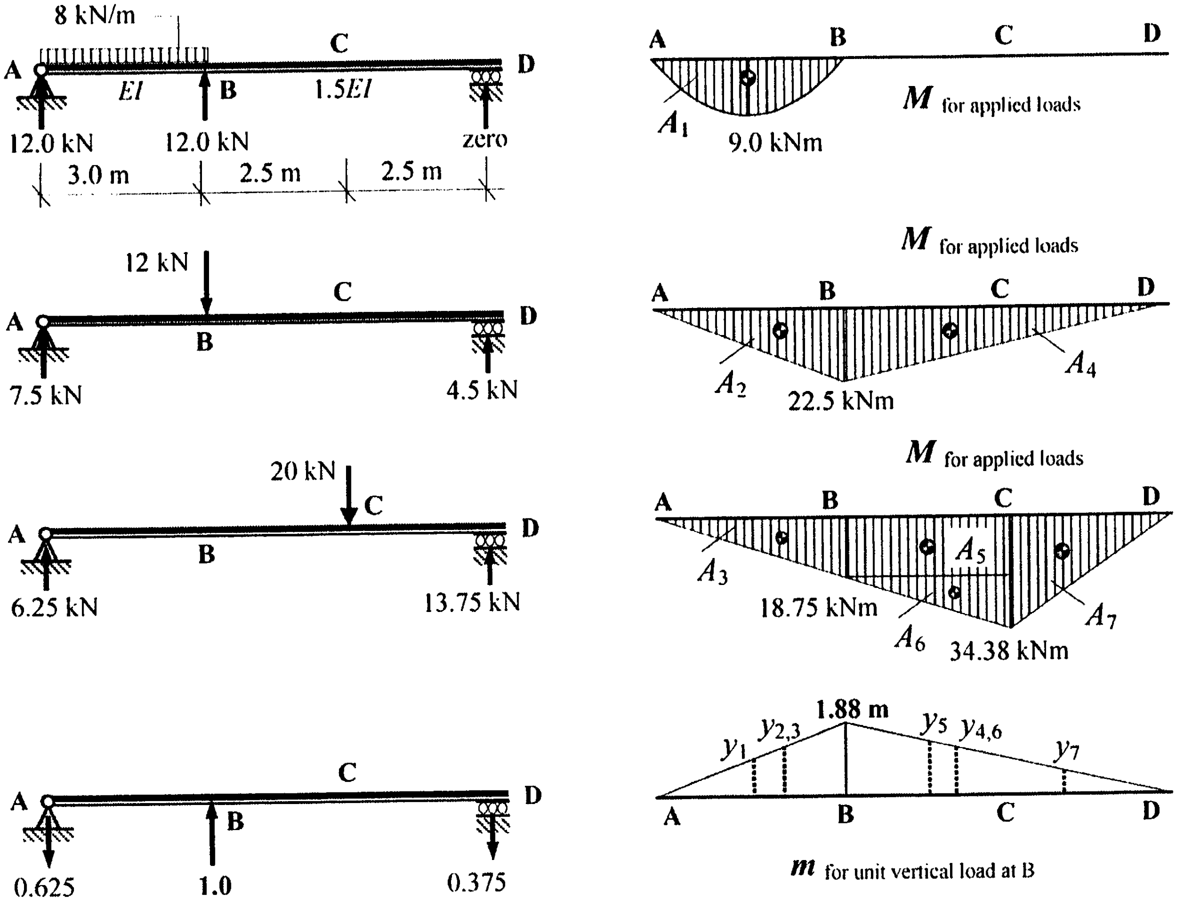


Figure 4.65

$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{EI}dx + \int_D^B \frac{Mm}{1.5EI}dx:$$

$A_1 = +(0.667 \times 3.0 \times 9.0) = +18.0\ \text{kNm}^2$, $y_1 = -0.94$ m, $\therefore A_1y_1 = -16.92\ \text{kNm}^3$

$A_2 = +(0.5 \times 3.0 \times 22.5) = +33.75\ \text{kNm}^2$, $y_2 = -1.25$ m, $\therefore A_2y_2 = -41.29\ \text{kNm}^3$

$A_3 = +(0.5 \times 3.0 \times 18.75) = +28.13\ \text{kNm}^2$, $y_3 = -1.25$ m, $\therefore A_3y_3 = -35.16\ \text{kNm}^3$

$A_4 = +(0.5 \times 5.0 \times 22.5) = +52.25\ \text{kNm}^2$, $y_4 = -1.25$ m, $\therefore A_4y_4 = -65.31\ \text{kNm}^3$

$A_5 = +(2.5 \times 18.75) = +46.88\ \text{kNm}^2$, $y_5 = -1.41$ m, $\therefore A_5y_5 = -66.10\ \text{kNm}^3$

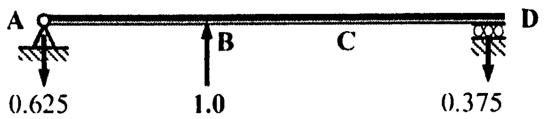
$A_6 = +(0.5 \times 2.5 \times 15.63) = +19.54\ \text{kNm}^2$, $y_6 = -1.25$ m, $\therefore A_6y_6 = -24.43\ \text{kNm}^3$

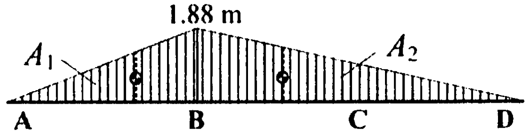
$A_7 = +(0.5 \times 2.5 \times 34.38) = +42.98\ \text{kNm}^2$, $y_7 = -0.63$ m, $\therefore A_7y_7 = -27.08\ \text{kNm}^3$

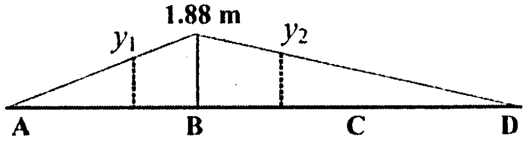
$$\int_0^L \frac{Mm}{EI}dx = \sum_{n=1}^{n=7} \frac{(A_n y_n)}{EI}$$

$$= -[(16.92 + 41.29 + 35.16)/EI + (65.31 + 66.10 + 24.43 + 27.08)/1.5EI]$$

$$= -216.13/EI$$







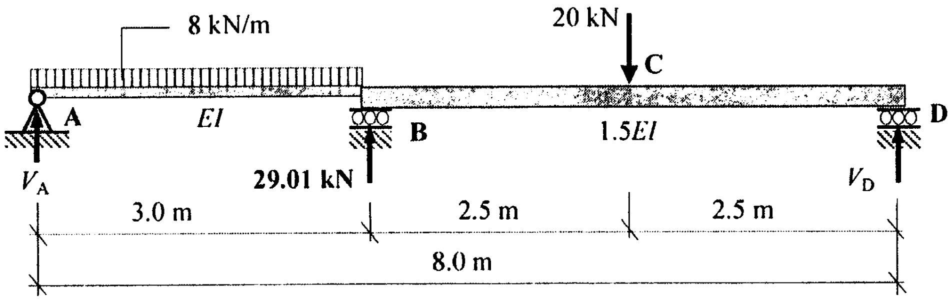
$$\int_0^L \frac{m^2}{EI}dx = \int_A^B \frac{m^2}{EI}dx + \int_D^B \frac{m^2}{1.5EI}dx:$$

$A_1 = -(0.5 \times 3.0 \times 1.88) = -2.82\ m^2$, $\quad y_1 = -1.25$ m, $\quad \therefore A_1y_1 = +3.53\ m^3$

$A_2 = -(0.5 \times 5.0 \times 1.88) = -4.70\ kNm^2$, $\quad y_2 = -1.25$ m, $\quad \therefore A_2y_2 = +5.88\ m^3$

$$\int_0^L \frac{m^2}{EI}dx = \sum_{n=1}^{n=2}\frac{(A_n y_n)}{EI} = [\,+(3.53/EI) + (5.88/1.5EI)\,] = +7.45/EI$$

$$V_B = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -(-216.13/EI)/(7.45/EI) = +29.01\text{ kN} \uparrow$$

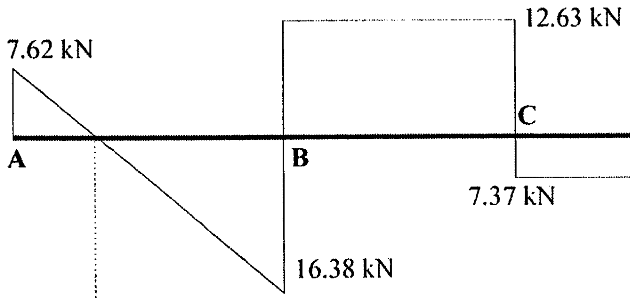


$V_A = +12.0 + 7.5 + 6.25 - (0.625 \times 29.01)$ $\quad \therefore$ **$V_A = +7.62$ kNm** $\uparrow$

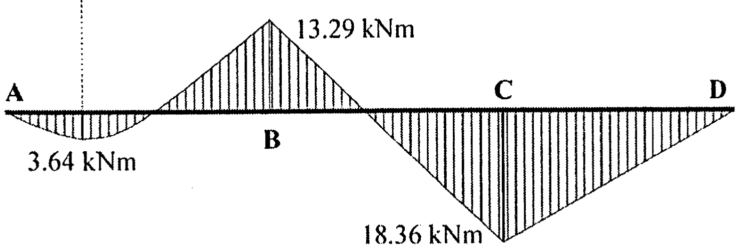
$V_D = \text{zero} + 4.5 + 13.75 - (0.375 \times 29.01)$ $\quad \therefore$ **$V_D = +7.37$ kNm** $\uparrow$

$M_B = +22.5 + 18.75 - (1.88 \times 29.01)$ $\quad \therefore$ **$M_B = -13.29$ kN**

$M_C = +11.25 + 34.38 - (0.94 \times 29.01)$ $\quad \therefore$ **$M_C = -18.36$ kN**



Shear Force Diagram



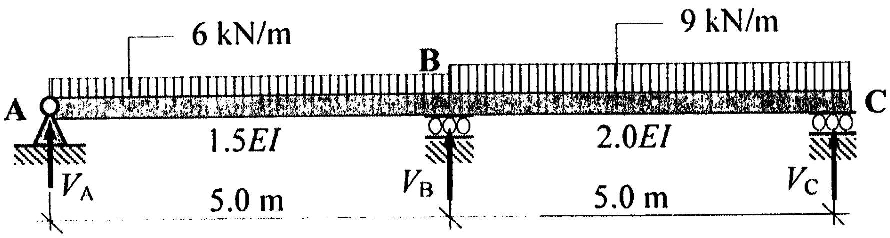
Bending Moment Diagram

Figure 4.66

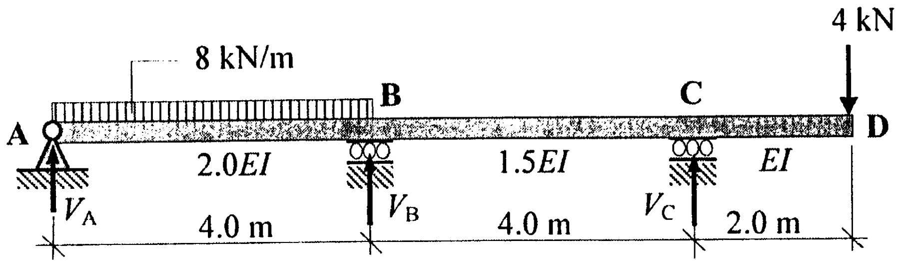
4.6.4 Problems: Unit Load Method for Singly-Redundant Beams

A series of singly-redundant beams are indicated in Problems 4.24 to 4.27. Using the applied loading given in each case:

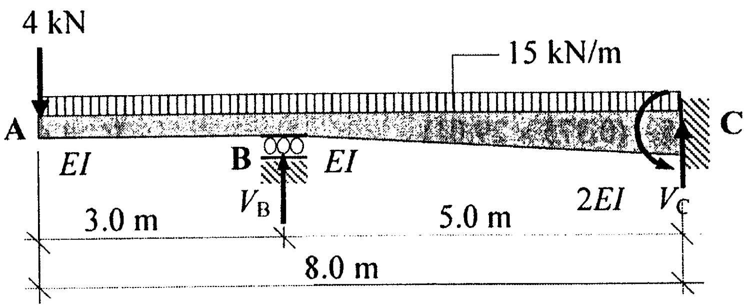
i) determine the support reactions,
ii) sketch the shear force diagram and
iii) sketch the bending moment diagram.



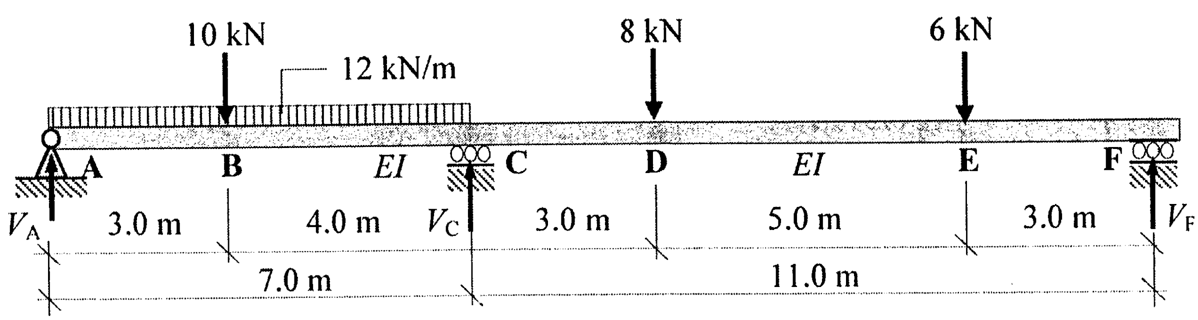
Problem 4.24



Problem 4.25



Problem 4.26



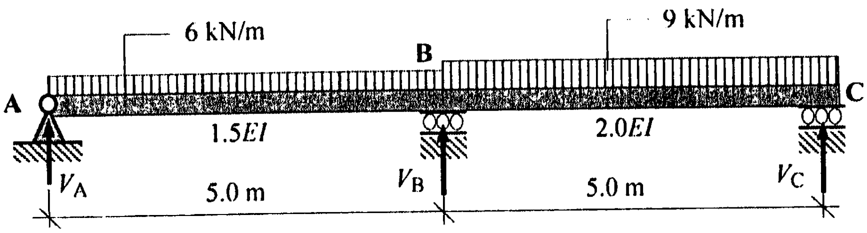
Support C settles by 4.0 mm and $EI = 100.0 \times 10^3$ kNm^2

Problem 4.27

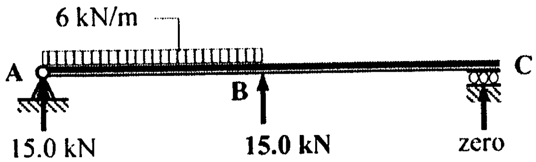
4.6.5 Solutions: Unit Load Method for Singly-Redundant Beams

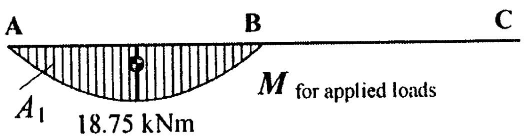
Solution

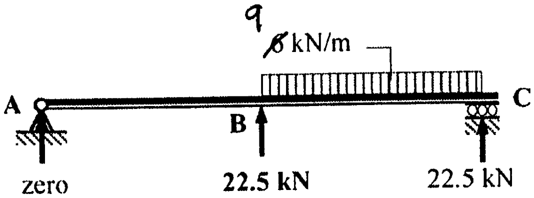
Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.24 **Page No. 1**

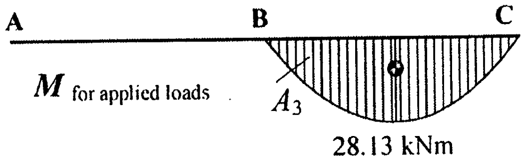


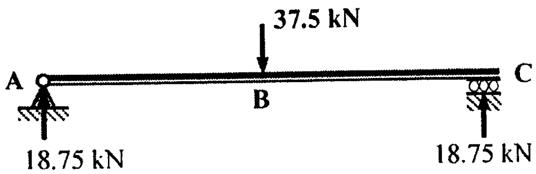
Determine the value of the support reactions and sketch the shear force and bending moment diagrams.
Assume that the reaction at B is the redundant reaction.

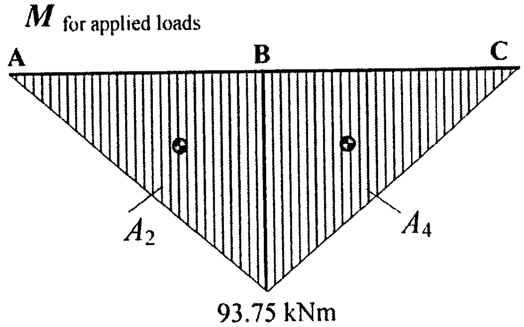


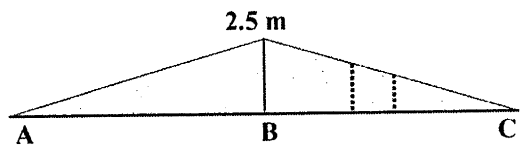




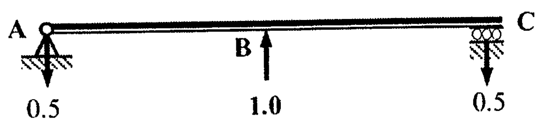








m for unit vertical load at B



$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{1.5EI}dx + \int_C^B \frac{Mm}{2.0EI}dx$$

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.24 **Page No. 2**

Using the coefficients given in Table 4.1:

$$\int_0^L \frac{Mm}{EI}dx = \sum_0^L (Coefficient \times a \times b \times L)/EI$$

$$\int_0^L \frac{Mm}{EI}dx = [-(0.333 \times 18.75 \times 2.5 \times 5.0) - (0.333 \times 93.75 \times 2.5 \times 5.0)]/1.5EI$$

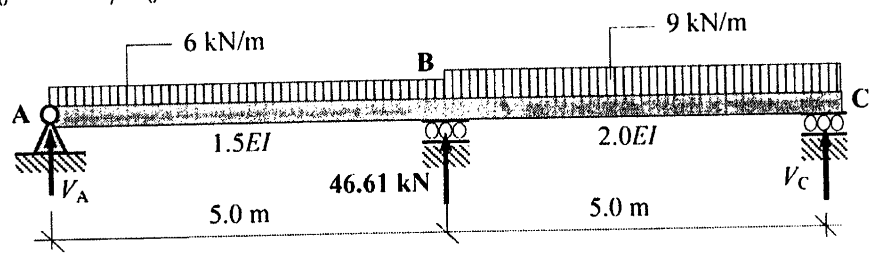
$$+ [-(0.333 \times 28.13 \times 2.5 \times 5.0) - (0.333 \times 93.75 \times 2.5 \times 5.0)]/2.0EI$$

$$= \mathbf{-565.85/EI}$$

$$\int_0^L \frac{m^2}{EI}dx = +(0.333 \times 2.5 \times 2.5 \times 5.0)/1.5EI + (0.333 \times 2.5 \times 2.5 \times 5.0)/2.0EI$$

$$= \mathbf{+12.14/EI}$$

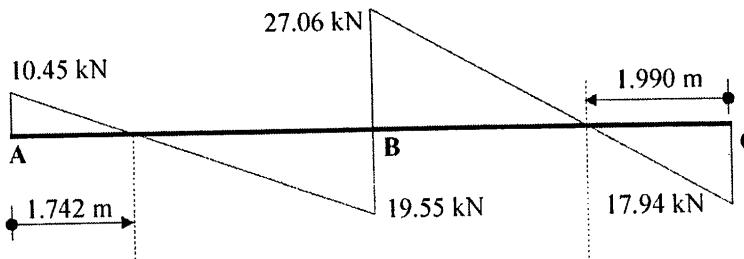
$$V_B = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -(-565.85/EI)/(12.14/EI) = \mathbf{+46.61\ kN} \uparrow$$



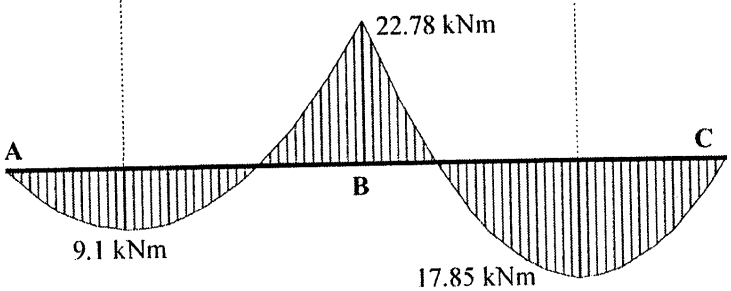
$V_A = +15.0 + 18.75 - (0.5 \times 46.61)$ $\therefore$ $V_A = \mathbf{+10.45\ kN}$ ↑

$V_C = +22.5 + 18.75 - (0.5 \times 46.61)$ $\therefore$ $V_C = \mathbf{+17.94\ kN}$ ↑

$M_B = +93.75 - (2.5 \times 46.61)$ $\therefore$ $M_B = \mathbf{-22.78\ kNm}$



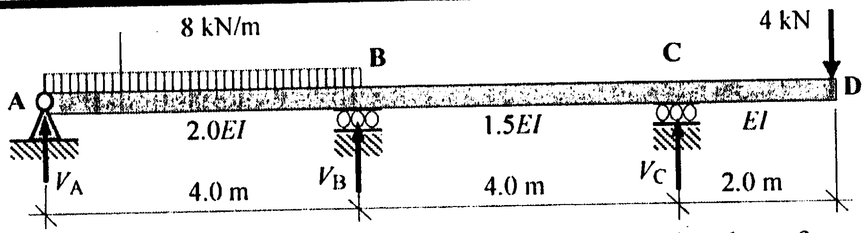
Shear Force Diagram



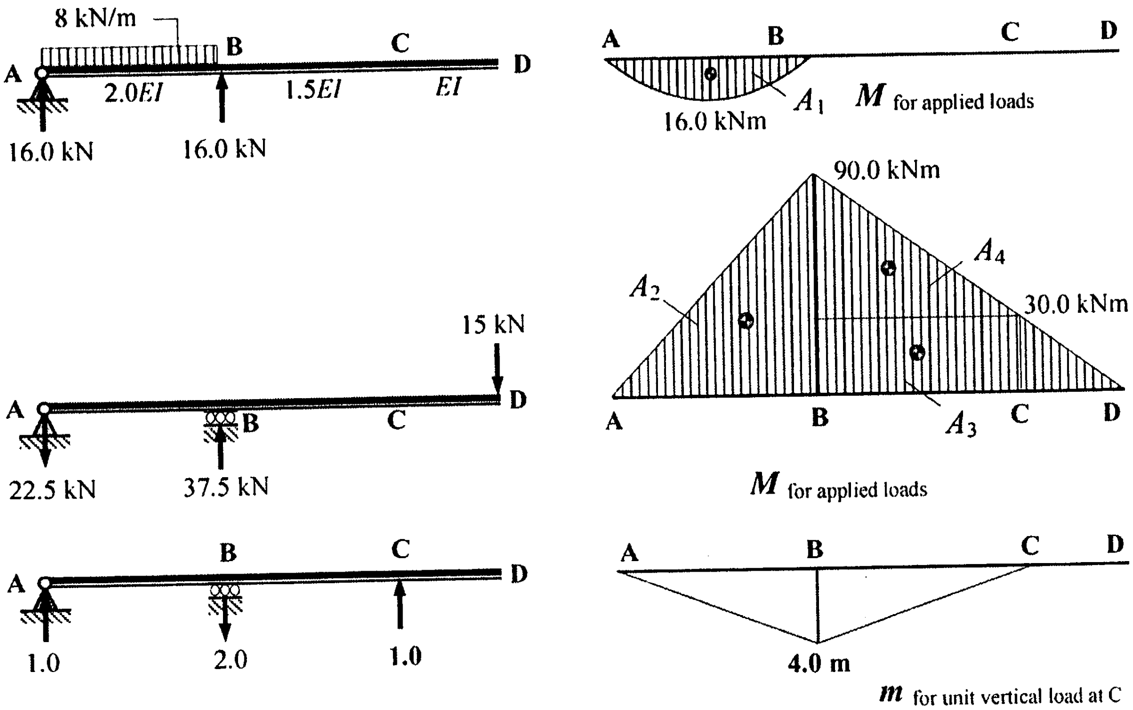
Bending Moment Diagram

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.25 **Page No. 1**



Determine the value of the support reactions and sketch the shear force and bending moment diagrams.
Assume that the reaction at C is the redundant reaction.



$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{2.0EI}dx + \int_C^B \frac{Mm}{2.0EI}dx + \int_D^C \frac{Mm}{EI}dx$$

↘ zero since m is equal to zero

Using the coefficients given in Table 4.1:

$$\int_0^L \frac{Mm}{EI}dx = \sum_0^L (Coefficient \times a \times b \times L)/EI$$

$$\int_0^L \frac{Mm}{EI}dx = [+ (0.333 \times 16.0 \times 4.0 \times 4.0) - (0.333 \times 90.0 \times 4.0 \times 4.0)]/2.0EI$$

$$+ [- (0.5 \times 30.0 \times 4.0 \times 4.0) - (0.333 \times 60.0 \times 4.0 \times 4.0)]/1.5EI$$

$$= \mathbf{-570.26/EI}$$

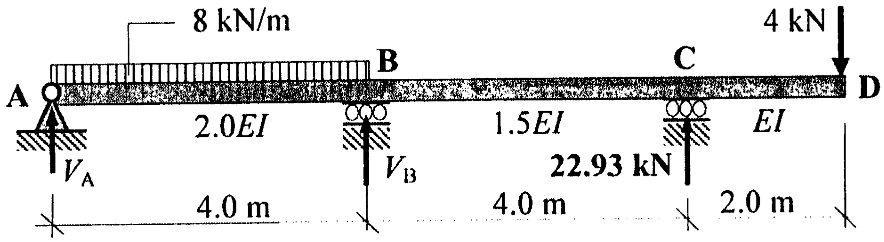
Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.25 **Page No. 2**

$$\int_0^L \frac{m^2}{EI} dx = +(0.333 \times 4.0 \times 4.0 \times 4.0)/2.0EI + (0.333 \times 4.0 \times 4.0 \times 4.0)/1.5EI$$

$$= \mathbf{+\ 24.87/EI}$$

$$V_C = -\int_0^L \frac{Mm}{EI} dx \bigg/ \int_0^L \frac{m^2}{EI} dx = -(-570.26/EI)/(24.87/EI) = \mathbf{+\ 22.93\ kN} \uparrow$$

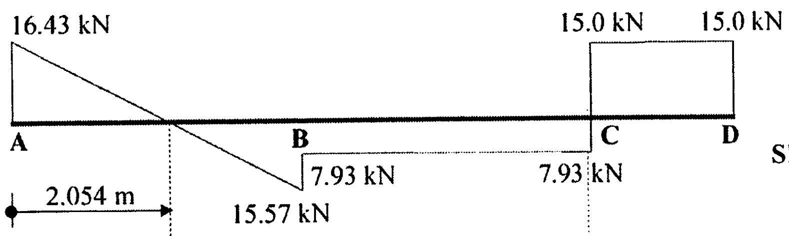


$V_A = +16.0 - 22.5 + (1.0 \times 22.93)$ $\therefore$ $\mathbf{V_A = +\ 16.43\ kN}$ $\uparrow$

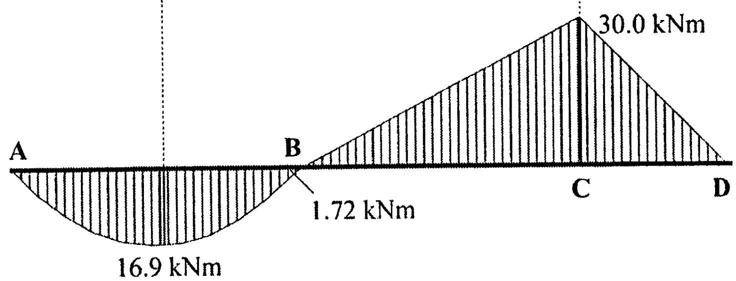
$V_B = +16.0 + 37.5 - (2.0 \times 22.93)$ $\therefore$ $\mathbf{V_C = +\ 7.64\ kN}$ $\uparrow$

$M_B = -90.0 + (4.0 \times 22.93)$ $\therefore$ $\mathbf{M_B = +\ 1.72\ kNm}$

$M_C = -30.0$ $\therefore$ $\mathbf{M_C = -\ 30.0\ kNm}$



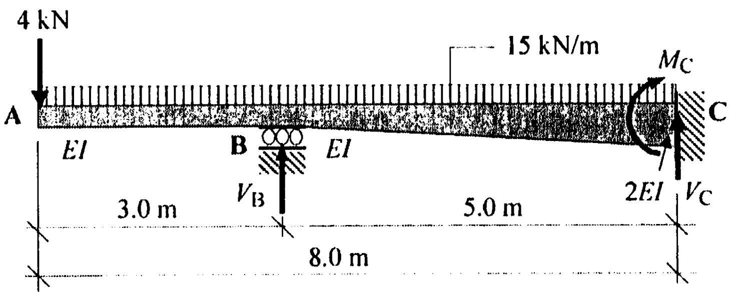
Shear Force Diagram



Bending Moment Diagram

Solution

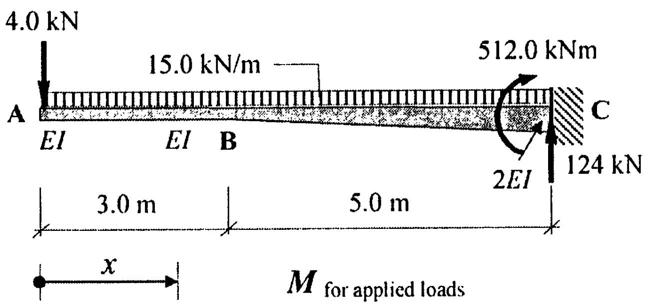
Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.26 **Page No. 1**



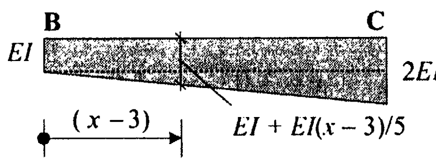
The EI value of the beam BC varies linearly from EI at support B to $2.0EI$ at C.

Determine the value of the support reactions and sketch the shear force and bending moment diagrams.

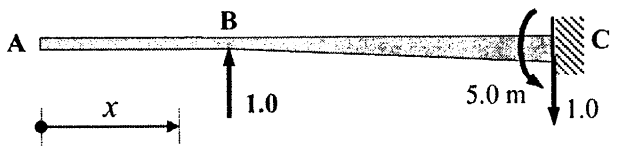
Assume that the reaction at B is the redundant reaction.



M for applied loads



The value of EI at a distance of x m from A is given by: $EI\,(0.4 + 0.2x)$



m for unit vertical load at B

(Mm/EI) is not a continuous function the product integral must be evaluated between each of the discontinuities, i.e. A to B and B to C.

The value of I at position 'x' along the beam between B and C is given by: $EI\,(0.4 + 0.2x)$

$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{EI}dx + \int_B^C \frac{Mm}{EI(0.4+0.2x)}dx$$

Solution

Topic: Unit Load – Singly-Redundant Beams
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Consider the section from A to B: $\mathbf{0 \le x \le 3.0\ m}$

m = zero $\qquad \therefore\ Mm$ = zero

$$\int_A^B \frac{Mm}{EI}dx = \text{zero}$$

Consider the section from B to A: $\mathbf{3.0 \le x \le 8.0\ m}$

$M = -4.0x - 15.0x^2/2 = -4.0x - 7.5x^2$

$m = +1.0(x-3)$

$Mm = (x-3)(-4.0x - 7.5x^2) = 12.0x + 18.5x^2 - 7.5x^3$

$m^2 = +(x-3)^2$

$$\int_B^C \frac{Mm}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{12.0x + 18.5x^2 - 7.5x^3}{EI(0.4+0.2x)}dx$$

Let $v = (0.4 + 0.2x)$ $\qquad \therefore\ x = (5v - 2)$ $\qquad$ and $\qquad dx = 5dv$

$x^2 = (25v^2 - 20v + 4.0)$

$x^3 = (125v^3 - 150v^2 + 60v - 8.0)$

when $\quad x = 3.0 \quad v = 1.0 \qquad$ and when $\quad x = 8.0 \quad v = 2.0$

$Mm = 12.0x + 18.5x^2 - 7.5x^3$

$\quad = 12.0(5v-2) + 18.5(25v^2 - 20v + 4.0) - 7.5(125v^3 - 150v^2 + 60v - 8.0)$

$\quad = (-760v + 110 + 1587.5v^2 - 937.5v^3)$

$$\int_B^C \frac{Mm}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{12.0x + 18.5x^2 - 7.5x^3}{EI(0.4+0.2x)}dx$$

$$= \int_{1.0}^{2.0} \frac{-760v + 110 + 1587.5v^2 - 937.5v^3}{EIv} 5.0dv$$

$$= \frac{5.0}{EI}\int_{1.0}^{2.0}\left(-760 + \frac{110}{v} + 1587.5v - 937.5v^2\right)dv$$

$$= \frac{5.0}{EI}\left[-760v + 110\,ln\,v + \frac{1587.5v^2}{2.0} - \frac{937.5v^3}{3.0}\right]_{1.0}^{2.0}$$

$$= \frac{5.0}{EI}\left[(-768.8) - (-278.8)\right] = +\frac{2450.0}{EI}\ \text{m}$$

Solution

Topic: Unit Load – Singly-Redundant Beams
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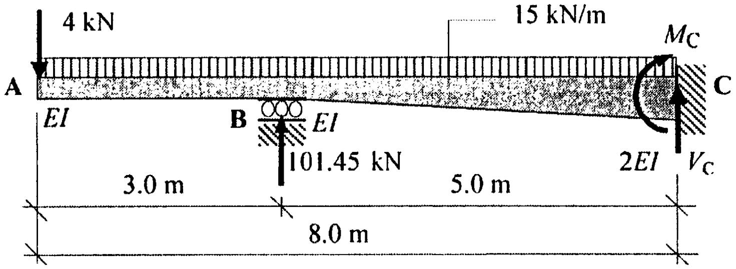
$m^2 = +(x-3)^2 \quad = x^2 - 6x + 9.0 \quad = 25v^2 - 50v + 25.0$

$$\int_B^C \frac{m^2}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{x^2 - 6.0x + 9.0}{EI(0.4+0.2x)}dx = \int_{1.0}^{2.0} \frac{25.0v - 50.0v + 25.0}{EIv}5.0dv$$

$$= \frac{5.0}{EI}\int_{1.0}^{2.0}\left(25.0v - 50.0 + \frac{25.0}{v}\right)dv = \frac{5.0}{EI}\left[\frac{25.0v^2}{2.0} - 50.0v + 25.0 \ln v\right]_{1.0}^{2.0}$$

$$= \frac{5.0}{EI}\left[(-32.67) - (-37.5)\right] = +\frac{24.15}{EI}\text{m}$$

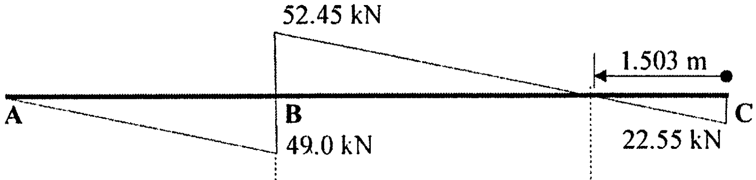
$$V_B = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -(-2450/EI)/(24.15/EI) = +\mathbf{101.45\ kN} \uparrow$$



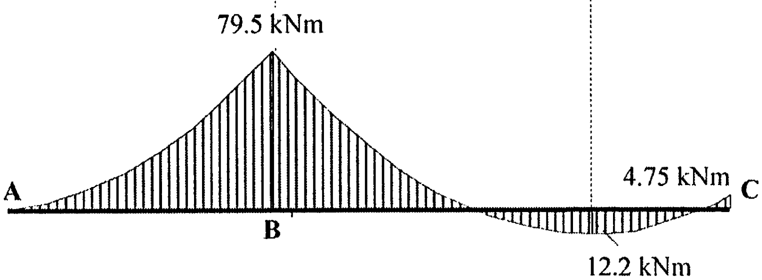
$V_C = +124.0 - (1.0 \times 101.45)$ $\quad\therefore$ $\mathbf{V_A = +22.55\ kN}$ ↑

$M_C = -512.0 + (5.0 \times 101.45)$ $\quad\therefore$ $\mathbf{M_C = -4.75\ kNm}$

$M_B = -(4.0 \times 3.0) - (15.0 \times 3.0)(1.5)$ $\quad\therefore$ $\mathbf{M_B = -79.5\ kNm}$



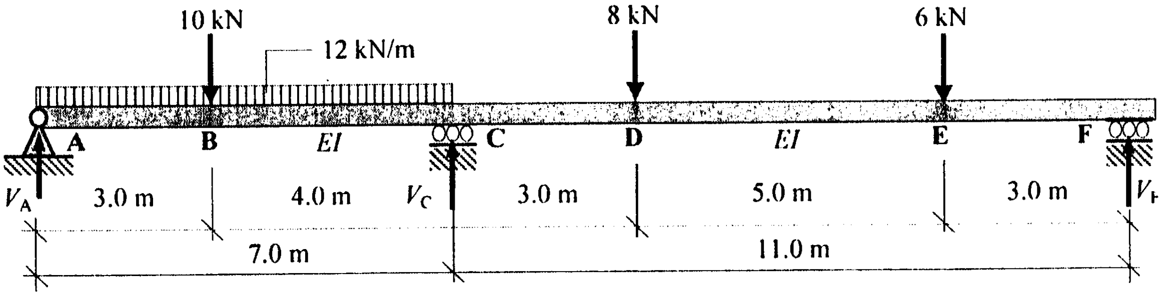
Shear Force Diagram



Bending Moment Diagram

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.27 **Page No. 1**

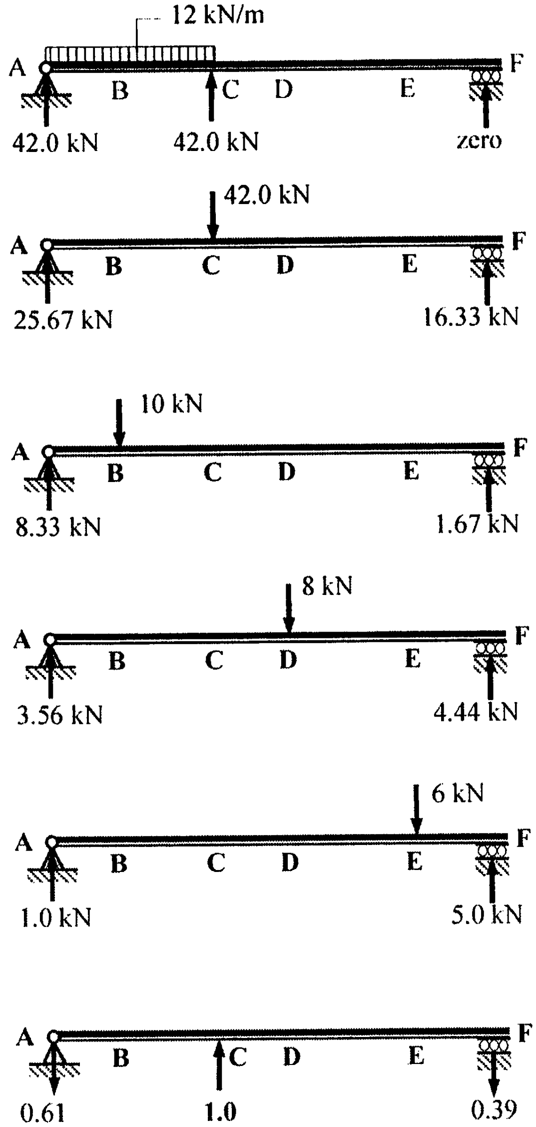


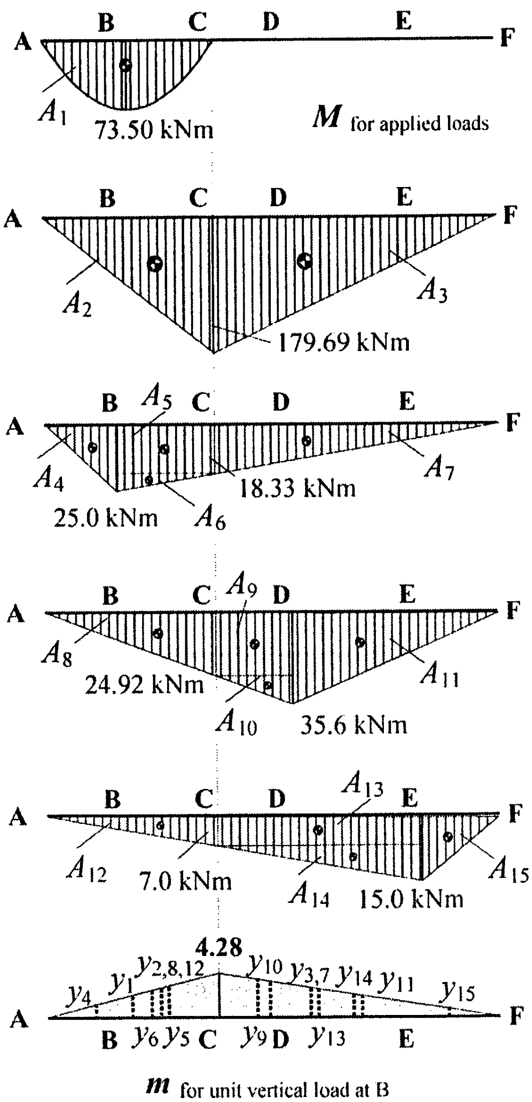
Support C settles by 4.0 mm and $EI = 100.0 \times 10^3$ kNm^2

Determine the value of the support reactions and sketch the shear force and bending moment diagrams.

Assume that the reaction at C is the redundant reaction.-

Note: B.M. diagrams not to scale





Solution

Topic: Unit Load – Singly-Redundant Beams
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$$\int_0^L \frac{Mm}{EI}dx + \left\{\int_0^L \frac{mm}{EI}dx\right\} \times V_C = -0.004 \qquad V_C = -\left(0.004 + \int_0^L \frac{Mm}{EI}dx\right) \Big/ \int_0^L \frac{m^2}{EI}dx$$

$$\int_0^L \frac{Mm}{EI}dx = \int_A^F \frac{Mm}{EI}dx$$

(**Note:** The reader should check this using the coefficients given in Table 4.1).

$A_1 = +(0.67 \times 7.0 \times 73.5) = +344.72$ kNm^2
y_1 (3.5 m from A) $= -2.14$ m $\quad \therefore A_1y_1 = -737.70$ kNm^3
$A_2 = +(0.5 \times 7.0 \times 179.69) = +628.92$ kNm^2
y_2 (4.67 m from A) $= -2.85$ m $\quad \therefore A_2y_2 = -1792.42$ kNm^3
$A_3 = +(0.5 \times 11.0 \times 179.69) = +988.30$ kNm^2
y_3 (7.33 m from F) $= -2.85$ m $\quad \therefore A_3y_3 = -2816.66$ kNm^3
$A_4 = +(0.5 \times 3.0 \times 25.0) = +37.50$ kNm^2
y_4 (2.0 m from A) $= -1.22$ m $\quad \therefore A_4y_4 = -45.75$ kNm^3
$A_5 = +(4.0 \times 18.33) = +73.32$ kNm^2
y_5 (5.0 m from A) $= -3.05$ m $\quad \therefore A_5y_5 = -223.63$ kNm^3
$A_6 = +(0.5 \times 4.0 \times 6.67) = +13.34$ kNm^2
y_6 (4.33 m from A) $= -2.64$ m $\quad \therefore A_6y_6 = -35.22$ kNm^3
$A_7 = +(0.5 \times 11.0 \times 18.33) = +100.82$ kNm^2
y_7 (7.33 m from F) $= -2.85$ m $\quad \therefore A_7y_7 = -287.34$ kNm^3
$A_8 = +(0.5 \times 7.0 \times 24.92) = +87.22$ kNm^2
y_8 (4.67 m from A) $= -2.85$ m $\quad \therefore A_8y_8 = -248.58$ kNm^3
$A_9 = +(3.0 \times 24.92) = +74.76$ kNm^2
y_9 (9.50 m from F) $= -3.71$ m $\quad \therefore A_9y_9 = -277.36$ kNm^3
$A_{10} = +(0.5 \times 3.0 \times 10.68) = +16.02$ kNm^2
y_{10} (9.0 m from F) $= -3.51$ m $\quad \therefore A_{10}y_{10} = -56.23$ kNm^3
$A_{11} = +(0.5 \times 8.0 \times 35.6) = +142.4$ kNm^2
y_{11} (5.33 m from F) $= -2.08$ m $\quad \therefore A_{11}y_{11} = -296.19$ kNm^3
$A_{12} = +(0.5 \times 7.0 \times 7.0) = +24.50$ kNm^2
y_{12} (4.67 m from A) $= -2.85$ m $\quad \therefore A_{12}y_{12} = -69.83$ kNm^3
$A_{13} = +(8.0 \times 7.0) = +56.0$ kNm^2
y_{13} (7.0 m from F) $= -2.73$ m $\quad \therefore A_{13}y_{13} = -152.88$ kNm^3
$A_{14} = +(0.5 \times 8.0 \times 8.0) = +32.0$ kNm^2
y_{14} (5.67 m from F) $= -2.21$ m $\quad \therefore A_{14}y_{14} = -70.72$ kNm^3
$A_{15} = +(0.5 \times 3.0 \times 15.0) = +22.5$ kNm^2
y_{15} (2.0 m from F) $= -0.78$ m $\quad \therefore A_{15}y_{15} = -17.55$ kNm^3

$$\int_0^L \frac{Mm}{EI}dx = \sum_{n=1}^{n=15} \frac{(A_n y_n)}{EI} = -7128.06/EI \text{ m}$$

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.27 **Page No. 3**

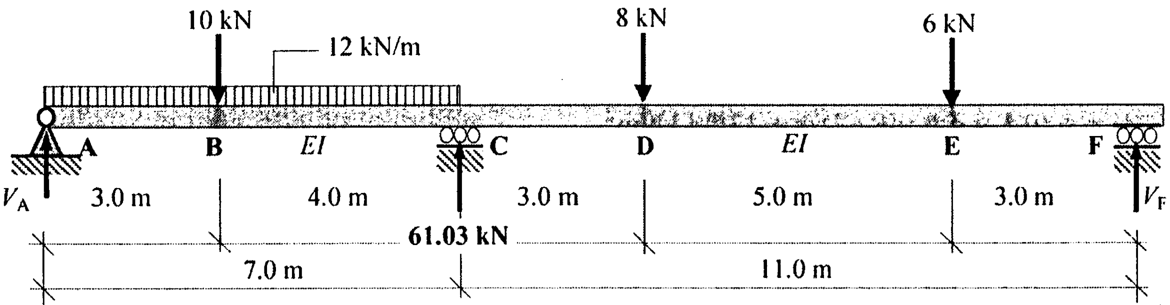
$$\int_0^L \frac{m^2}{EI}dx = \int_A^F \frac{m^2}{EI}dx$$

$A_1 = +(0.5 \times 7.0 \times 4.28) = -14.98\ \text{kNm}^2$
y_1 (4.67 m from A) $= -2.85$ m $\quad \therefore A_1y_1 = +42.69\ \text{kNm}^3$
$A_2 = +(0.5 \times 11.0 \times 4.28) = -23.57\ \text{kNm}^2$
y_2 (7.33 m from A) $= -2.85$ m $\quad \therefore A_2y_2 = +67.09\ \text{kNm}^3$

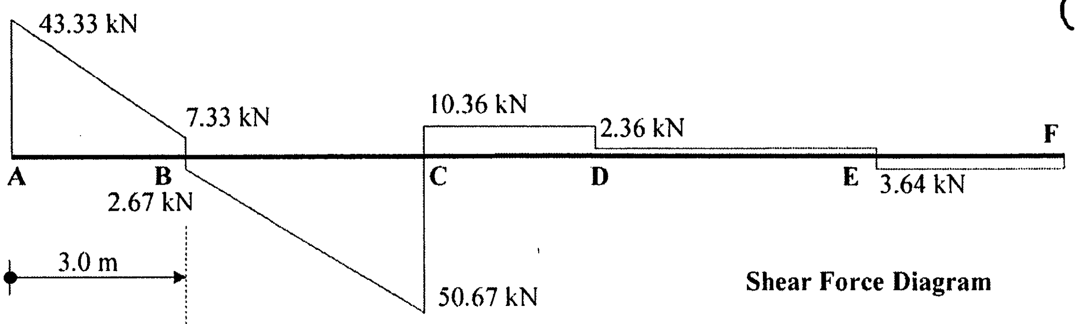
$$\int_0^L \frac{m^2}{EI}dx = \sum_{n=1}^{n=2}\frac{(A_n y_n)}{EI} = [+(42.69/EI) + (67.09/EI)] = +109.78/EI$$

$$V_C = -\left(0.004 + \int_0^L \frac{Mm}{EI}dx\right)\Big/\int_0^L \frac{m^2}{EI}dx = -(0.004 - 7128.06/EI)/\ 109.78/EI$$

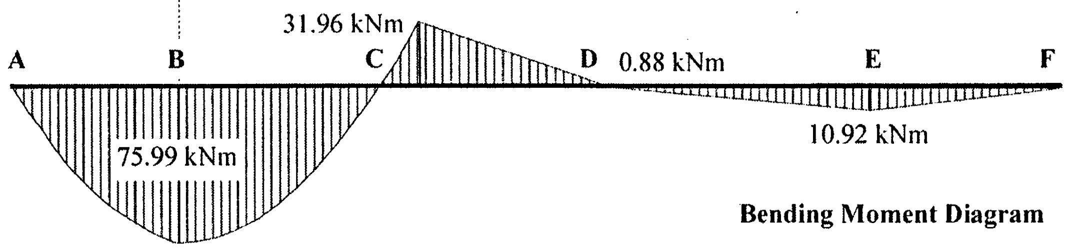
$$= +61.03\ \text{kN}$$



$V_A = +42.0 + 25.67 + 8.33 + 3.56 + 1.0 - (0.61 \times 61.03)$ $\quad \therefore$ $\mathbf{V_A = +43.33\ kN}$
$V_F = +16.33 + 1.67 + 4.44 + 5.0 - (0.39 \times 61.03)$ $\quad \therefore$ $\mathbf{V_F = +3.64\ kN}$
$M_C = +179.69 + 18.33 + 24.92 + 7.0 - (4.28 \times 61.03)$ $\quad \therefore$ $\mathbf{M_C = +31.27\ kNm}$



Shear Force Diagram



Bending Moment Diagram