CE573

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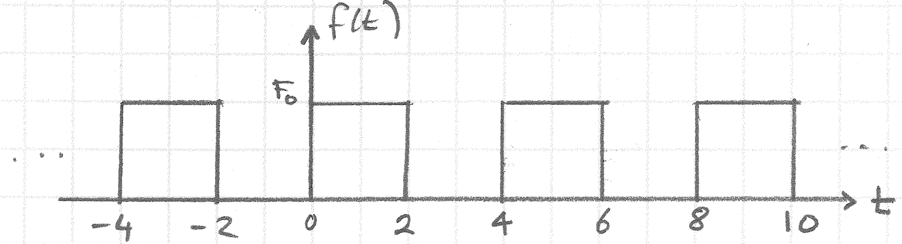
Fourier Series Example



Find the Fourier series expansion for $f(t)$.

$$T_0 = 4 \text{ sec} \quad , \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Fourier series representation

$$f(t) = a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t) + a_2 \cos(2 \cdot \omega_0 t) + b_2 \sin(2 \cdot \omega_0 t) + \ldots$$

$$a_0 = \frac{1}{T_0} \int_0^4 f(t) = \frac{2F_0}{4} = \frac{F_0}{2}$$

NB: we could have picked any 4 sec wide time window. $0 \sim 4$ sec is just convenient

a_i , $i \neq 0$:

$$a_i = \frac{2}{T_0} \int_0^{T_0} f(t) \cos(i \omega_0 t)\, dt$$

$$a_i = \frac{2}{4} \underbrace{\int_0^4 f(t) \cos(i \omega_0 t)\, dt}_{= \int_0^2 F_0 \cos(i \cdot \omega_0 t) dt} = \frac{F_0}{2} \left. \frac{\sin(i \overset{\frac{\pi}{2}}{\omega_0} t)}{i\omega_0} \right|_0^2 = 0$$

because $f(t) = 0$ over $2 \sim 4$ sec

$\Rightarrow a_i = 0$ for all $i > 0$, i.e. $i = 1, 2, \ldots$

b_i :

$$b_i = \frac{2}{T_0} \int_0^{T_0} f(t) \sin(i\omega_0 t)\, dt$$

$$b_i = \frac{2}{4} \underbrace{\int_0^4 f(t) \sin(i\omega_0 t) dt}_{= \int_0^2 F_0 \sin(i\omega_0 t) dt + \int_2^4 0 \cdot \cancel{\sin(i\omega_0 t)} dt} = \frac{F_0}{2} \left. \left(\frac{-\cos(i\omega_0 t)}{i\omega_0} \right) \right|_0^2$$

$$b_i = \frac{F_0}{i\pi} \left(1 - \cos(i\pi)\right) \Rightarrow \text{for } i = \text{odd \#},\ b_i = \frac{2F_0}{i\pi}$$

for i = even #, $b_i = 0$

$$\therefore f(t) = \frac{F_0}{2} + \sum_{i=1,3,5\ldots}^{\infty} \frac{2F_0}{i\pi} \sin\left(i \frac{\pi}{2} t\right)$$