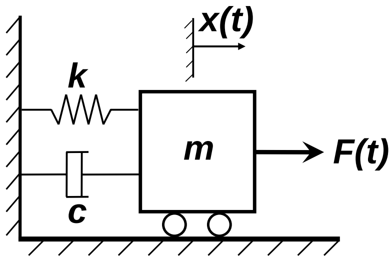
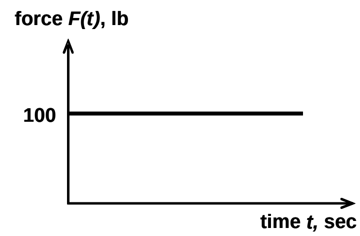
CE 573 – Structural Dynamics
Homework # 1

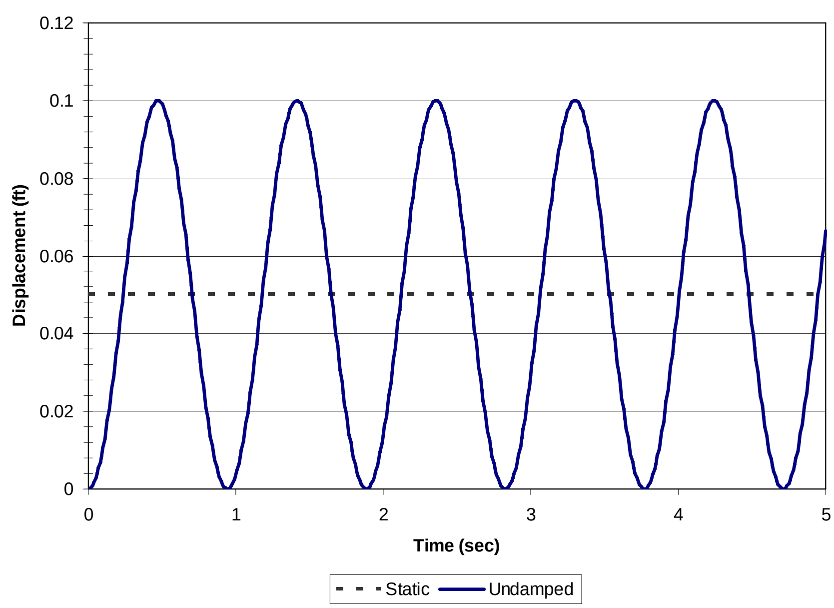




I used average acceleration method. Period of the system is 0.94 sec, so step-size of 0.01 sec is acceptable.

a) undamped case

i) See figure below for displacement vs. time graph.



ii) - The hypothetical "static" deflection (at any instant) is simply *F/k* as in "static" solutions, we ignore time-dependent resistance in the system (i.e. ignore inertial and velocity-induced resistance).
- Dynamic response oscillates about the static response. Since there is no damping in the system the oscillation about the static response is perpetual.

$$x_{static}^{\max} = 100/2000 = 0.05\,ft$$

and

$$x_{dynamic}^{\max} = 0.1\,ft$$

Note that the maximum dynamic response is twice that of the "static" response.

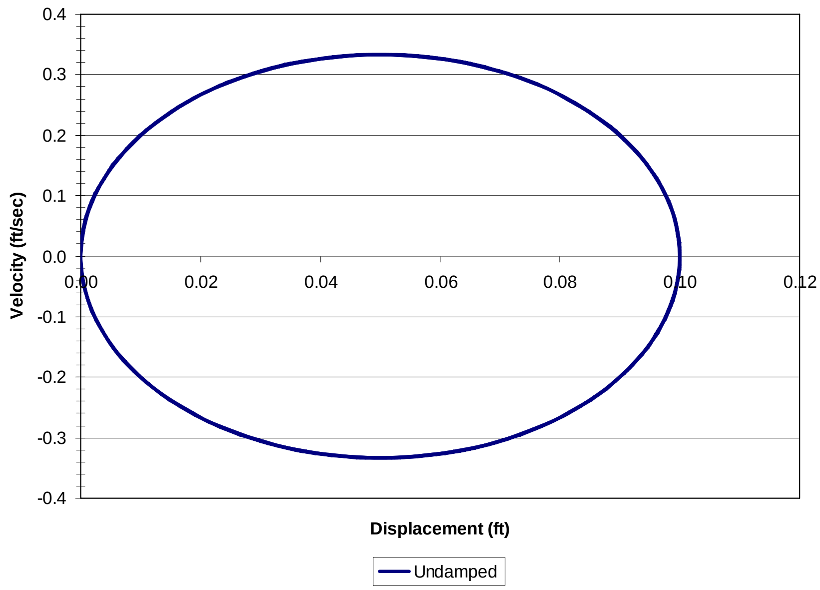
iii) Force in the linear spring is proportional to the displacement from its "unstretched" position (let *x=0* be that position).

$$x^{max}_{dynamic} = 0.1\,ft \;\; F^{max}_{static} = k\,g x^{max}_{static} = 2000 g 0.05 = 100 kip$$

and

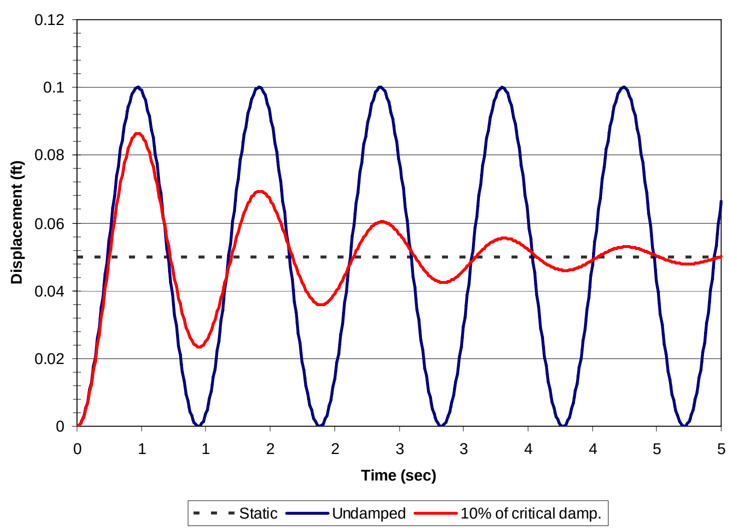
$$F^{max}_{dynamic} = k \bullet x^{max}_{dynamic} = 2000 \bullet 0.1 = 200 kip$$

iv) See figure below for displacement vs. velocity graph. This graph is also known as the *phase-plane* plot of the dynamic system.



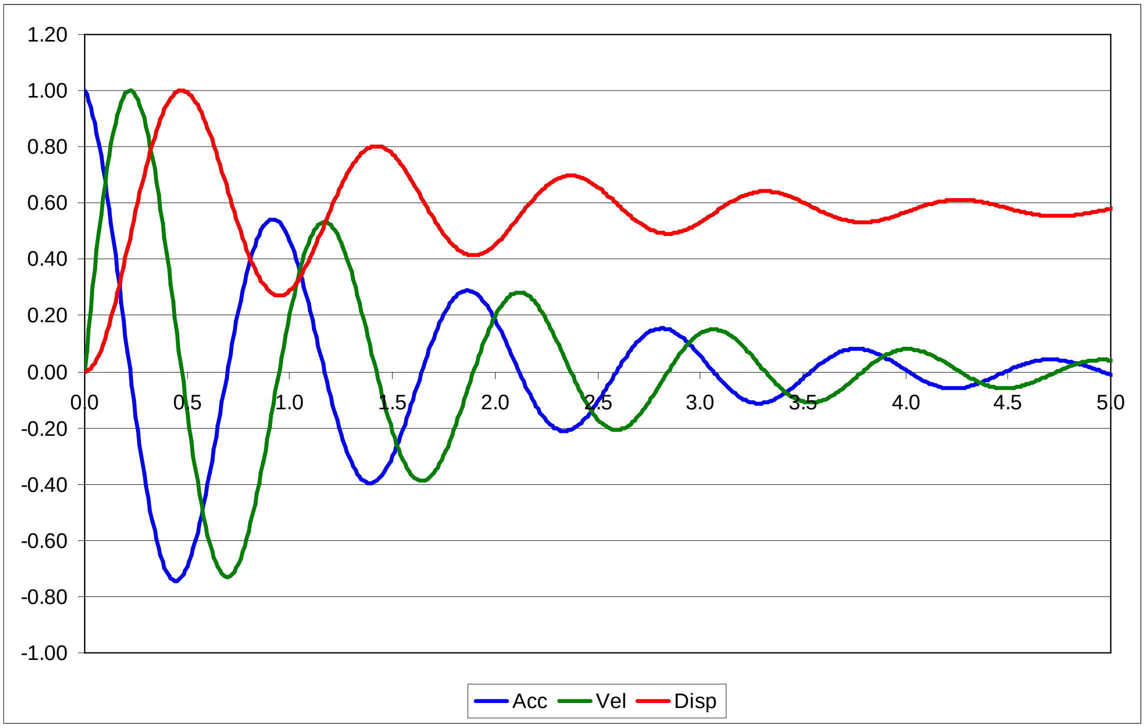
The continuous path (ellipse) indicates that the response is not convergent but perpetually oscillatory. The center of the ellipse gives us the static solution (i.e. 0.05 ft displacement and zero velocity) about which the dynamic solution varies.

b) Let the system have 10% of the critical damping, i.e. let *c* = 60 lb/(ft/sec). Superimposed graphs of undamped and 10% damped systems are given below.



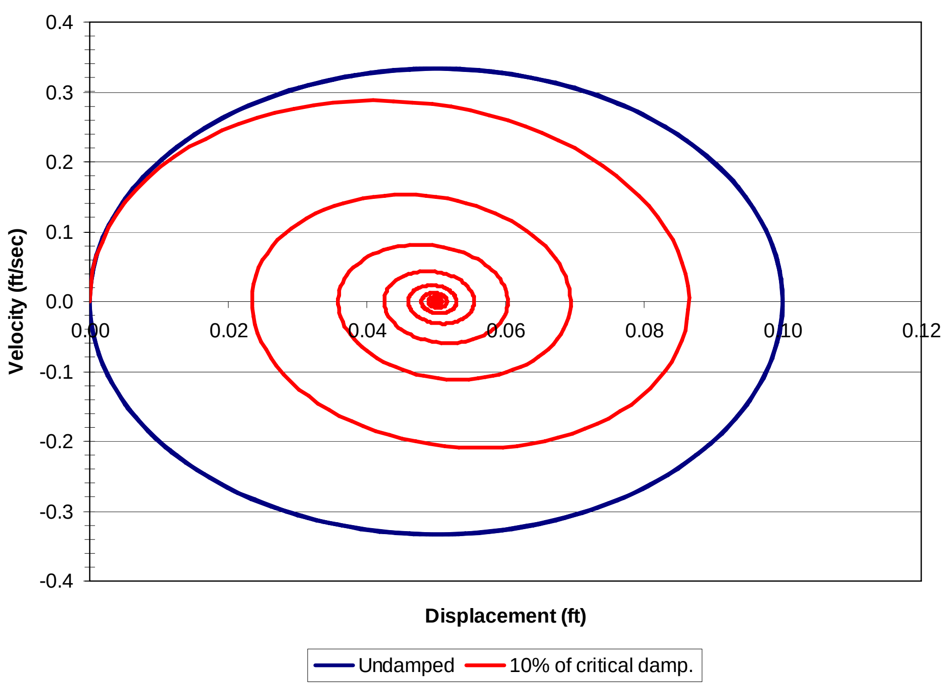
There is hardly any change in the oscillation period (the damping is small). But the decay in oscillations is obvious.

iii) Normalized acceleration, velocity, and displacement responses are graphed below.



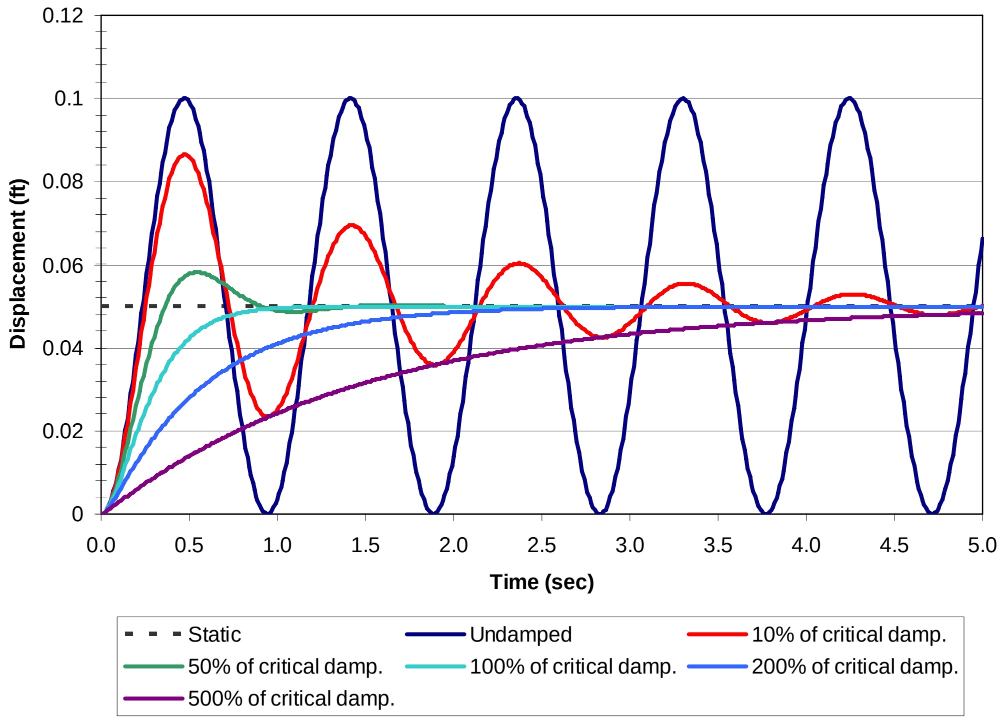
Note the time-lag between peaks of the various responses (i.e. peak acceleration vs. peak velocity vs. peak displacement). Acceleration leads velocity by $T/4$ (~0.24 sec) and leads displacement by $T/2$.

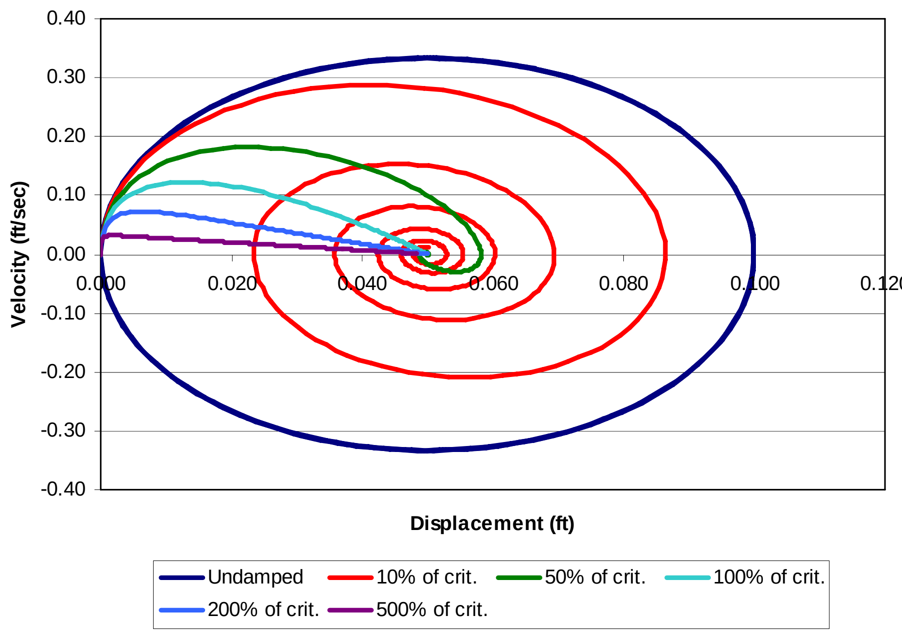
iv) Phase-plane plot for the 10% damped system (formally, it should be called "system with 10% of the critical damping") and the undamped system are given below. Note how the 10% damped system closes in on a solution ("static" solution).



c) Displacement vs. time and displacement vs. velocity (phase-plane) histories for damping ratios of 0%, 10%, 50%, 100%, 200%, 500% of the critical damping are given below.

Note that the first non-oscillatory response is observed at 100% of the critical damping (i.e. when the system is critically damped – guess why it is called "critically" damped?). Systems with damping higher than the critical damping (they are also known as supercritically damped systems) converge to static solution but at a slower pace. In fact, the fastest converging system is the critically damped one.





All systems, except the undamped one, converge to the static solution. Note that the critically damped SDOF as well as the supercritically damped ones do not oscillate about the convergence point.

d)

Case	**Max. disp. (ft)**	**Max. force in spring (lb)**
Undamped	0.100	200
10% of critical damping	0.086	176
50% of critical damping	0.058	116
100% of critical damping	0.050	100
200% of critical damping	0.041	82
500% of critical damping	0.024	48