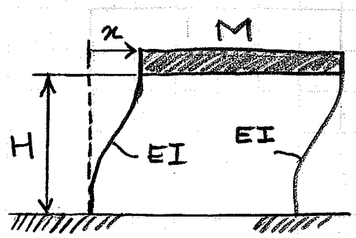
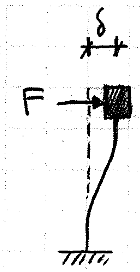
CE 573 – Structural Dynamics
Homework #3

1)

a)





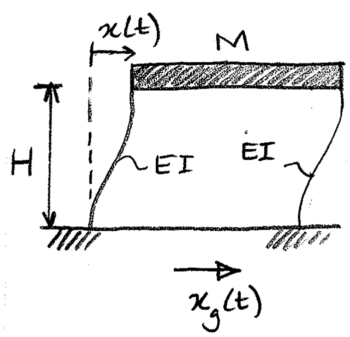
$$\delta = \frac{FH^3}{12EI}$$

$$k_{eq} = \frac{F}{\delta} = \frac{12EI}{H^3}$$

Eqn of motion: $M\ddot{x} + \underbrace{2k_{eq}}_{\text{two columns in parallel}} x = 0$

$$\omega_n = \sqrt{\frac{2k_{eq}}{M}} = \sqrt{\frac{24EI}{MH^3}} \cong 4.9\sqrt{\frac{EI}{MH^3}}$$

b)



Let $x(t)$ be the <u>absolute</u> displacement of M.

Eqn of motion: $M\ddot{x} + \underbrace{2k_{eq}}_{k^*}(x - x_g) = 0$ $\qquad k^* = \sqrt{\frac{24EI}{MH^3}}$

$$\Rightarrow \quad M\ddot{x} + k^* x = k^* x_g$$

$$\ddot{x} + \omega_n^2 x = \omega_n^2 x_0 \sin\omega t \qquad \omega_n = \sqrt{\frac{k^*}{M}}$$

undamped SDOF, forced by $\omega_n^2 x_0 \sin\omega t$. Particular Soln is in the form of $A\sin\omega t$.

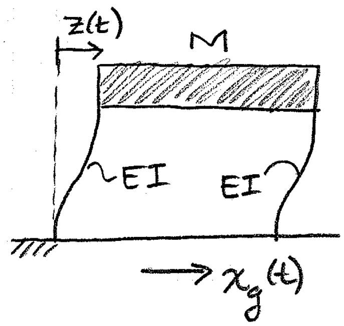
$$\Rightarrow -A\omega^2 \sin\omega t + \omega_n^2 A \sin\omega t = \omega_n^2 x_0 \sin\omega t$$

$$\Rightarrow \quad A = \frac{\omega_n^2 x_0}{\omega_n^2 - \omega^2} = \frac{x_0}{1-(\omega/\omega_n)^2}$$

$$\Rightarrow \quad x_p(t) = \frac{x_0}{1-(\omega/\omega_n)^2} \sin\omega t$$

NOTE THAT $x(t)$ DEFINES THE "ABSOLUTE" MOTION OF MASS M.

ALTERNATIVE APPROACH:



Let $z(t)$ be the displacement of M relative to the ground.

$$k^* = 2 \times \frac{12EI}{H^3} = \frac{24EI}{H^3}$$

Eqn of motion: $M(\ddot{z} + \ddot{x}_g) + k^* z(t) = 0$

$$\Rightarrow \quad M\ddot{z} + k^* z = -M\ddot{x}_g$$

$$\Rightarrow \quad \ddot{z} + \omega_n^2 z = -\ddot{x}_g = \omega^2 x_0 \sin\omega t$$

again, part. soln is in the form of $z_p(t) = B\sin\omega t$

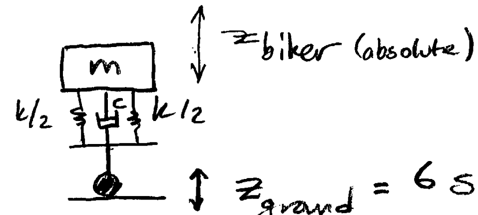
$$\Rightarrow -B\omega^2 \sin\omega t + \omega_n^2 B\sin\omega t = \omega^2 x_0 \sin\omega t$$

$$\Rightarrow \quad B = \frac{\omega^2 x_0}{\omega_n^2 - \omega^2} \quad \Rightarrow \quad z_p(t) = \frac{x_0}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} \sin\omega t$$

Total displacement of M is $z_p(t) + x_g(t) = \dfrac{x_0}{1-(\omega/\omega_n)^2} \sin\omega t$

Can you show this to be the case?

2)



$z_{ground} = 6\sin(\omega t)$ [cm]

Base displacement frequency, $\omega = \dfrac{2\pi}{2m/2.5\,m/sec} = 2.5\pi$ rad/sec

a)

Natural frequency of the rider + bike system, ω_n

$$\delta_{st,\,self\,weight} = 5\text{ cm}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{mg}} = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{981}{5}} = 14 \text{ rad/sec}$$

Damping ratio, $\zeta = 10\%$

Transmissibility Ratio $\left(= \dfrac{|z_{biker}|_{max}}{|z_{ground}|_{max}}\right) = \dfrac{\sqrt{1 + (2\zeta\, \omega/\omega_n)^2}}{\sqrt{(1-(\omega/\omega_n)^2)^2 + (2\zeta\,\omega/\omega_n)^2}}$

$$= \frac{\sqrt{1 + \left(2\times 0.1\times 2.5\pi/14\right)^2}}{\sqrt{\left(1 - \left(2.5\pi/14\right)^2\right)^2 + \left(2\times 0.1\times \frac{2.5\pi}{14}\right)^2}}$$

$$= 1.45$$

$$|z_{biker}|_{max} = 1.45 \times 6^{cm} = 8.7\text{ cm}$$

b) mass increased by 20%

$$\omega_n = \sqrt{\frac{k}{m}} \rightarrow \omega_n^{new} = \frac{\overset{14\ rad/sec}{\cancel{\omega_n}}}{\sqrt{1.2}} = 12.79\ rad/sec$$

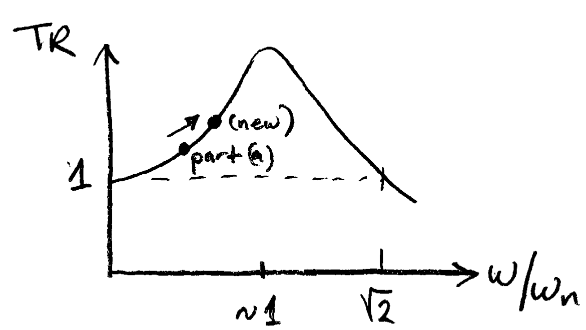
$$\xi = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} \rightarrow \xi^{new} = \frac{\overset{10\%}{\cancel{\xi}}}{\sqrt{1.2}} = 9.1\%$$

$$T.R = \frac{\sqrt{1 + \left(2 \times 0.091 \times {}^{2.5\pi}/_{12.79}\right)^2}}{\sqrt{\left(1 - \left({}^{2.5\pi}/_{12.79}\right)^2\right)^2 + \left(2 \times 0.1 \times \frac{2.5\pi}{12.79}\right)^2}} = 1.6$$

$$|z_{biker}^{new}|_{max} = 1.6 \times 6^{cm} = 9.6\ cm > 8.7\ cm$$

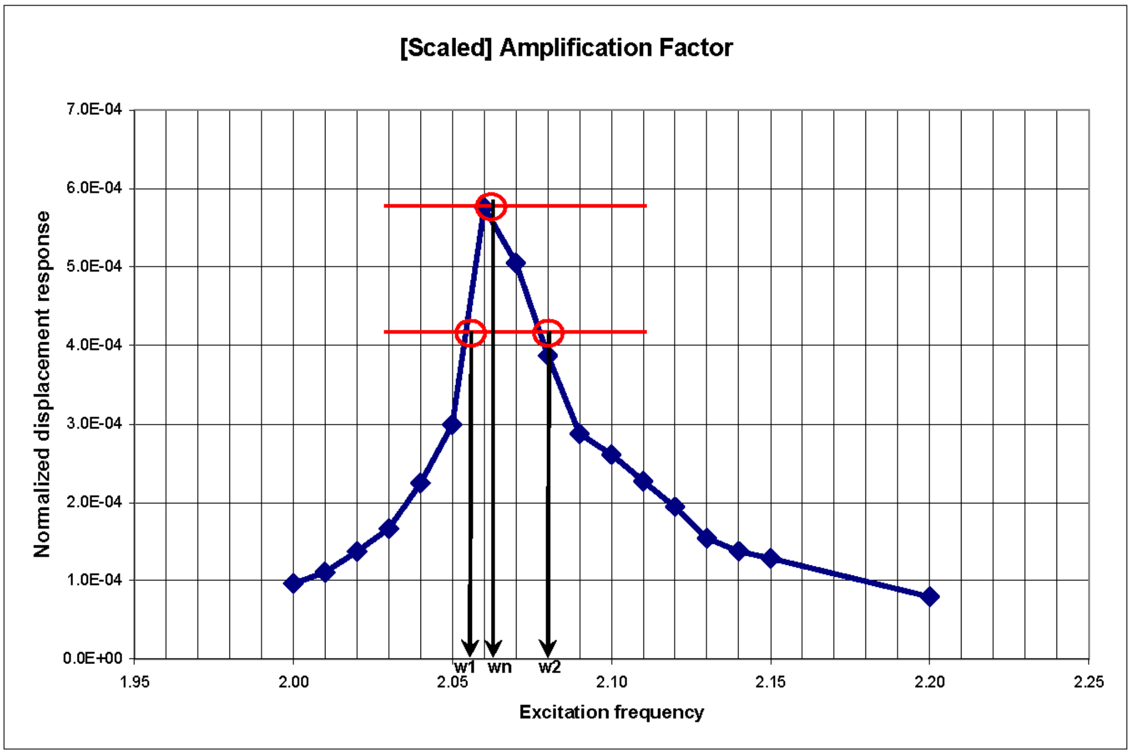
$\therefore$ Less comfortable

note that $\frac{\omega}{\omega_n} = \frac{2.5\pi}{14} = 0.56$ and $\frac{\omega}{\omega_n^{new}} = 0.61$



3)

Excitation Frequency [Hz]	Actuator Displ. [+ -] [in]	Accel. in Actuator [in/s^2]	Accel. in Actuator [g]	Displacement at Roof [+ -] [10^-3 in]	Force Applied by the Actuator (lbf)	Disp. at Roof Normalized by Force
2.00	0.1	15.8	0.04	2.75	29	**9.6E-05**
2.01	0.1	15.9	0.04	3.2	29	**1.1E-04**
2.02	0.1	16.1	0.04	4	29	**1.4E-04**
2.03	0.1	16.3	0.04	4.9	30	**1.7E-04**
2.04	0.1	16.4	0.04	6.7	30	**2.2E-04**
2.05	0.1	16.6	0.04	9	30	**3.0E-04**
2.06	0.1	16.8	0.04	17.5	30	**5.8E-04**
2.07	0.1	16.9	0.04	15.5	31	**5.1E-04**
2.08	0.1	17.1	0.04	12	31	**3.9E-04**
2.09	0.1	17.2	0.04	9	31	**2.9E-04**
2.10	0.1	17.4	0.05	8.25	32	**2.6E-04**
2.11	0.1	17.6	0.05	7.25	32	**2.3E-04**
2.12	0.1	17.7	0.05	6.25	32	**1.9E-04**
2.13	0.1	17.9	0.05	5	32	**1.5E-04**
2.14	0.1	18.1	0.05	4.5	33	**1.4E-04**
2.15	0.1	18.2	0.05	4.25	33	**1.3E-04**
2.20	0.1	19.1	0.05	2.75	35	**7.9E-05**



Peak amplification occurs at w = 2.06 Hz therefore assuming that damping ratio is small, fundamental frequency **wn** of the system is around **2.06 Hz**, giving a fundamental period of about 0.49 sec.

Half-power (the amplification factor is reduced to 1/sqrt(2) of the max) frequencies:
w1 = 2.053 Hz and w2 = 2.078 Hz. (Δw = w2 - w1 = 0.025 Hz)

Continuing with the assumption that the damping is very little, we can use the approximation that Δw / wn = 2ξ
2ξ = 0.025 / 2.06 gives **ξ = 0.6%**