CE 573 – Structural Dynamics
Homework #4

1)

Assuming that damping is small enough to justify the approximation that the resonant frequency is ω_n and the resonant amplitude $A_{res} = 1/2\xi$, the given data implies the following.

$$x_{\omega=\omega_n} = \frac{x_{static}}{2\xi}$$

$$x_{\omega=1.3\omega_n} = \frac{x_{static}}{\sqrt{[1-(\omega/\omega_n)^2]^2+(2\xi\,\omega/\omega_n)^2}}$$

$$x_{\omega=1.3\omega_n} = \frac{x_{static}}{\sqrt{(1-1.3^2)^2+(2\xi(1.3))^2}} = \frac{1}{8}\,x_{\omega=\omega_n} = \frac{1}{8}\,\frac{x_{static}}{2\xi}$$

$$\Rightarrow \frac{\cancel{x_{static}}}{\sqrt{0.4761+6.76\xi^2}} = \frac{1}{8}\,\frac{\cancel{x_{static}}}{2\xi}$$

$$\Rightarrow 256\xi^2 = 0.4761 + 6.76\xi^2$$

$$\Rightarrow \xi^2 = 1.91 \times 10^{-3}$$

$$\Rightarrow \xi \cong 0.04 \Longleftarrow$$ this is indeed a small damping so approximations utilized are permissible.

Note: If one wanted to be exact or if the damping was not small, we would have to use the exact resonant frequency, i.e. $\omega_n\sqrt{1-2\xi^2}$ and resonant ampl. factor $\dfrac{x_{static}}{2\xi\sqrt{1-\xi^2}}$

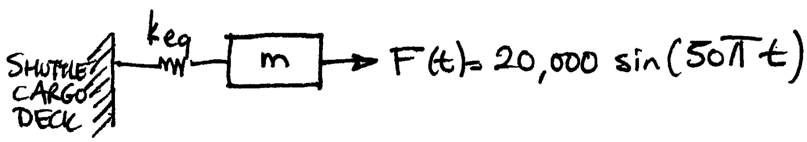
2)

Trying to reduce the transmitted force.

$m = 500\ kg \quad ; \quad c = 0$ (undamped)

$\omega = 2\pi \dfrac{1500}{60} = 50\pi$ rad/sec

$F_o = 20{,}000\,N$



$$F^{max}_{transmitted} \leq 10{,}000\ N$$

$$T.R. = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2}} = \frac{F^{max}_{transmitted}}{F_o}$$

need $TR \leq \frac{1}{2} \Rightarrow \dfrac{1}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|} \leq \dfrac{1}{2}$

$$\Rightarrow \quad 2 \leq \left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|$$

$$\Rightarrow \quad 1-\left(\frac{\omega}{\omega_n}\right)^2 \geq 2 \Rightarrow \times$$

$$1-\left(\frac{\omega}{\omega_n}\right)^2 \leq -2 \Rightarrow \left(\frac{\omega}{\omega_n}\right)^2 \geq 3$$

$$\frac{\omega}{\omega_n} \geq \sqrt{3} = 1.73$$

$$\Rightarrow \quad \omega_n \leqslant \frac{\omega}{\sqrt{3}} = 90.7 \text{ rad/sec}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_{eq}}{500}} \leqslant 90.7 \text{ rad/sec}$$

$$k_{eq} \leqslant 4.11 \times 10^6 \text{ N/m}$$

springs : $k = 8 \times 10^6$ N/m each

need 2 or 3 springs in series

k_{eq} for 2 springs in series $= 4 \times 10^6$ N/m

k_{eq} for 3 spring in series : $\frac{8}{3} \times 10^6$ N/m

$$T.R._{\text{2 springs in series}} = 0.48 \longrightarrow F^{max}_{transmitted} = 9{,}600 \text{ N}$$

$$T.R._{\text{3 springs in series}} = 0.28 \longrightarrow F^{max}_{transmitted} = 5{,}600 \text{ N}$$

3)

Given $W = 1{,}000 \text{ lb} \rightarrow m = \dfrac{1{,}000 \text{ lb}}{386 \text{ in/sec/sec}} = 2.59 \dfrac{\text{lb.sec}^2}{\text{in}}$

$f = 600 \text{ cpm} = 10 \text{ Hz} \rightarrow w = \dfrac{2\pi}{f} = 62.83 \text{ rad/sec}$

$$\frac{X^{max}_{\substack{\text{isolated}\\ \text{system}}}}{\text{Base Disp}} = \frac{\sqrt{1 + \left(2\xi \frac{w}{w_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{w}{w_n}\right)^2\right)^2 + \left(2\xi \frac{w}{w_n}\right)^2}} \qquad (*)$$

since the isolation block is undamped, using $(*)$ we can find the ratio of maximum displacement of the block to that of the ground as

$$\frac{X^{max}_{\text{iso. block}}}{\text{Base Disp}} = \frac{1}{\left|1 - \left(\frac{w}{w_n}\right)^2\right|} \underbrace{\leq 0.10}_{\text{design expectation}}$$

$$\Rightarrow \frac{1}{\left|1 - \left(\frac{w}{w_n}\right)^2\right|} \leq 0.10 \longrightarrow 10 \leq \left|1 - (w/w_n)^2\right|$$

$$1 - (w/w_n)^2 \leq -10 \qquad\qquad 1 - (w/w_n)^2 \geq 10$$

$$\Downarrow \qquad\qquad \downarrow$$

$$\left(\frac{w}{w_n}\right)^2 \geq 11 \qquad\qquad -9 \geq (w/w_n)^2$$

impossible

$$\left(\frac{w}{w_n}\right)^2 \geq 11 \longrightarrow w_n \leq 18.94 \text{ rad/se}$$

$$w_n = \sqrt{k/m} \Rightarrow k = m \cdot w_n^2 \Rightarrow k \leq 2.59 \times 18.94^2 \approx 930 \text{ lb/in}$$

so $k \leq 930$ lb/in to reduce the motion by 90% or more.