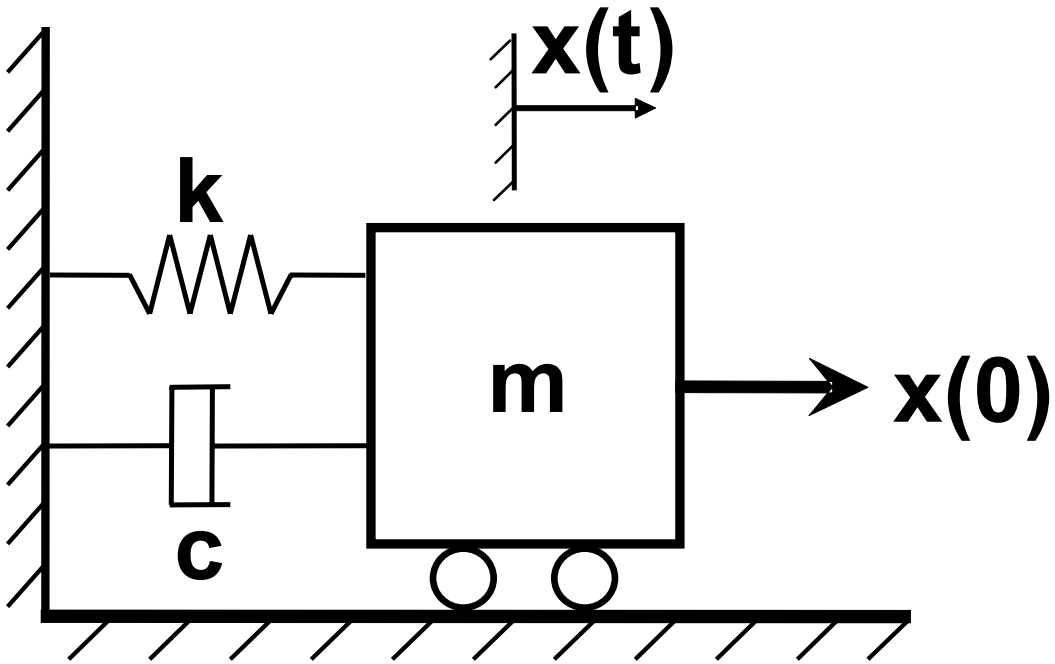
NUMERICAL SOLUTION BASED ON APPROXIMATING DERIVATIVES

constant average-acceleration method



k =	1	lb/ft
m =	1	lb/(ft/sec/sec)
c =	0	lb/(ft/sec)
x(0) =	1	ft
v(0) =	0	ft/sec

EQUATION OF MOTION (continuous-time)

$$m \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + k \cdot x(t) = F(t)$$

EQUATION OF MOTION (discrete-time)

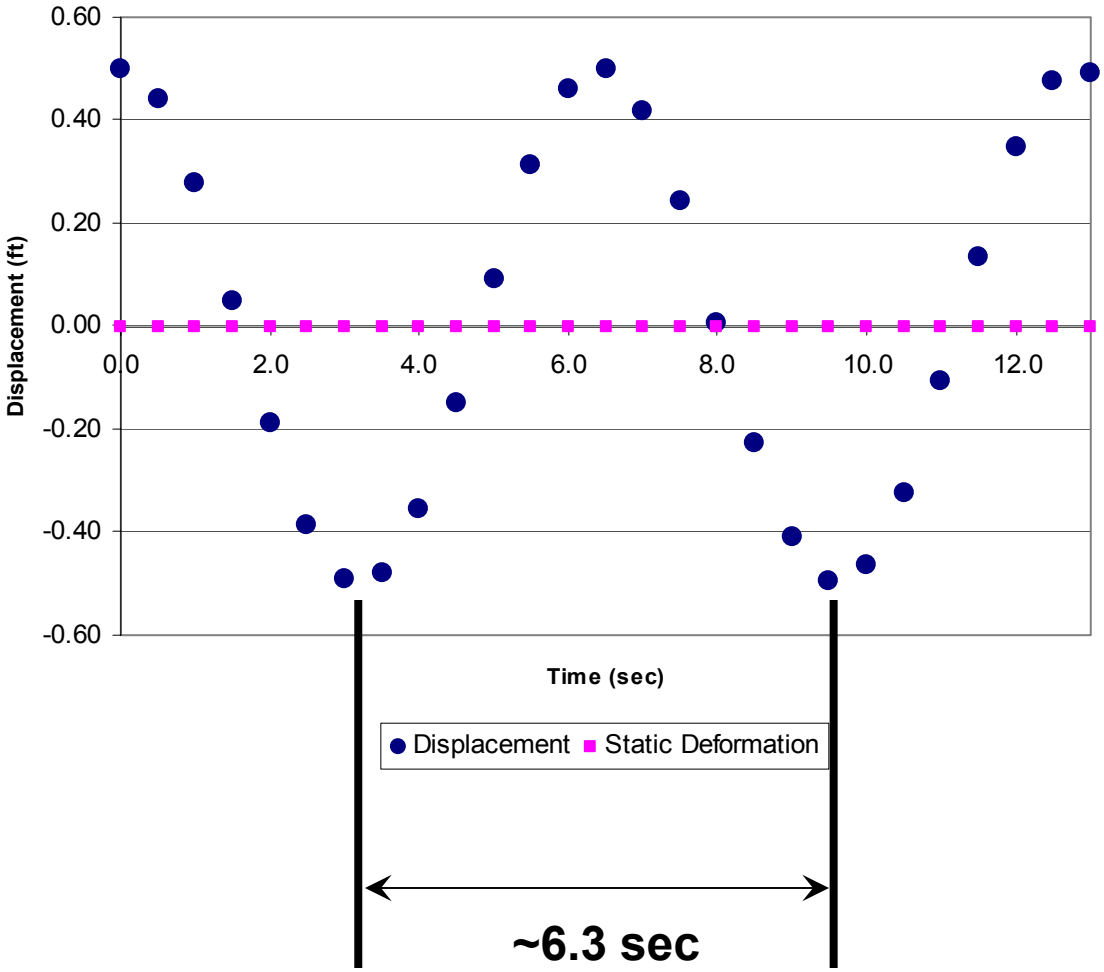
$$m \cdot \ddot{x}_i + c \cdot \dot{x}_i + k \cdot x_i = F_i$$

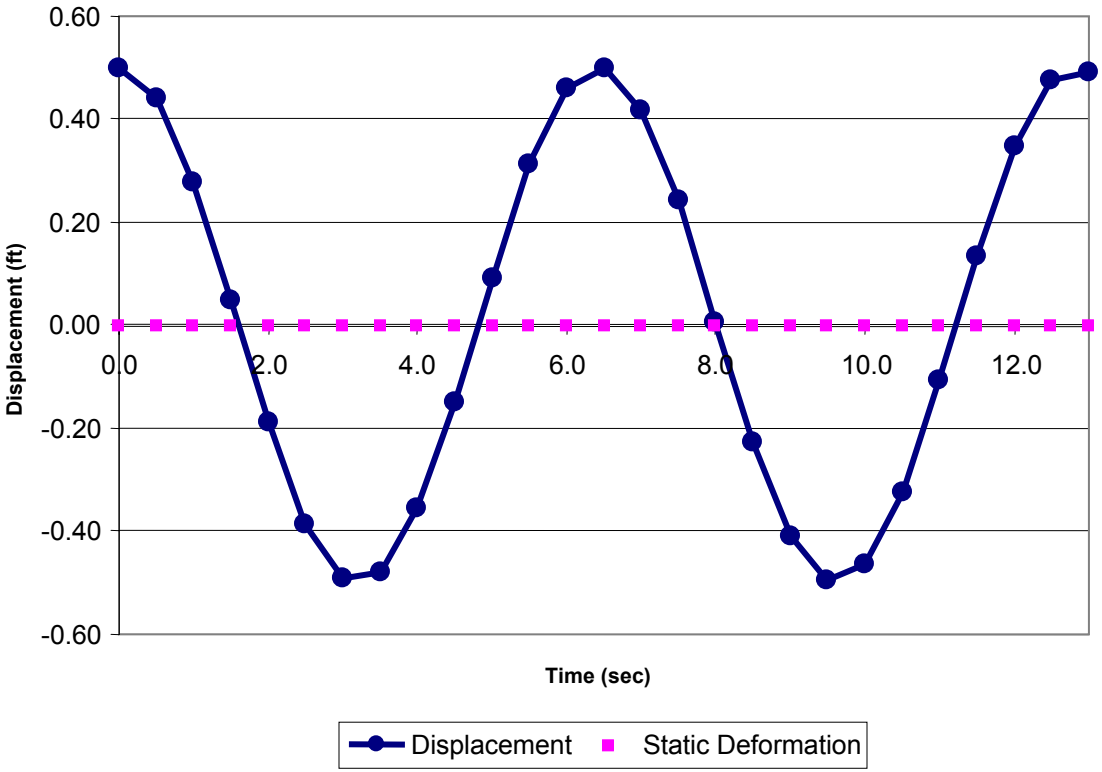
$$T = \frac{2\pi}{w} = 2\pi \cdot \sqrt{\frac{m}{k}} \quad \Rightarrow \quad \mathbf{T =} \ 6.3 \text{ sec}$$

Choose $\Delta t = 0.5$ sec

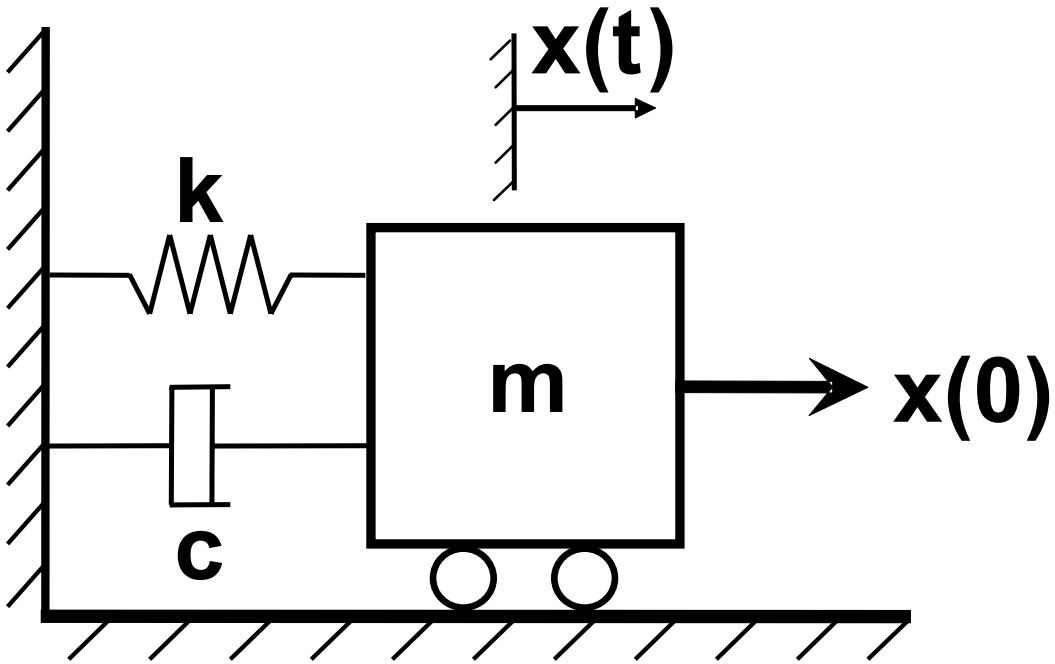
TABULATION

t_i (sec)	F_i (lb)	a_i (ft/sec/sec)	v_i (ft/sec)	x_i (ft)	ΔF*_i	k*	Δx_i	Δv_i	Δa_i	xstatic_i
0.00	0	-0.50	**0.00**	**0.50**	-1.0	17.0	-0.06	-0.24	0.06	0
0.50	0	-0.44	-0.24	**0.44**	-2.8	17.0	-0.16	-0.18	0.16	0
1.00	0	-0.28	-0.42	**0.28**	-3.9	17.0	-0.23	-0.08	0.23	0
1.50	0	-0.05	-0.50	**0.05**	-4.1	17.0	-0.24	0.03	0.24	0
2.00	0	0.19	-0.46	**-0.19**	-3.3	17.0	-0.20	0.14	0.20	0
2.50	0	0.39	-0.32	**-0.39**	-1.8	17.0	-0.10	0.22	0.10	0
3.00	0	0.49	-0.10	**-0.49**	0.2	17.0	0.01	0.24	-0.01	0
3.50	0	0.48	0.14	**-0.48**	2.1	17.0	0.12	0.21	-0.12	0
4.00	0	0.36	0.35	**-0.36**	3.5	17.0	0.21	0.13	-0.21	0
4.50	0	0.15	0.48	**-0.15**	4.1	17.0	0.24	0.01	-0.24	0
5.00	0	-0.09	0.49	**0.09**	3.7	17.0	0.22	-0.10	-0.22	0
5.50	0	-0.31	0.39	**0.31**	2.5	17.0	0.15	-0.19	-0.15	0
6.00	0	-0.46	0.20	**0.46**	0.7	17.0	0.04	-0.24	-0.04	0
6.50	0	-0.50	-0.04	**0.50**	-1.3	17.0	-0.08	-0.23	0.08	0
7.00	0	-0.42	-0.27	**0.42**	-3.0	17.0	-0.18	-0.17	0.18	0
7.50	0	-0.24	-0.44	**0.24**	-4.0	17.0	-0.23	-0.06	0.23	0
8.00	0	-0.01	-0.50	**0.01**	-4.0	17.0	-0.24	0.06	0.24	0
8.50	0	0.23	-0.44	**-0.23**	-3.1	17.0	-0.18	0.16	0.18	0
9.00	0	0.41	-0.28	**-0.41**	-1.5	17.0	-0.09	0.23	0.09	0
9.50	0	0.50	-0.06	**-0.50**	0.5	17.0	0.03	0.24	-0.03	0
10.00	0	0.47	0.18	**-0.47**	2.4	17.0	0.14	0.20	-0.14	0
10.50	0	0.32	0.38	**-0.32**	3.7	17.0	0.22	0.11	-0.22	0
11.00	0	0.11	0.49	**-0.11**	4.1	17.0	0.24	-0.01	-0.24	0
11.50	0	-0.14	0.48	**0.14**	3.6	17.0	0.21	-0.12	-0.21	0
12.00	0	-0.35	0.36	**0.35**	2.2	17.0	0.13	-0.21	-0.13	0
12.50	0	-0.48	0.16	**0.48**	0.3	17.0	0.02	-0.24	-0.02	0
13.00	0	-0.49	-0.09	**0.49**	-1.7	17.0	-0.10	-0.22	0.10	0


0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
Displacement (ft)
0.0
2.0
4.0
6.0
8.0
10.0
12.0
Time (sec)
Displacement
Static Deformation
~6.3 sec


0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
Displacement (ft)
0.0
2.0
4.0
6.0
8.0
10.0
12.0
Time (sec)
Displacement
Static Deformation

Damped SDOF case

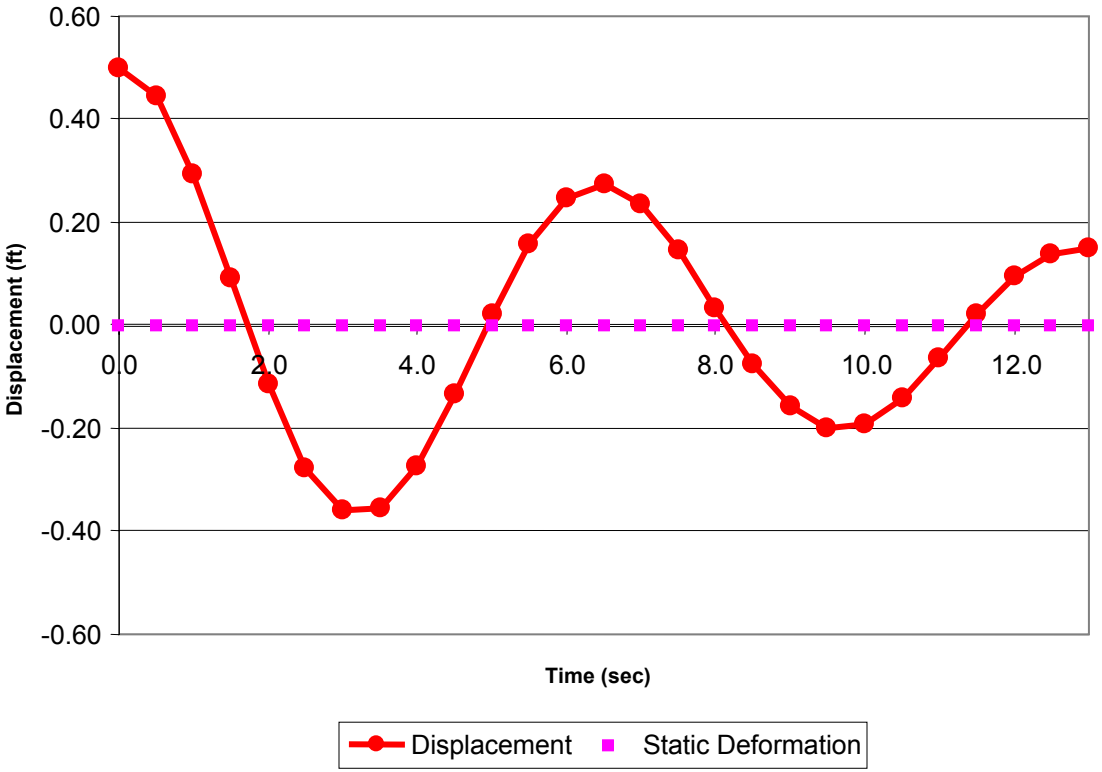


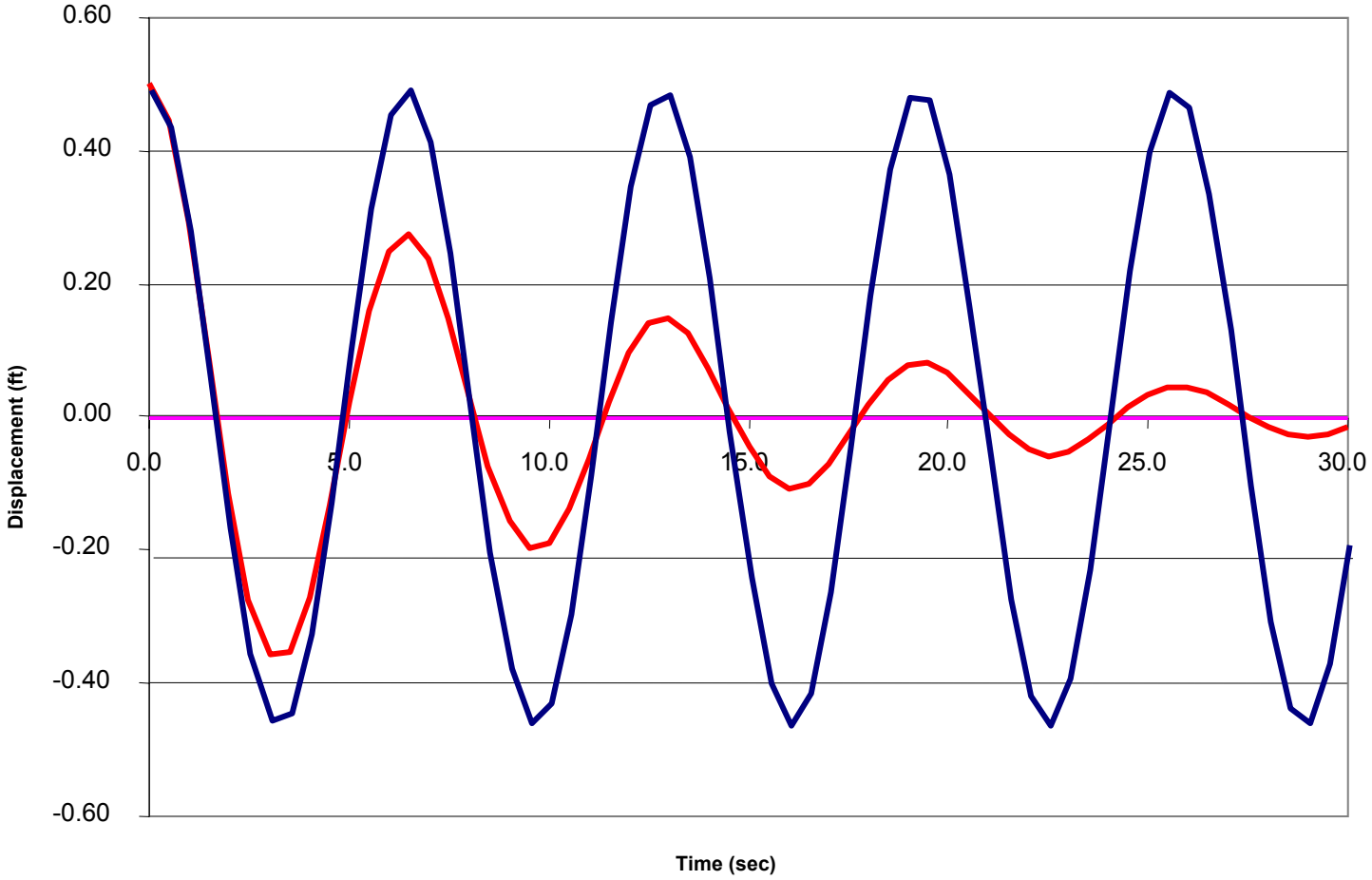
k =	1	lb/ft
m =	1	lb/(ft/sec/sec)
c =	**0.2**	lb/(ft/sec)
x(0) =	1	ft
v(0) =	0	ft/sec

Critical damping, $c_{cr} = 2 \cdot \sqrt{k \cdot m}$

TABULATION

t_i (sec)	F_i (lb)	a_i (ft/sec/sec)	v_i (ft/sec)	x_i (ft)	ΔF*_i	k*	Δx_i	Δv_i	Δa_i	xstatic_i
0.00	0	-0.50	0.00	**0.50**	-1.0	17.8	-0.06	-0.22	0.10	0
0.50	0	-0.40	-0.22	**0.44**	-2.7	17.8	-0.15	-0.15	0.18	0
1.00	0	-0.22	-0.38	**0.29**	-3.6	17.8	-0.20	-0.06	0.21	0
1.50	0	0.00	-0.43	**0.09**	-3.7	17.8	-0.21	0.05	0.20	0
2.00	0	0.19	-0.39	**-0.12**	-2.9	17.8	-0.16	0.13	0.13	0
2.50	0	0.33	-0.26	**-0.28**	-1.5	17.8	-0.08	0.18	0.05	0
3.00	0	0.38	-0.08	**-0.36**	0.1	17.8	0.00	0.18	-0.04	0
3.50	0	0.34	0.10	**-0.36**	1.5	17.8	0.08	0.14	-0.11	0
4.00	0	0.22	0.24	**-0.27**	2.4	17.8	0.14	0.07	-0.15	0
4.50	0	0.07	0.31	**-0.13**	2.8	17.8	0.16	0.00	-0.15	0
5.00	0	-0.08	0.31	**0.02**	2.4	17.8	0.14	-0.07	-0.12	0
5.50	0	-0.21	0.24	**0.16**	1.6	17.8	0.09	-0.12	-0.07	0
6.00	0	-0.27	0.12	**0.25**	0.5	17.8	0.03	-0.13	0.00	0
6.50	0	-0.27	-0.02	**0.27**	-0.7	17.8	-0.04	-0.12	0.06	0
7.00	0	-0.21	-0.14	**0.23**	-1.6	17.8	-0.09	-0.08	0.10	0
7.50	0	-0.10	-0.21	**0.15**	-2.0	17.8	-0.11	-0.02	0.12	0
8.00	0	0.01	-0.24	**0.03**	-2.0	17.8	-0.11	0.03	0.10	0
8.50	0	0.12	-0.20	**-0.08**	-1.5	17.8	-0.08	0.08	0.07	0
9.00	0	0.18	-0.13	**-0.16**	-0.7	17.8	-0.04	0.10	0.02	0
9.50	0	0.21	-0.03	**-0.20**	0.1	17.8	0.01	0.10	-0.03	0
10.00	0	0.18	0.06	**-0.19**	0.9	17.8	0.05	0.07	-0.06	0
10.50	0	0.11	0.14	**-0.14**	1.4	17.8	0.08	0.04	-0.08	0
11.00	0	0.03	0.17	**-0.06**	1.5	17.8	0.08	-0.01	-0.08	0
11.50	0	-0.05	0.17	**0.02**	1.3	17.8	0.07	-0.04	-0.06	0
12.00	0	-0.12	0.12	**0.09**	0.8	17.8	0.04	-0.07	-0.03	0
12.50	0	-0.15	0.06	**0.14**	0.2	17.8	0.01	-0.07	0.01	0
13.00	0	-0.14	-0.02	**0.15**	-0.4	17.8	-0.02	-0.06	0.04	0

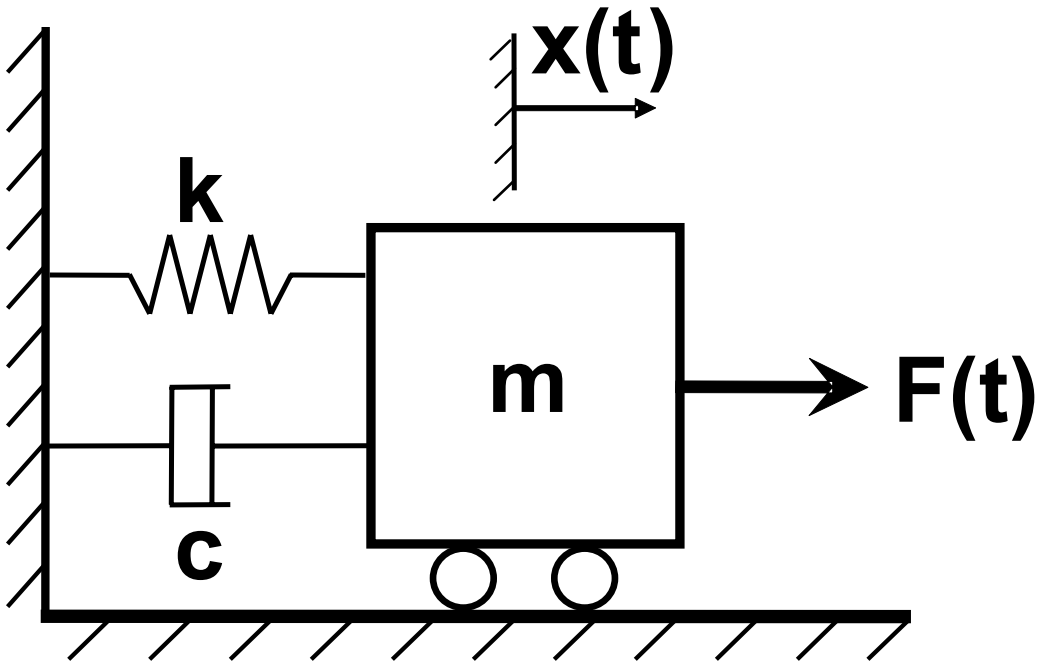

0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
0.0
2.0
4.0
6.0
8.0
10.0
12.0
Displacement (ft)
Time (sec)
Displacement
Static Deformation


0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
Displacement (ft)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
Time (sec)

NUMERICAL SOLUTION BASED ON APPROXIMATING DERIVATIVES

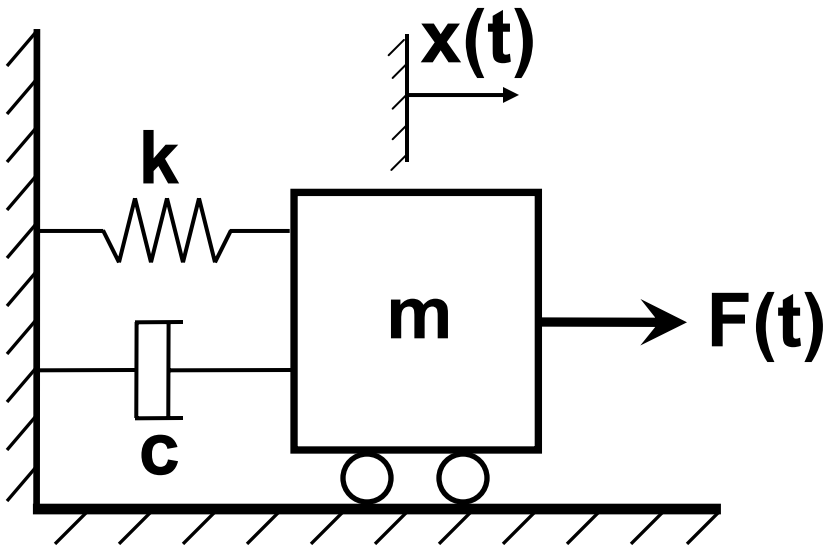
constant average-acceleration method

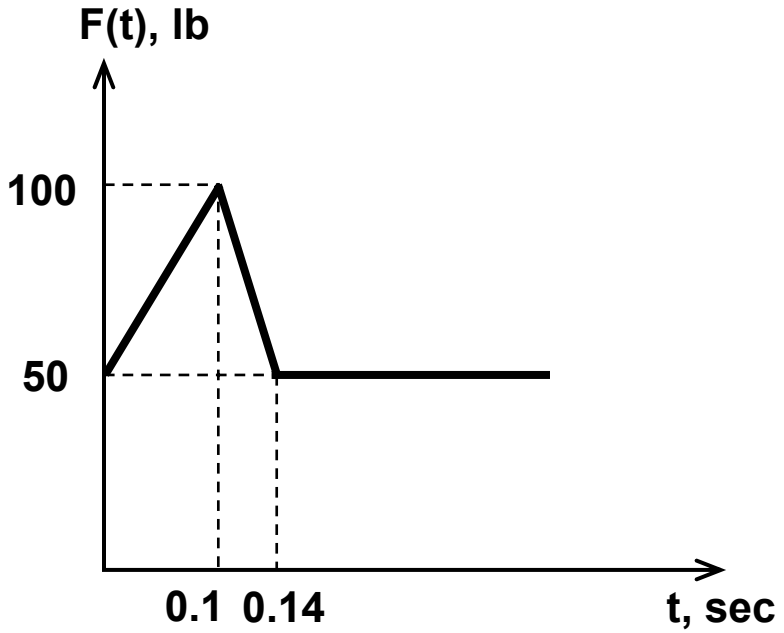
~nonlinear analysis~



k* =	2000	lb/ft
m =	2	lb/(ft/sec/sec)
c =	0	lb/(ft/sec)
x(0) =	0	ft
v(0) =	0	ft/sec
Δt =	0.01	sec

(*) Spring is elasto-plastic w/ maximum resistance 110 lb

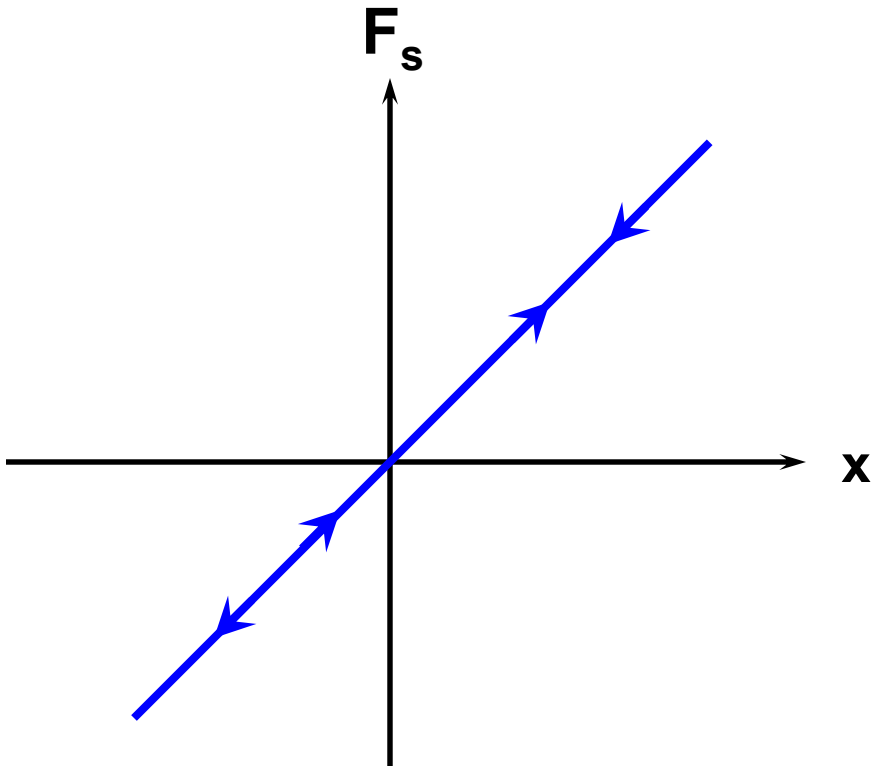

x(t)
k
m
F(t)
c


F(t), lb
100
50
0.1
0.14
t, sec

Linear elastic spring case

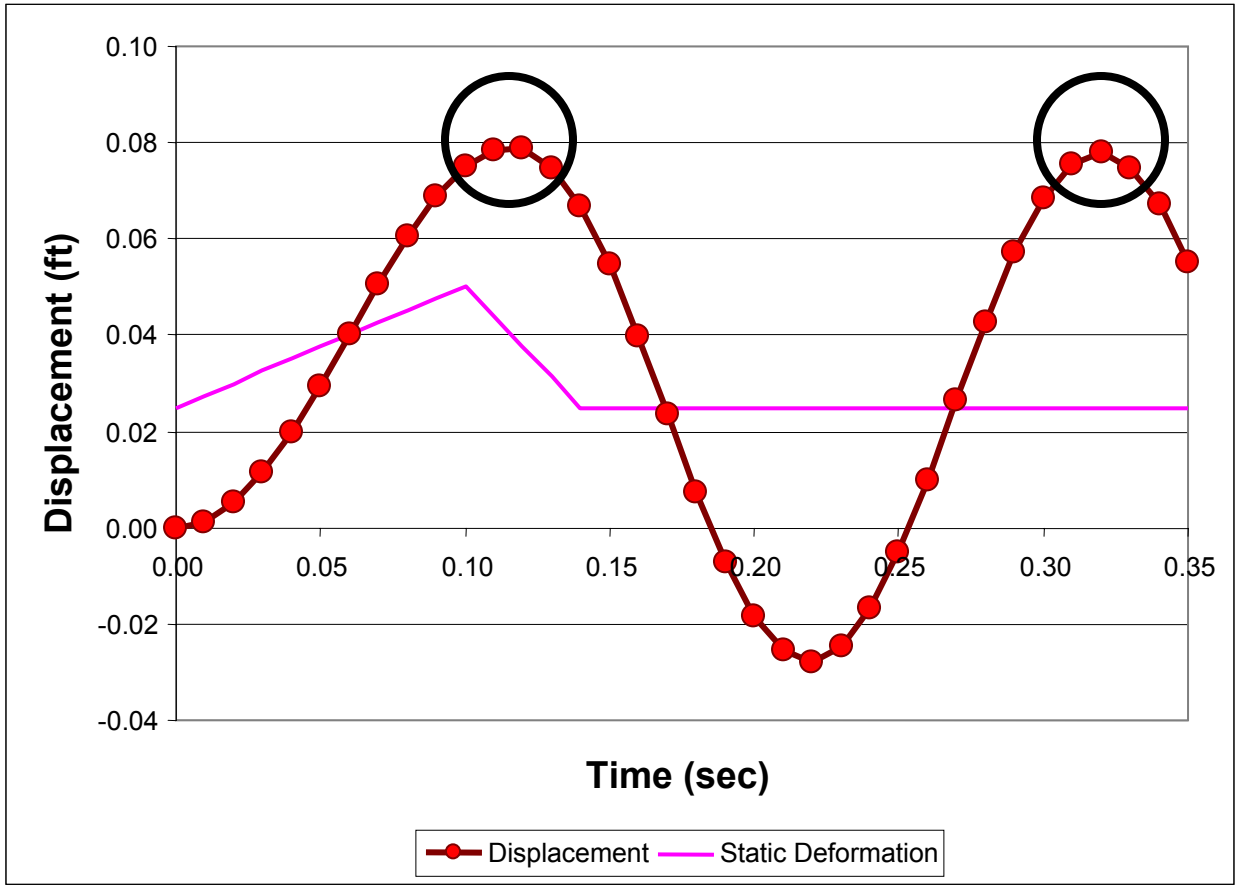
k=2000 lb/ft

Linear Elastic Spring



t_i (sec)	F_i (lb)	a_i (ft/sec/sec)	v_i (ft/sec)	x_i (ft)	ΔF*_i	k*	Δx_i	Δv_i	Δa_i	xstatic_i	fspring
0.00	50	25.00	0.00	**0.00**	105.0	82000	0.00	0.26	1.22	0.025	0.0
0.01	55	26.22	0.26	**0.00**	314.8	82000	0.00	0.26	-1.34	0.0275	2.6
0.02	60	24.88	0.51	**0.01**	513.8	82000	0.01	0.23	-3.77	0.03	10.2
0.03	65	21.12	0.74	**0.01**	682.7	82000	0.01	0.18	-5.83	0.0325	22.8
0.04	70	15.29	0.92	**0.02**	805.0	82000	0.01	0.12	-7.32	0.035	39.4
0.05	75	7.97	1.04	**0.03**	868.8	82000	0.01	0.04	-8.10	0.0375	59.1
0.06	80	-0.12	1.08	**0.04**	867.8	82000	0.01	-0.04	-8.08	0.04	80.2
0.07	85	-8.21	1.04	**0.05**	802.2	82000	0.01	-0.12	-7.28	0.0425	101.4
0.08	90	-15.49	0.92	**0.06**	678.3	82000	0.01	-0.18	-5.77	0.045	121.0
0.09	95	-21.26	0.74	**0.07**	508.2	82000	0.01	-0.23	-3.70	0.0475	137.5
0.10	100	-24.96	0.50	**0.07**	291.0	82000	0.00	-0.30	-9.80	0.05	149.9
0.11	87.5	-34.76	0.21	**0.08**	13.0	82000	0.00	-0.38	-6.41	0.04375	157.0
0.12	75	-41.16	-0.17	**0.08**	-316.4	82000	0.00	-0.42	-2.39	0.0375	157.3
0.13	62.5	-43.56	-0.60	**0.07**	-664.8	82000	-0.01	-0.43	1.86	0.03125	149.6
0.14	50	-41.70	-1.02	**0.07**	-985.9	82000	-0.01	-0.36	12.02	0.025	133.4
0.15	50	-29.68	-1.38	**0.05**	-1223.3	82000	-0.01	-0.22	14.92	0.025	109.4
0.16	50	-14.76	-1.60	**0.04**	-1341.4	82000	-0.02	-0.07	16.36	0.025	79.5
0.17	50	1.60	-1.67	**0.02**	-1328.6	82000	-0.02	0.10	16.20	0.025	46.8
0.18	50	17.80	-1.57	**0.01**	-1186.1	82000	-0.01	0.25	14.47	0.025	14.4
0.19	50	32.27	-1.32	**-0.01**	-928.0	82000	-0.01	0.38	11.32	0.025	-14.5
0.20	50	43.59	-0.94	**-0.02**	-579.3	82000	-0.01	0.47	7.06	0.025	-37.2
0.21	50	50.65	-0.47	**-0.03**	-174.1	82000	0.00	0.52	2.12	0.025	-51.3
0.22	50	52.77	0.05	**-0.03**	248.1	82000	0.00	0.51	-3.03	0.025	-55.5
0.23	50	49.75	0.56	**-0.02**	646.1	82000	0.01	0.46	-7.88	0.025	-49.5
0.24	50	41.87	1.02	**-0.02**	981.0	82000	0.01	0.36	-11.96	0.025	-33.7
0.25	50	29.91	1.38	**0.00**	1220.3	82000	0.01	0.22	-14.88	0.025	-9.8
0.26	50	15.02	1.60	**0.01**	1340.5	82000	0.02	0.07	-16.35	0.025	20.0
0.27	50	-1.32	1.67	**0.03**	1329.9	82000	0.02	-0.09	-16.22	0.025	52.6
0.28	50	-17.54	1.57	**0.04**	1189.6	82000	0.01	-0.25	-14.51	0.025	85.1
0.29	50	-32.05	1.33	**0.06**	933.2	82000	0.01	-0.38	-11.38	0.025	114.1
0.30	50	-43.43	0.95	**0.07**	585.8	82000	0.01	-0.47	-7.14	0.025	136.9
0.31	50	-50.57	0.48	**0.08**	181.2	82000	0.00	-0.52	-2.21	0.025	151.1
0.32	50	-52.78	-0.04	**0.08**	-241.1	82000	0.00	-0.51	2.94	0.025	155.6
0.33	50	-49.84	-0.55	**0.07**	-639.8	82000	-0.01	-0.46	7.80	0.025	149.7
0.34	50	-42.04	-1.01	**0.07**	-976.1	82000	-0.01	-0.36	11.90	0.025	134.1
0.35	50	-30.14	-1.37	**0.06**	-1217.2	82000	-0.01	-0.23	14.84	0.025	110.3

Max spring force = max deflection * k = ~0.08 ft * 2000 lb/ft = ~ 160 lb



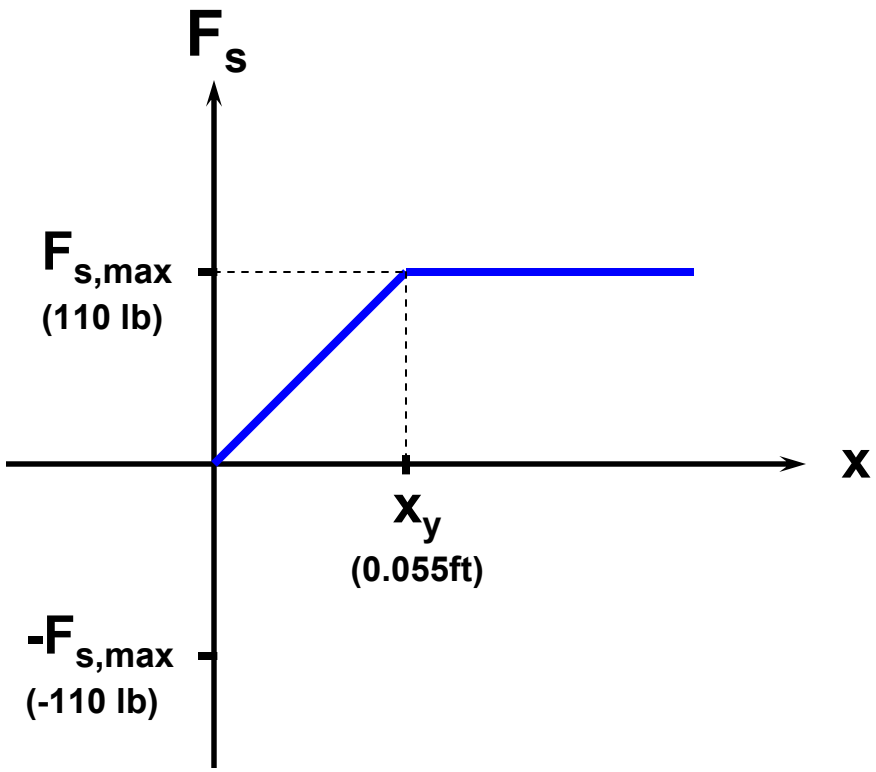
Linear elastic – plastic (elasto-plastic) spring case

$k^* = 2{,}000$ lb/ft

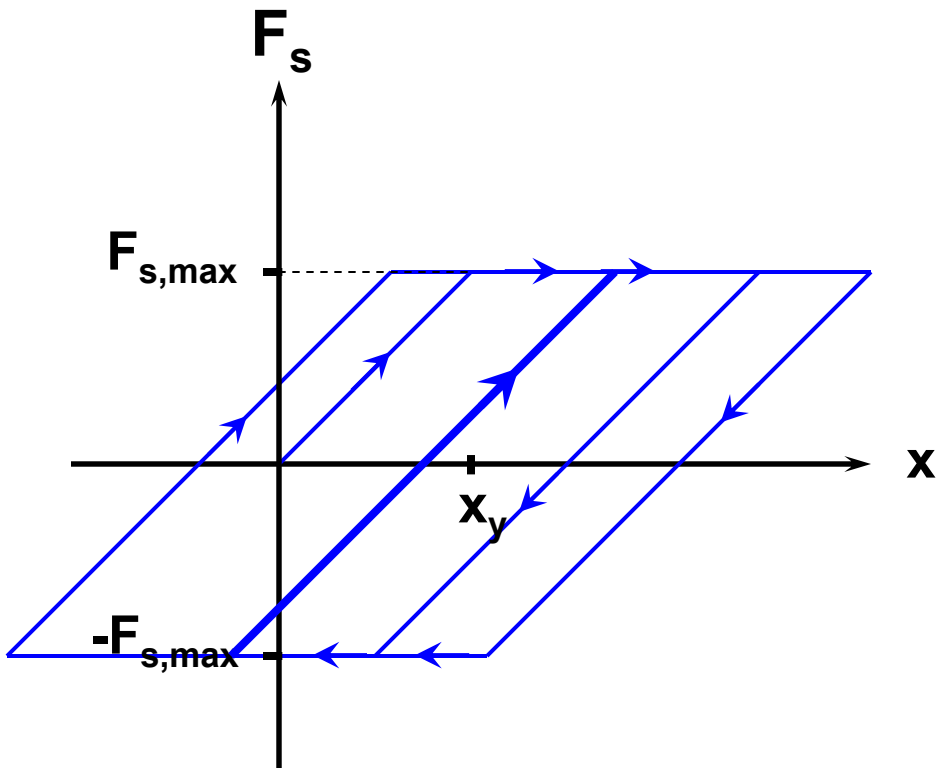
$F_{s,max} = 110$ lb

→ **spring yield def. = 0.055 ft**

Elasto-Plastic Spring

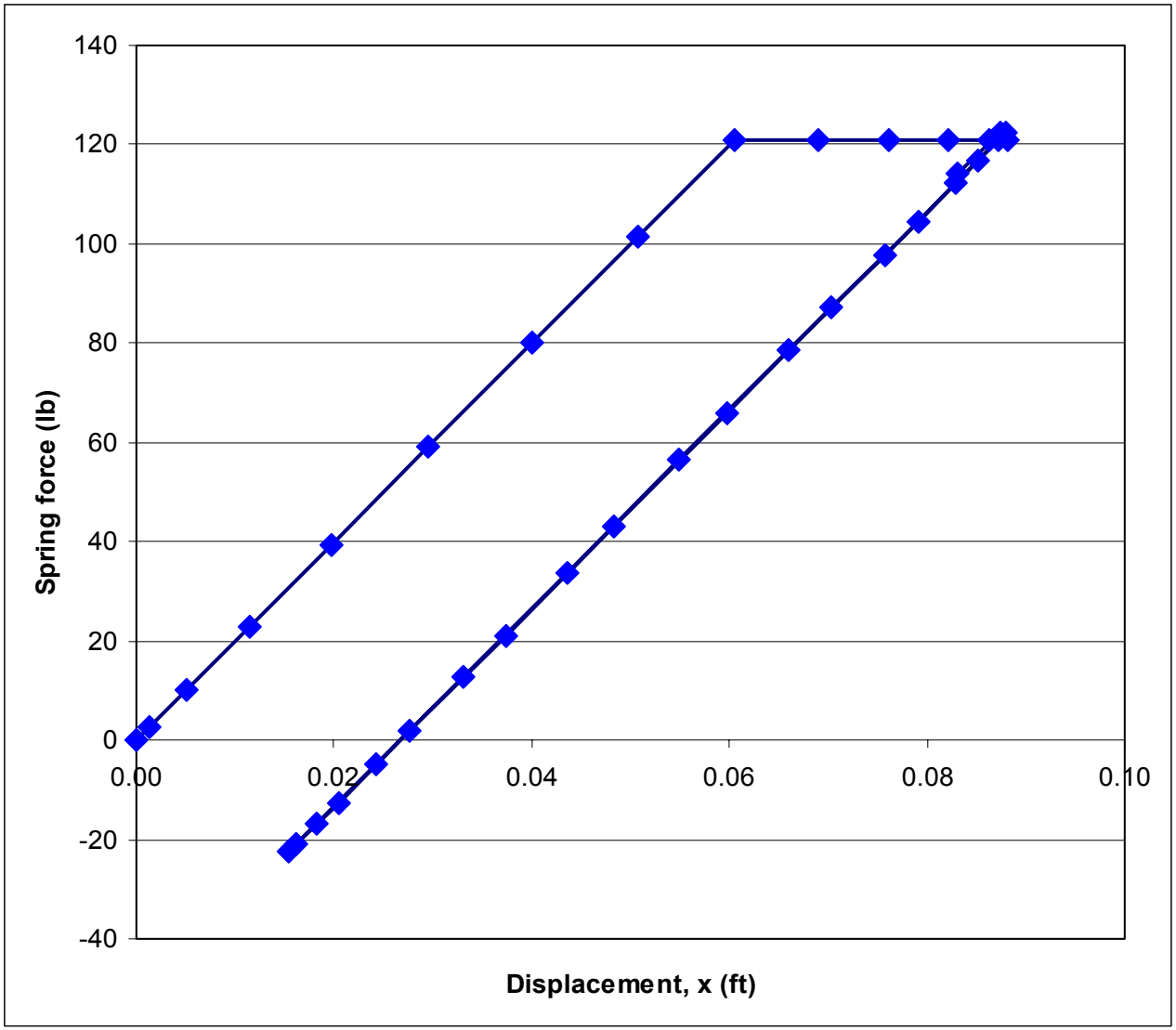


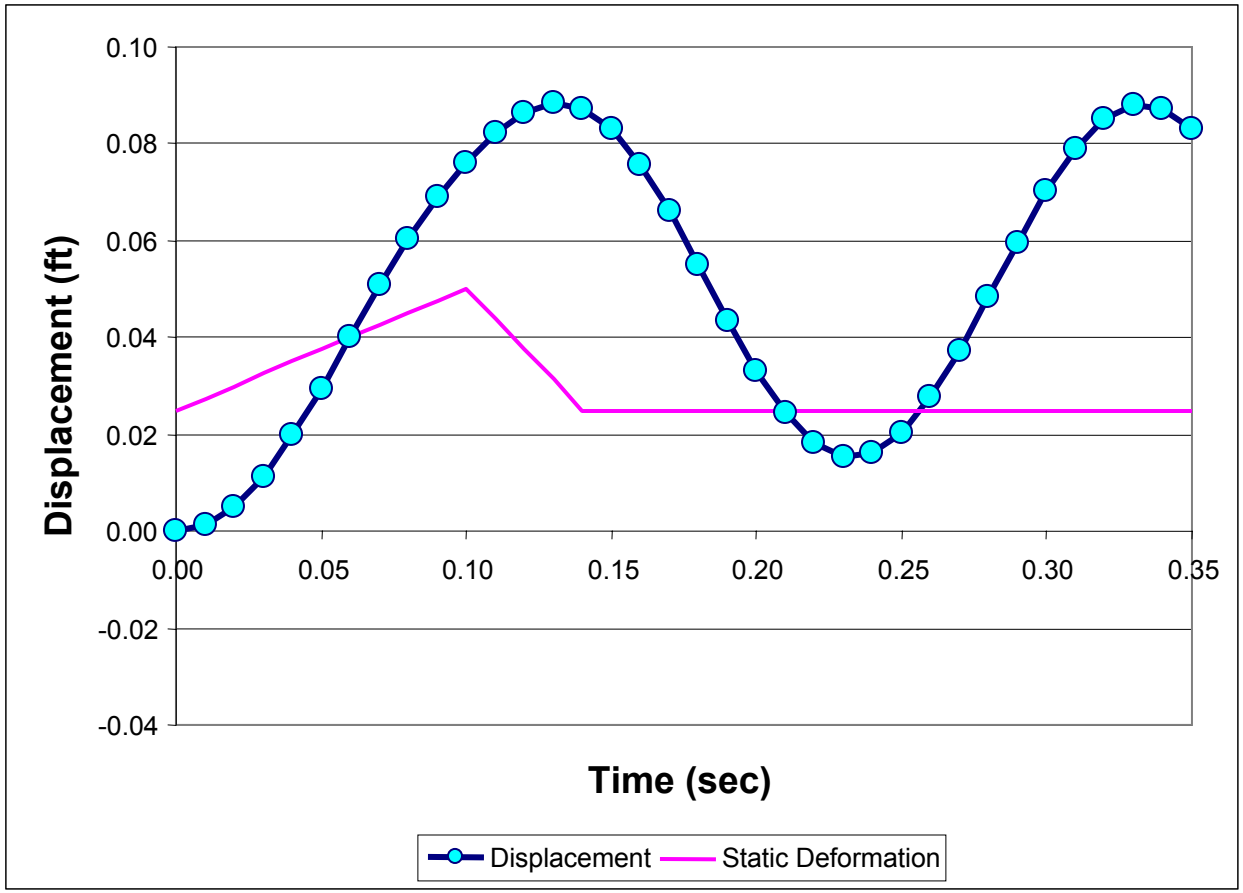
Elasto-Plastic Spring



t_i (sec)	F_i (lb)	a_i (ft/sec/sec)	v_i (ft/sec)	x_i (ft)	ΔF*_i	k*	Δx_i	Δv_i	Δa_i	xstatic_i	fspring
0.00	50	25.00	0.00	**0.000**	105.0	82000	0.00	0.26	1.22	0.025	0
0.01	55	26.22	0.26	**0.001**	314.8	82000	0.00	0.26	-1.34	0.0275	3
0.02	60	24.88	0.51	**0.005**	513.8	82000	0.01	0.23	-3.77	0.03	10
0.03	65	21.12	0.74	**0.011**	682.7	82000	0.01	0.18	-5.83	0.0325	23
0.04	70	15.29	0.92	**0.020**	805.0	82000	0.01	0.12	-7.32	0.035	39
0.05	75	7.97	1.04	**0.030**	868.8	82000	0.01	0.04	-8.10	0.0375	59
0.06	80	-0.12	1.08	**0.040**	867.8	82000	0.01	-0.04	-8.08	0.04	80
0.07	85	-8.21	1.04	**0.0507**	802.2	82000	0.01	-0.12	-7.28	0.0425	101
0.08	90	-15.49	0.92	**0.060**	678.3	80000	0.01	-0.14	2.50	0.045	121
0.09	95	-12.99	0.78	**0.069**	574.3	80000	0.01	-0.12	2.50	0.0475	121
0.10	100	-10.49	0.66	**0.076**	472.9	80000	0.01	-0.14	-6.25	0.05	121
0.11	87.5	-16.74	0.52	**0.082**	339.0	80000	0.00	-0.20	-6.25	0.04375	121
0.12	75	-22.99	0.32	**0.086**	155.1	80000	0.00	-0.26	-6.25	0.0375	121
0.13	62.5	-29.24	0.06	**0.088**	-78.8	80000	0.00	-0.32	-6.25	0.03125	121
0.14	50	-35.49	-0.26	**0.087**	-350.2	82000	0.00	-0.33	4.27	0.025	121
0.15	50	-31.22	-0.59	**0.083**	-600.0	82000	-0.01	-0.28	7.32	0.025	112
0.16	50	-23.90	-0.87	**0.076**	-791.2	82000	-0.01	-0.19	9.65	0.025	98
0.17	50	-14.25	-1.06	**0.0660**	-905.2	82000	-0.01	-0.09	11.04	0.025	79
0.18	50	-3.21	-1.15	**0.0550**	-930.9	82000	-0.01	0.02	11.35	0.025	56
0.19	50	8.14	-1.12	**0.044**	-865.8	82000	-0.01	0.13	10.56	0.025	34
0.20	50	18.70	-0.99	**0.033**	-716.2	82000	-0.01	0.23	8.73	0.025	13
0.21	50	27.43	-0.76	**0.024**	-496.8	82000	-0.01	0.30	6.06	0.025	-5
0.22	50	33.49	-0.45	**0.018**	-228.8	82000	0.00	0.35	2.79	0.025	-17
0.23	50	36.28	-0.10	**0.015**	61.4	82000	0.00	0.36	-0.75	0.025	-23
0.24	50	35.53	0.25	**0.016**	345.7	82000	0.00	0.33	-4.22	0.025	-21
0.25	50	31.32	0.59	**0.020**	596.2	82000	0.01	0.28	-7.27	0.025	-13
0.26	50	24.05	0.87	**0.028**	788.6	82000	0.01	0.19	-9.62	0.025	2
0.27	50	14.43	1.06	**0.037**	904.0	82000	0.01	0.09	-11.02	0.025	21
0.28	50	3.40	1.15	**0.048**	931.2	82000	0.01	-0.02	-11.36	0.025	43
0.29	50	-7.95	1.12	**0.060**	867.6	82000	0.01	-0.13	-10.58	0.025	66
0.30	50	-18.53	0.99	**0.070**	719.4	82000	0.01	-0.23	-8.77	0.025	87
0.31	50	-27.31	0.76	**0.079**	500.9	82000	0.01	-0.30	-6.11	0.025	105
0.32	50	-33.41	0.46	**0.085**	233.6	82000	0.00	-0.35	-2.85	0.025	117
0.33	50	-36.26	0.11	**0.088**	-56.5	80000	0.00	-0.36	0.00	0.025	123
0.34	50	-36.26	-0.25	**0.087**	-346.6	82000	0.00	-0.34	4.23	0.025	123
0.35	50	-32.04	-0.59	**0.083**	-602.9	82000	-0.01	-0.28	7.35	0.025	114

Elasto-Plastic Spring

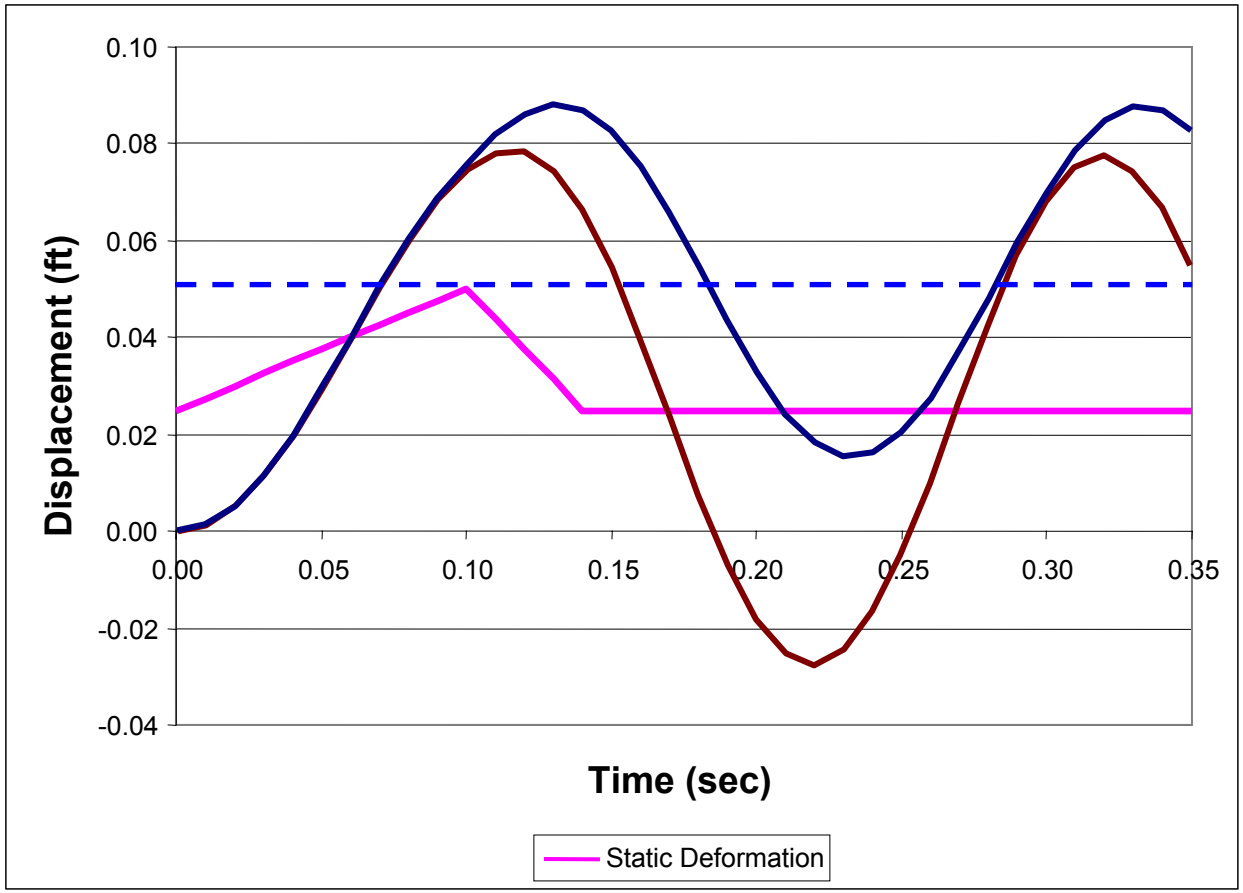



0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
Displacement (ft)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Time (sec)
Displacement
Static Deformation

Linear elastic

vs.

Linear elastic-plastic (elasto-plastic)



Max deflections:

Linear elastic: ~0.08’

Elasto-plastic: ~0.09’

Max spring forces:

Linear elastic: ~160 lb

Elasto-plastic: ~110 lb