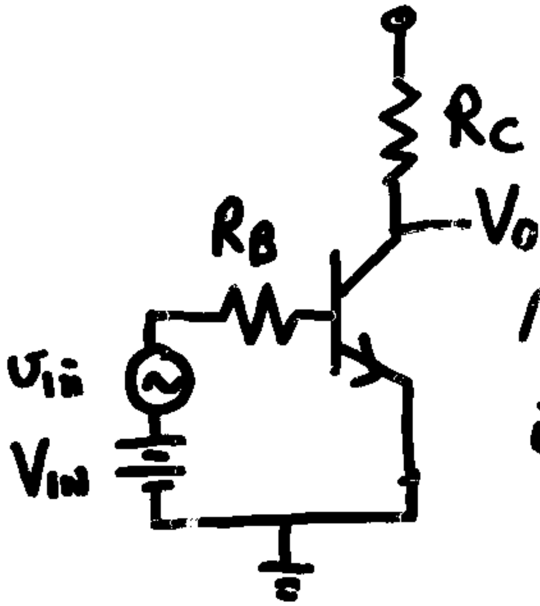
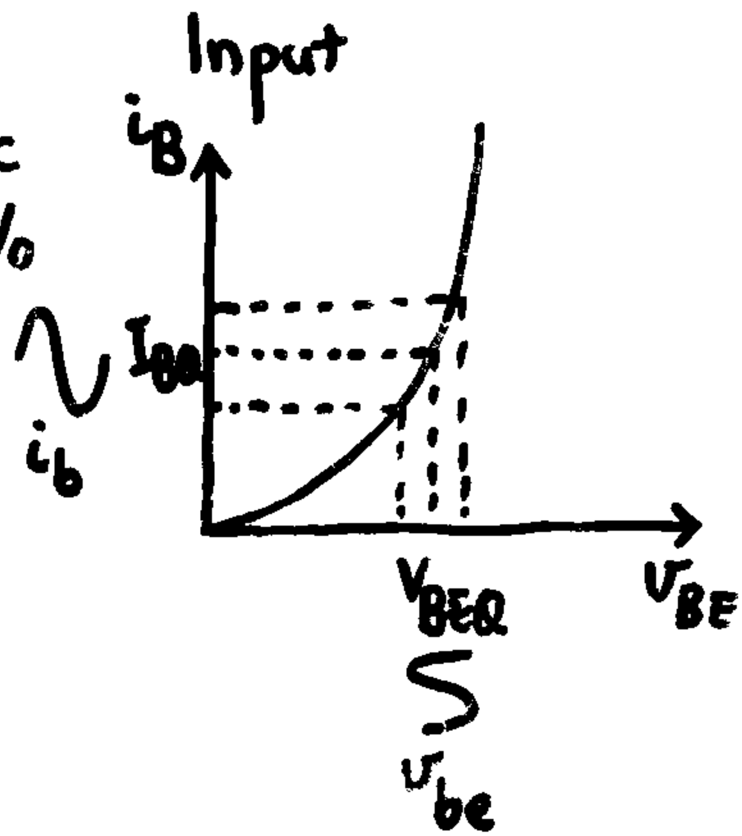
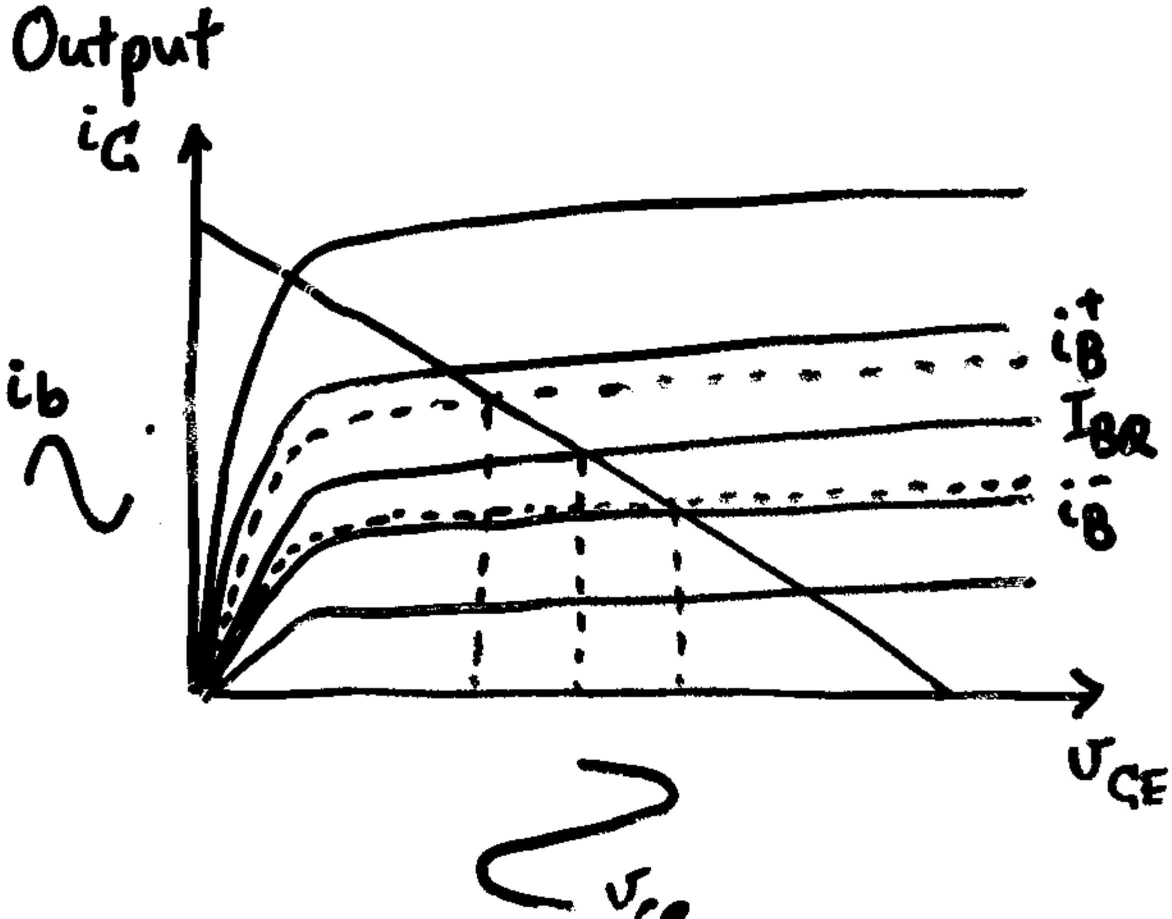
Small-signal model for BJT







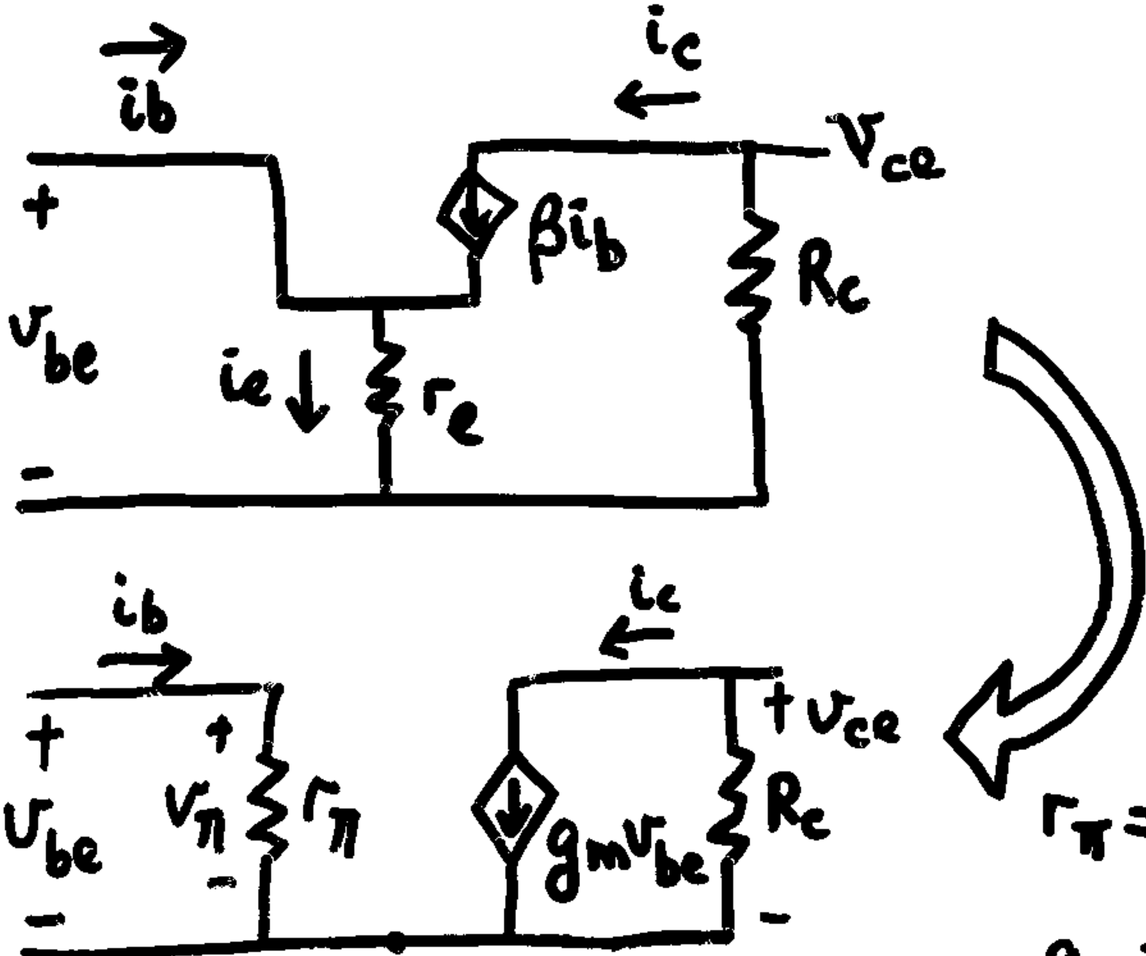
$$i_E = I_s e^{v_{BE}/V_T}$$

$$\doteq I_s e^{V_{BEQ}/V_T} + I_s e^{V_{BEQ}/V_T}\left(\frac{1}{V_T} v_{be}\right.$$

$$\downarrow \qquad\qquad \downarrow$$

$$I_{EQ} \qquad\qquad i_e = v_{be}\frac{I_E}{V_T}$$

$$\Rightarrow r_e = \frac{V_T}{I_E}$$



$$r_\pi = (\beta+1)r_e$$

$$g_m = \frac{\beta}{r_\pi}$$

Another derivation

$$i_C = \alpha I_s e^{v_{BE}/V_T}$$

$$\doteq \alpha I_s e^{V_{BEQ}/V_T} + \alpha I_s e^{V_{BEQ}/V_T} \frac{v_{be}}{V_T}$$

$\downarrow$ I_{CQ}

$\downarrow$ $i_c = v_{be} \frac{I_C}{V_T}$

$$= g_m v_{be}$$

Transconductance $g_m = \frac{I_C}{V_T}$

$$= \frac{\beta}{r_\pi} \; ?$$

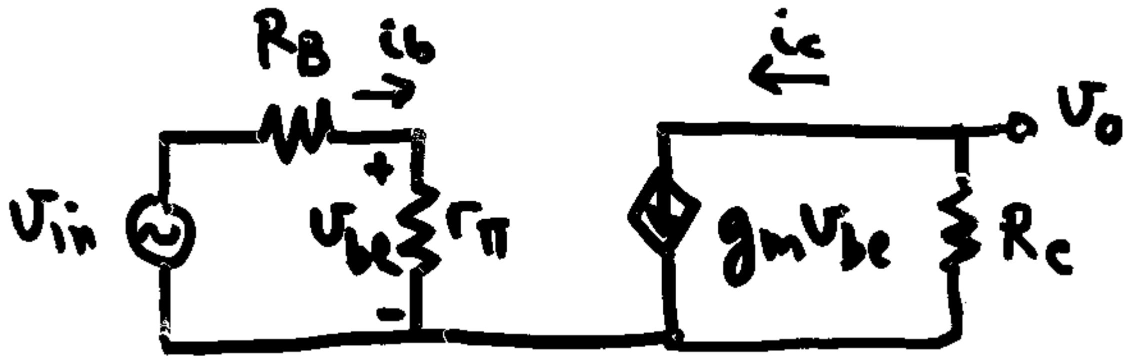
$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}\beta}{i_c}$$

$$= \frac{\beta}{g_m}$$

$$\frac{\beta}{(\beta+1)r_e}$$

$$= \frac{\alpha}{V_T/I_E} = \frac{I_C}{V_T}$$

Equivalent (ac) circuit



$$v_{be} = \frac{r_\pi}{R_B + r_\pi} v_{in}$$

$$i_c = \frac{g_m r_\pi}{R_B + r_\pi} v_{in} = \frac{\beta}{r_\pi + R_B} v_{in}$$

$$v_o = -i_c R_C = -\frac{\beta}{r_\pi + R_B} R_C \, v_{in}$$

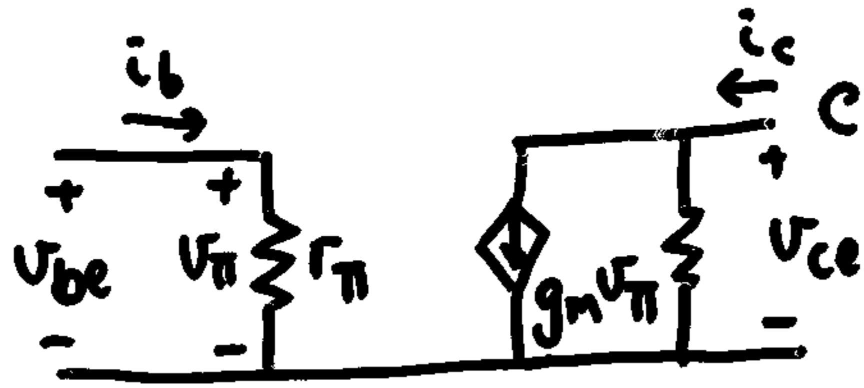
Voltage gain $A_v = \frac{v_o}{v_{in}}$

$$= \frac{-\beta R_c}{r_\pi + R_B}$$

Early effect

$$I_C = \alpha I_S e^{V_{BE}/V_T}\left[1 + \frac{V_{CE}}{V_A}\right]$$

$$\left.\frac{di_C}{dv_{CE}}\right|_{V_{CEQ}} = \frac{I_{CQ}}{V_A} \Rightarrow r_o = \frac{V_A}{I_C}$$



Voltage gain $A_V = -g_m r_o r_\pi$

With R_B & R_C ?

$$A_V = -g_m (r_o \| R_C) \frac{r_\pi}{r_\pi + R_B}$$

$$= -\frac{\beta (r_o \| R_C)}{r_\pi + R_B}$$

Early effect

$$i_C = \left[\alpha I_s e^{V_{BE}/V_T}\right]\left[1 + \frac{v_{CE}}{V_A}\right]$$

$$= \left[\alpha I_s e^{V_{BEQ}/V_T} \cdot e^{v_{be}/V_T}\right]\left[1 + \frac{V_{CEQ}}{V_A} + \frac{v_{ce}}{V_A}\right]$$

$$= \alpha I_s e^{V_{BEQ}/V_T} \cdot \left[1 + \frac{v_{be}}{V_T}\right] \cdot \left[1 + \frac{V_{CEQ}}{V_A} + \frac{v_{ce}}{V_A}\right]$$

$$= \alpha I_s e^{V_{BEQ}/V_T}\left[1 + \frac{V_{CEQ}}{V_A}\right]$$

$$+ \alpha I_s e^{V_{BEQ}/V_T}\left[1 + \frac{V_{CEQ}}{V_A}\right] \cdot \frac{v_{be}}{V_A}$$

$$+ \alpha I_s^{V_{BEQ}/V_T} \frac{v_{ce}}{V_A}$$

$$+ \alpha I_s^{V_{BEQ}/V_T} \frac{v_{be}}{V_T} \frac{v_{ce}}{V_A}$$

Other effects

1. Series resistance (r_b) of semi-conductor material in base ($O(10\,\Omega)$)

2. Reverse-biased diffusion resistance (r_μ) of B-C junction ($O(M\Omega)$)

Typically ignored except for high frequencies