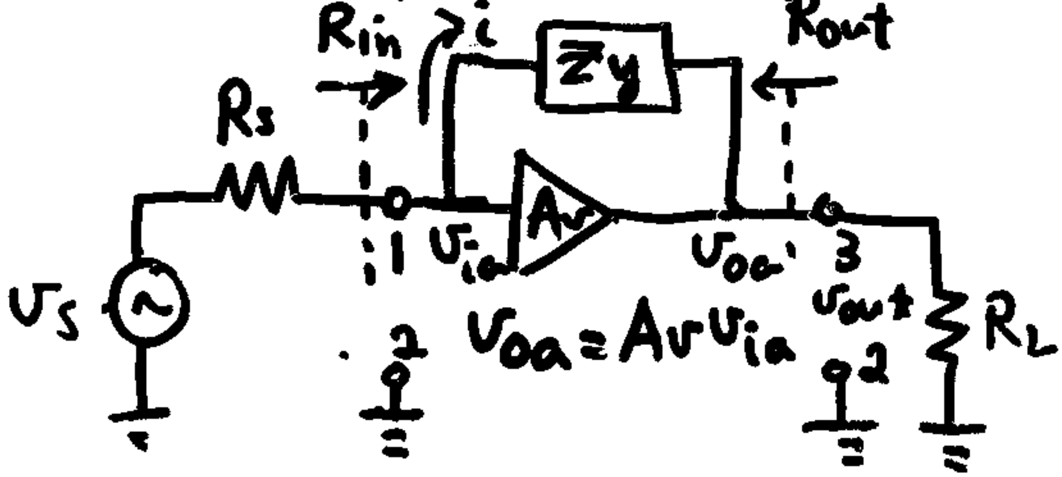
Miller Effect



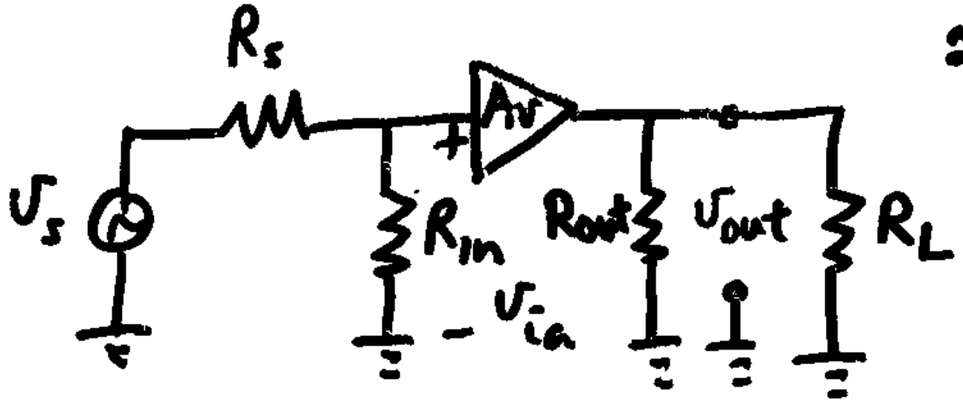
$$R_{in} = \frac{v_{ia}}{i}$$

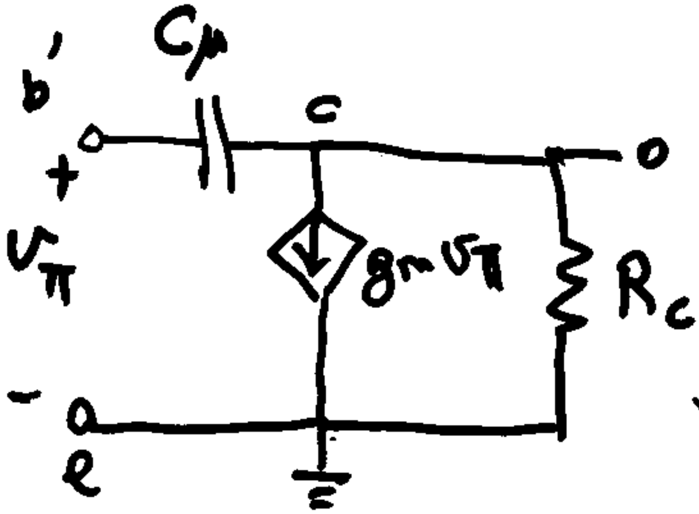
$$i = \frac{v_{ia} - v_{oa}}{Z_y} = \frac{v_{ia} - A_v v_{ia}}{Z_y} = \frac{(1-A_v)v_{ia}}{Z_y}$$

$$R_{in} = \frac{Z_y}{1-A_v} \qquad R_{out} = \frac{Z_y}{1-\frac{1}{A_v}}$$

$$\simeq Z_y$$

For large $|A_v|$





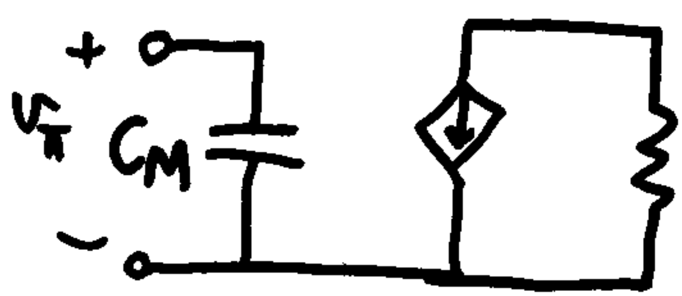
$$\frac{v_o}{v_\pi} = \frac{-g_m v_\pi R_c}{v_\pi} = -g_m R_c$$

$$v_c = -g_m v_\pi R_c$$

$$v_{b'e} = v_\pi(1 + g_m v_\pi R_c)$$

Charge supplied to C_μ

$$Q_1 = C_\mu v_{b'e} = C_\mu v_\pi(1 + g_m R_c)$$



$$Q_1 = Q_2$$

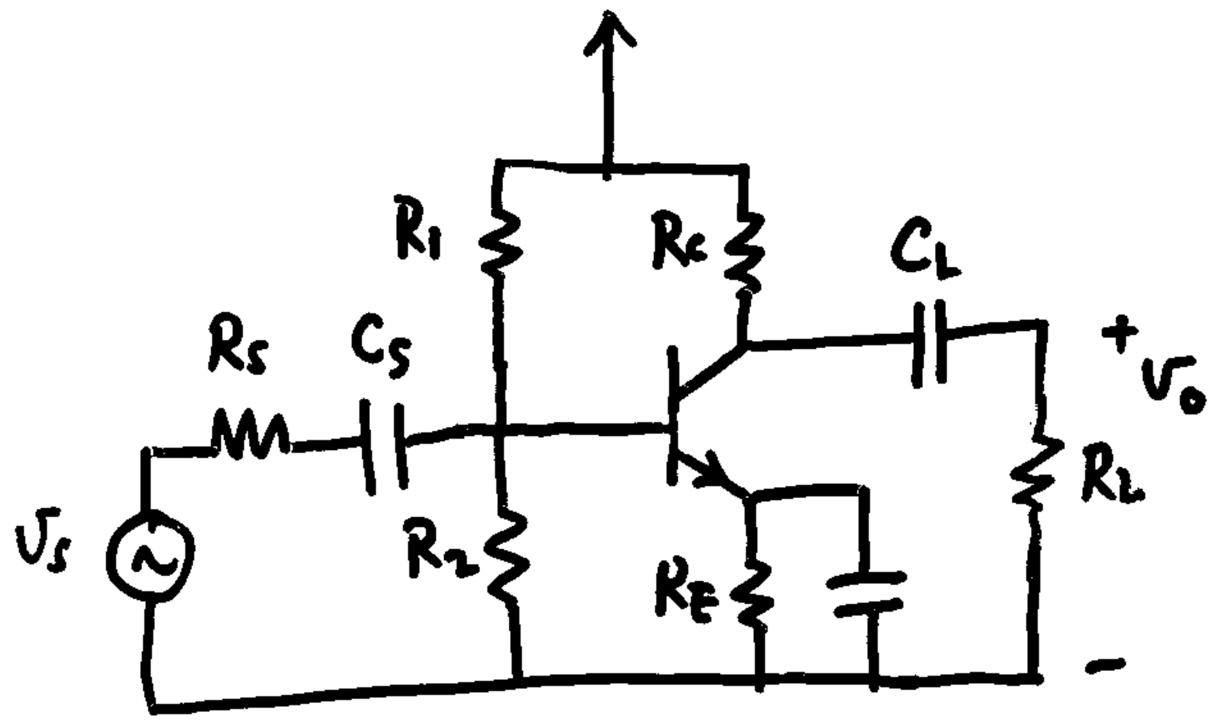
$$\Rightarrow C_M = (1 + g_m R_c)\, C_\mu$$

Charge supplied to C_M

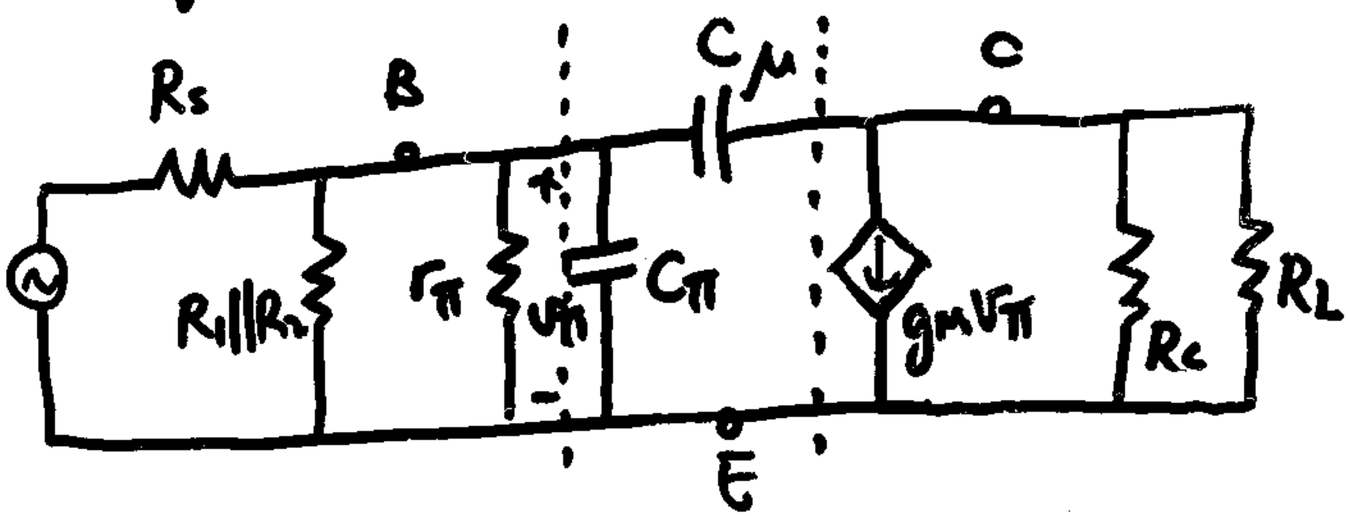
$$Q_2 = C_M v_\pi$$

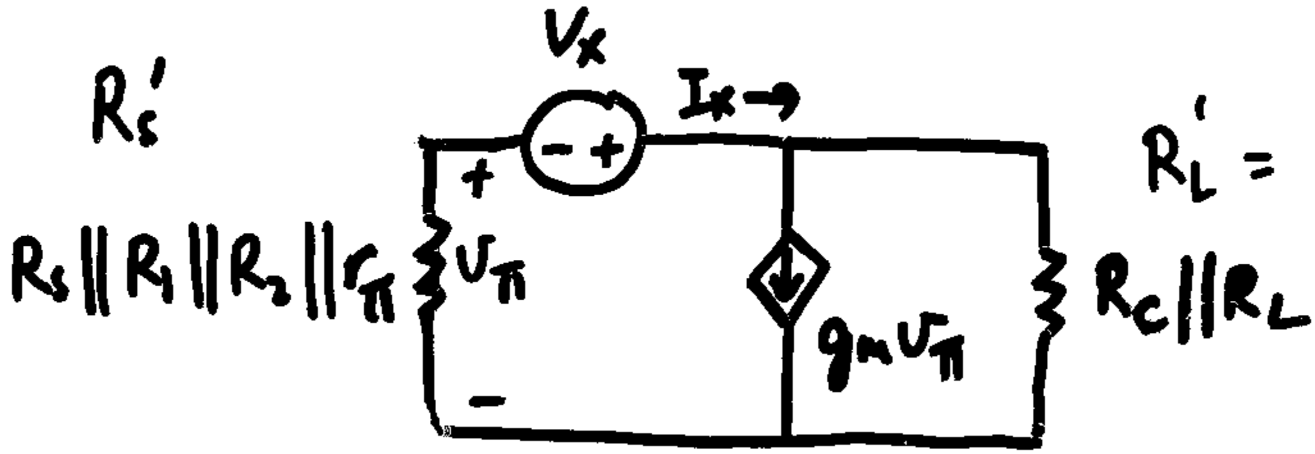
Unilateral equivalent ckt

High Frequency CE Amplifier



High-freq equiv. ckt





$$V_x = \left[I_x + g_m I_x R_s' \right] R_L' + I_x R_s'$$

$$\Rightarrow R_\mu = (1 + g_m R_s') R_L' + R_s'$$

$$= R_s' (1 + g_m R_L') + R_L'$$

$$\tau_\mu = R_s' \underbrace{(1 + g_m R_L') C_\mu}_{(1 - A_v) C_\mu = C_M} + R_L' C_\mu$$

$$R_\pi = R_s'$$

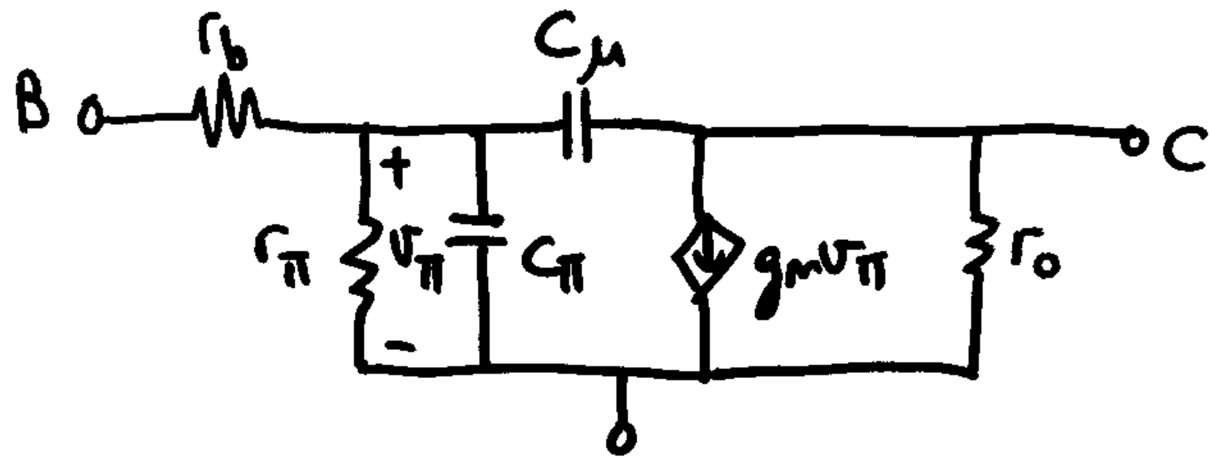
$$\tau_\pi = R_s' C_\pi$$

$$\omega_H \cong \frac{1}{\tau_\pi + \tau_\mu}$$

Short-circuit current gain

Manufacturer specifies cut-off frequency or unity-gain bandwith

f_T

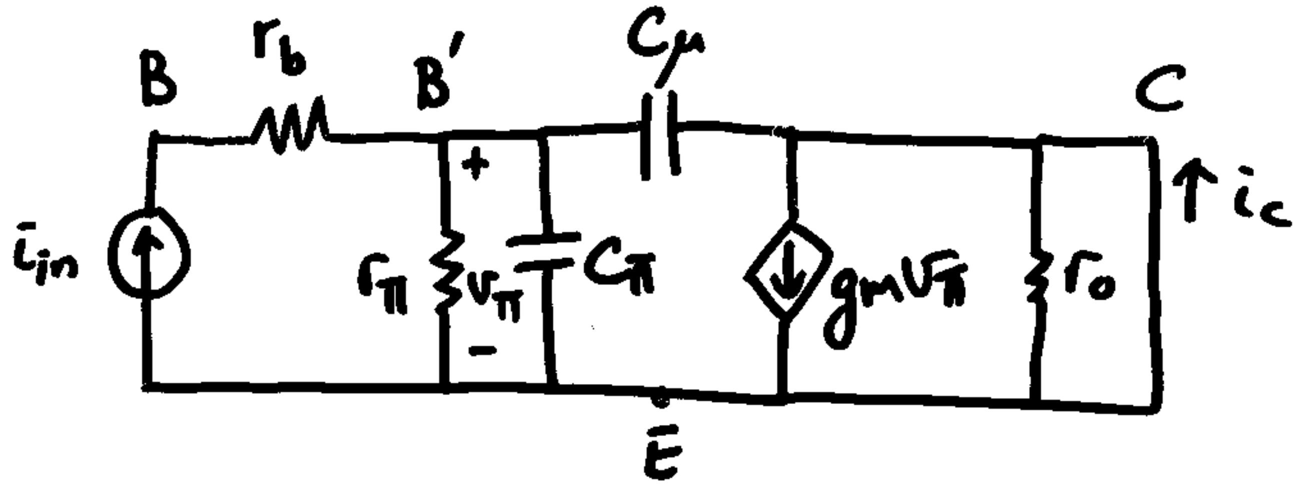


Low ω: C_μ, C_π open
current flows through r_π, leading to v_π, high $g_m v_\pi$ current

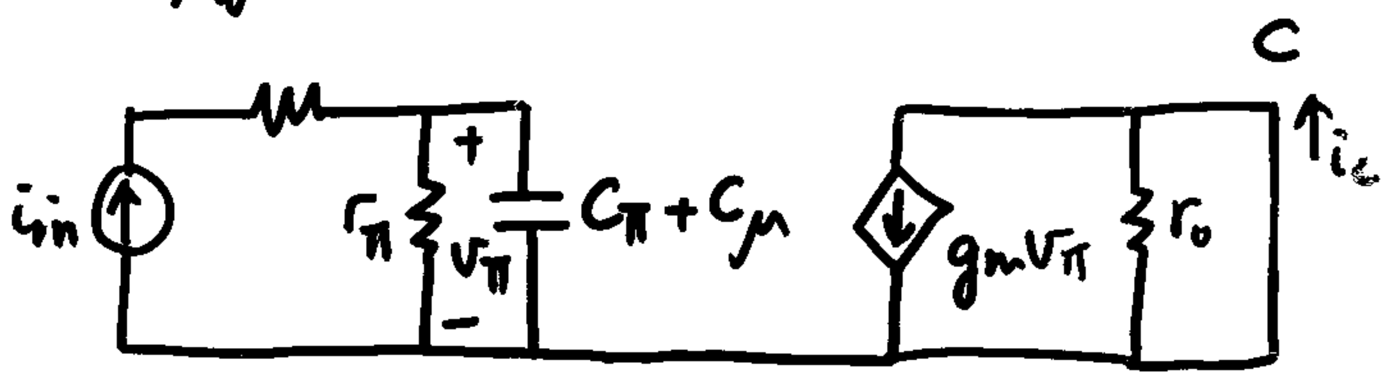
High ω: C_μ, C_π look more like short ckts
less current through r_π,
less $g_m v_\pi$ current

Dependance of β on ω

Short C, use a current source input



$A_v = 0$

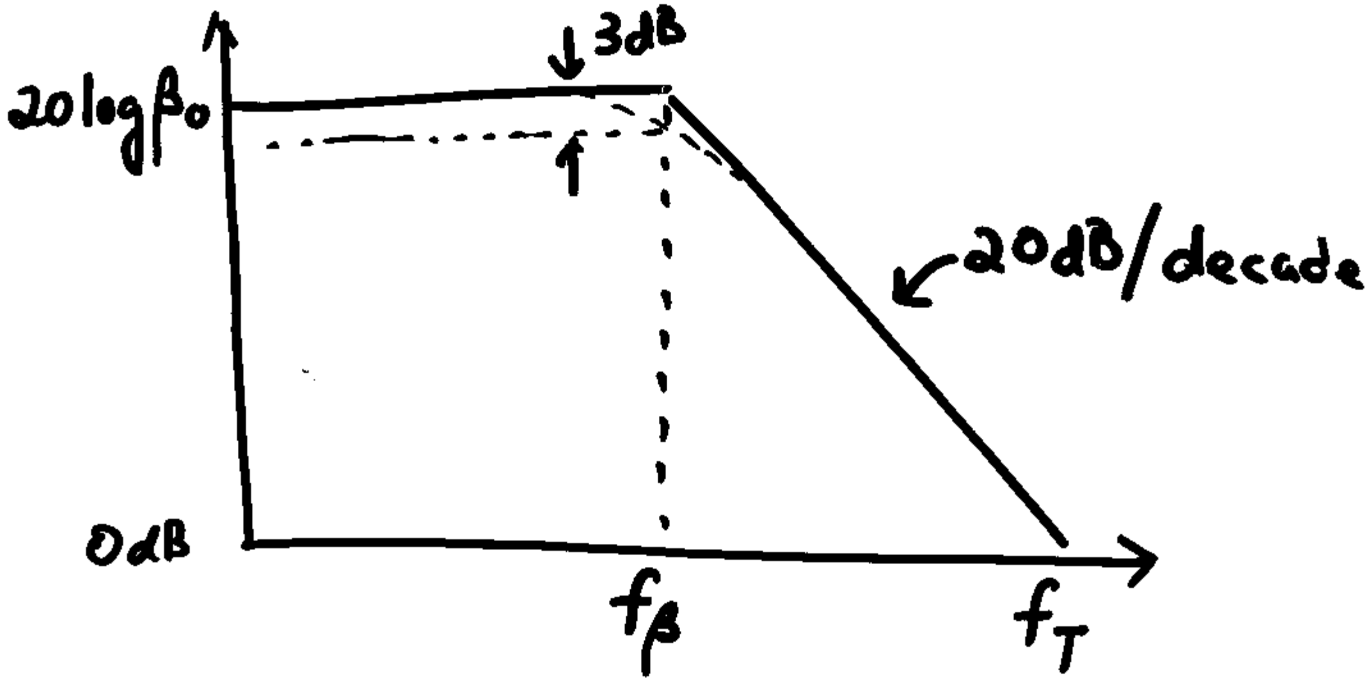


$$v_\pi = i_{in} \frac{r_\pi}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$

$$i_c = g_m v_\pi$$

(h_{fe}) Current gain $\dfrac{i_c}{i_{in}} = \dfrac{\beta_0}{1 + j\omega r_\pi (C_\pi + C_\mu)}$

f_β, β-cutoff frequency $= \dfrac{1}{2\pi r_\pi (C_\pi + C_\mu)}$
(3dB drop)



cut-off frequency when gain = 1

unity-gain bandwidth or gain-bandwidth product

$$20\log\frac{f_T}{f_\beta} = 20\log\beta_0$$

$$f_T = f_\beta\beta_0 \leftarrow \text{gain-bandwidth product}$$

$$= \frac{\beta_0}{2\pi r_\pi(C_\pi + C_\mu)}$$

$$= \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

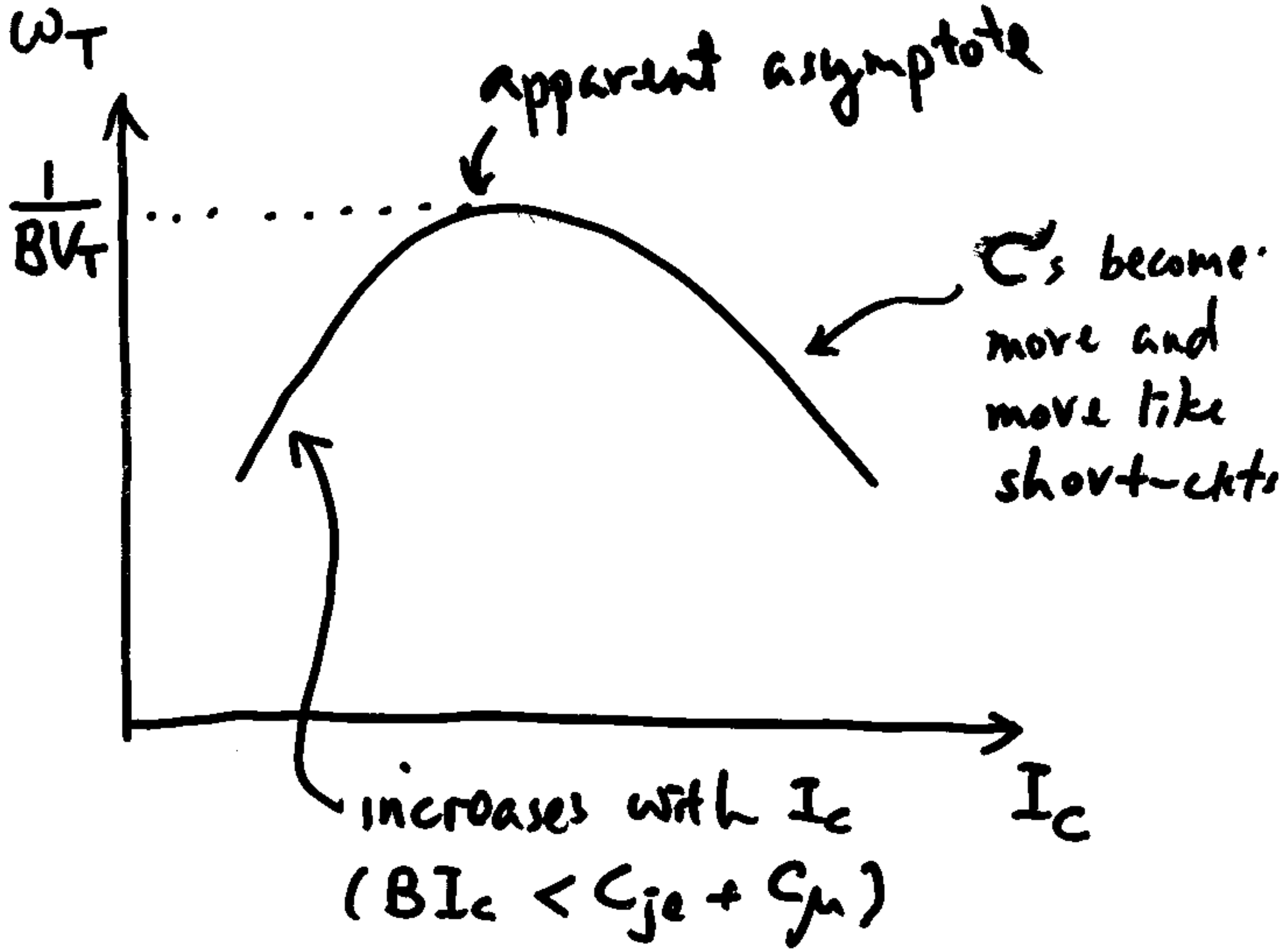
f_T given by manufacturer

F_T vs. I_c

$$\omega_T = \frac{g_m}{C_\pi + C_\mu}$$

$$= \frac{g_m}{C_{je} + BI_c + C_\mu}$$

$$= \frac{I_c}{V_T(C_{je} + BI_c + C_\mu)}$$



Example $\beta_o = 200$

$\omega_T = 500 \cdot 10^6$ rad/s

$n = 1$

$V_{bi} = 0.8V$

$V_{BE(ON)} = 0.65V$

$C_\mu = 3pF$

a) Find g_m, r_π, C_π, ω_β if $I_C = 1mA$

$V_{CE} = 10V$

$$g_m = \frac{I_C}{V_T} = 38.5 mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 5.2k\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = 74 pF$$

$$\omega_\beta = \frac{1}{r_\pi(C_\pi + C_\mu)} = 2.5 \text{ M rad/s}$$

b) Hf $I_c = 2mA$, $V_{CE} = 5V$?

g_m doubles

r_π halved

$$C_\mu = \frac{C_{jo}}{\left(1 + \frac{V_{CB}}{V_{bi}}\right)^N} \qquad \frac{1}{3} < N < \frac{1}{2}$$

$$\frac{C_{\mu 1}}{C_{\mu 2}} = \frac{(1 + 4.35/0.8)^N}{(1 + 9.35/0.8)^N} = \frac{3pF}{C_{\mu 2}}$$

Assume $N = 1/2$

$$C_{\mu 2} = 4.2pF$$

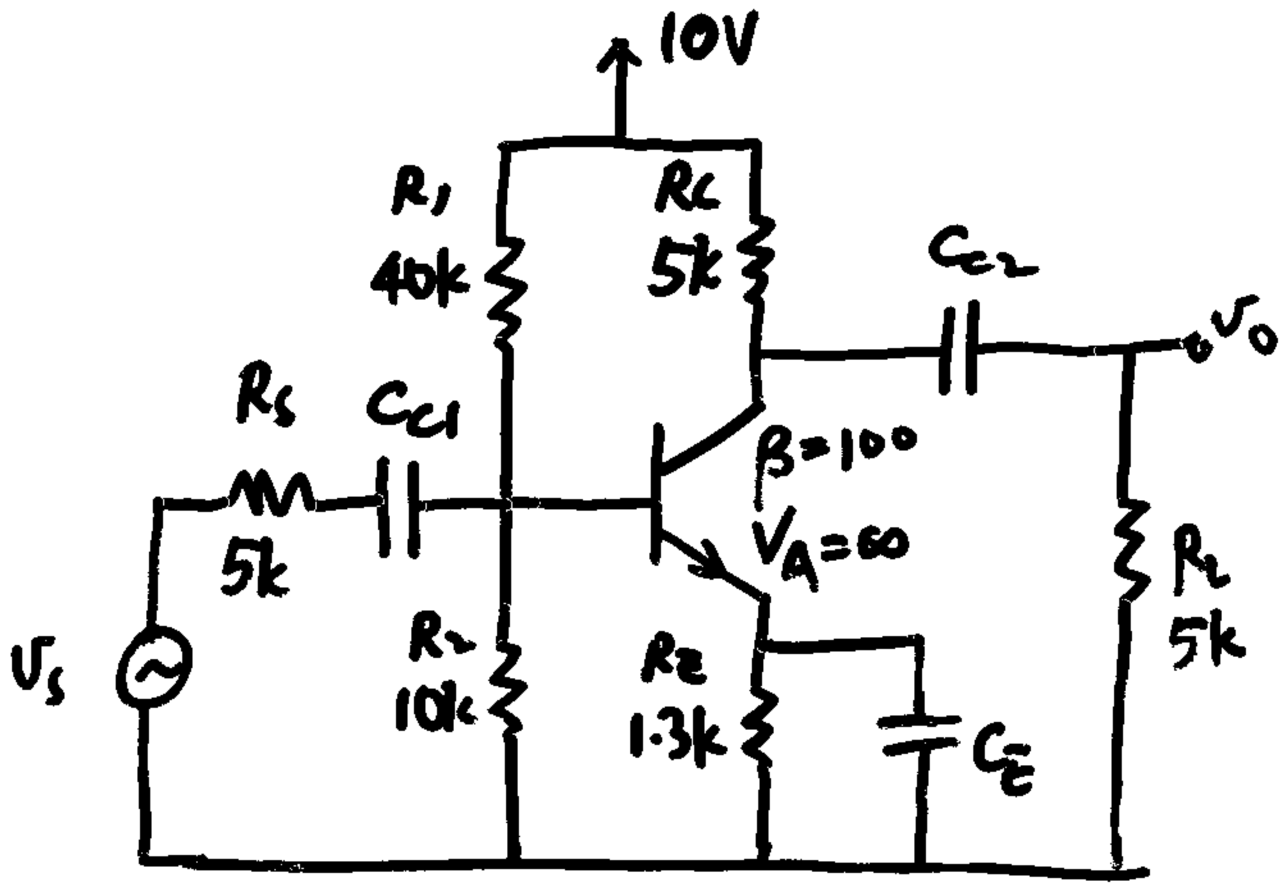
$$C_\pi = \frac{\omega_T}{g_m} - C_\mu$$

$$= 92pF$$

ω_T almost doubles

$\simeq 800.10^6$ rad/s

(from data sheet)



$V_{BE} = 0.7V$

$$I_B = \frac{V_{TH} - V_{BE}}{R_{TH} + (\beta+1)R_E} = 9.33\mu A$$

$I_C = 0.933\ mA$

$I_E = 0.943\ mA$

$V_{CB} = 3.41V$

$$C_\mu = \frac{C_{jo}}{\left(1 + \frac{V_{CB}}{V_{Bi}}\right)^{1/3}} \qquad C_{jo} = 3.54pF,\ V_{Bi} = .75V$$

$= 2pF$

$$g_m = \frac{I_C}{V_T} = 35.9\ mA/V$$

$$r_\pi = \frac{V_T}{I_B} = 2.79\ k\Omega$$

$$C_\mu = 2pF$$

given $f_T = 500\,MHz$

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu = 9.4\,pF$$

$$A_{MB} = -g_m(R_C \| R_L) = -89.8$$

$$A_{vs,MB} = A_{MB} \cdot \frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} = -26.3$$

$$R_\mu = (R_S \| R_1 \| R_2 \| r_\pi)(1 - A_{MB}) + R_C \| R_L$$
$$= 1.46\,k\Omega\,(90.8) + 2.5\,k\Omega$$
$$= 135\,k\Omega$$

$\tau_\mu = 270\,ns$ $\qquad \omega_H = \dfrac{1}{\tau_\mu + \tau_\pi}$

$R_\pi = 1.46\,k\Omega$ $\qquad = 3.52 \times 10^6\ rad/s$

$\tau_\pi = 13.7\,ns$ $\qquad f_H = 561\ kHz$

Gain-Bandwidth Product of CE

$$\omega_H \cong \frac{1}{\tau_\mu} \cong \frac{1}{R_s' C_M}$$

$$\cong \frac{1}{R_s' |A_{MB}| C_\mu}$$

$$f_H \cong \frac{1}{2\pi R_s' |A_{MB}| C_\mu}$$

$$f_H \times |A_{MB}| = \frac{1}{2\pi R_s' C_\mu} \cong \text{constant}$$

independent of V_{CE} or I_C