Exact Closed Form Formula for Partial Mutual Inductances of On-Chip Interconnects [*]

Guoan Zhong and Cheng-Kok Koh
School of Electrical and Computer Engineering
Purdue University, West Lafayette, IN 47907-1285
{zhongg,chengkok}@ecn.purdue.edu

Abstract

In this paper, we propose a new exact closed form mutual inductance equation for on-chip interconnects. We express the mutual inductance between two parallel rectangular conductors as a weighted sum of self-inductances. We do not place any restrictions on the alignment of the two parallel rectangular conductors. Moreover, they could be co-planar or reside on different layers. Most important, experimental results show that our formula is numerically more stable than that derived in [2] for long parallel on-chip interconnects.

1 Introduction

In modern VLSI design, it is prudent that inductance effect be considered in the timing and noise analysis of on-chip global interconnects. The concept of inductance is defined based on the magnetic fields caused by currents flowing through closed conductor loops. For general three-dimensional interconnects, however, the return paths of currents are distributed and not known *a priori*. An approach that obviates the need for prior knowledge of return paths in circuit simulation is the use of the *partial element equivalent circuit* (PEEC) model [6]. In this model, *partial inductances* are defined to represent the loop interactions among conductors, each forming its own return loop with infinity. In the following discourse, we use mutual inductance to refer strictly to partial mutual inductance and self-inductance to refer strictly to partial self-inductance.

In this paper, we derive the exact closed form formula for the mutual inductance of two parallel conductors; for two wires orthogonal to each other, the mutual inductance is zero. Exact formulas for the mutual inductance of two parallel conductors are available. For example, the mutual inductance between two parallel filaments with length l and spacing d is given by the following exact formula [5]:

$$M = \frac{\mu l}{2\pi}[\ln(\frac{l}{d} + \sqrt{1+\frac{l^2}{d^2}}) - \sqrt{1+\frac{d^2}{l^2}} + \frac{d}{l}], \qquad (1)$$

where μ is the permeability.

If $d \ll l$, a simpler approximate formula can be obtained through Taylor's expansion [1]:

$$M = \frac{\mu l}{2\pi}[\ln(\frac{2l}{d}) - 1]. \qquad (2)$$

If the length is not sufficiently larger than the distance, the accuracy could be affected. When that happens, Eqn. (2) is not a good approximation of Eqn. (1).

For two conductors with the cross-sectional dimensions comparable to their distance, which is typical of on-chip interconnects, they cannot be treated as filaments. In this case, the geometry mean distance (GMD) should be used in Eqn. (2) instead of d. Although exact formula for GMD of two rectangular area exists, it is common that only an approximation is used. In [1], pre-computed tables are used to obtain GMD. In [7], table-lookup and summation are used to calculate the GMD of two wires.

One major shortcoming of Eqns. (1) and (2) is that they do not apply to more general cases; the parallel conductors must be of the same length and their end points aligned. There are techniques that can be deployed to overcome this shortcoming [4].

In [2], the authors derived a closed form formula for the mutual inductance of any pair of parallel rectangular conductors even if they are not aligned. The formula is given below:

$$M = \frac{[[[f(X,Y,Z)]_{X=w_1+w_3,w_2+w_3}^{X=w_3,w_1+w_2+w_3}]_{Y=t_1+t_3,t_2+t_3}^{Y=t_3,t_1+t_2+t_3}]_{Z=l_3+l_2-l_1,l_3}^{Z=l_3-l_1,l_3+l_2}}{w_1 w_2 t_1 t_2}, \qquad (3)$$

where w_1, w_2, and w_3 are the widths and the distance between the two lines in the x-direction; t_1, t_2, and t_3 are the

---

[*] This research is supported in part by SRC (99-TJ-689), NSF (CAREER Award CCR-9984553), and a grant from Intel Corporation.

1063-6404/02 $17.00 © 2002 IEEE

thicknesses and the distance in the y-direction; l_1, l_2, and l_3 are the lengths and the offset in the z-direction; and

$$[[[f(X,Y,Z)]_{X=x_1,x_2}^{X=x_3,x_4}]_{Y=y_1,y_2}^{Y=y_3,y_4}]_{Z=z_1,z_2}^{Z=z_3,z_4} \equiv \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4}(-1)^{i+j+k+1} f(x_i, y_j, z_k).$$

The exact expression of $f(X,Y,Z)$ can be found in [2]. Unfortunately, the computation of the exact formula in Eqn. (3) is numerically unstable (see Section 6 for the numerical results).

Consider the special case when we calculate the mutual inductance between two identical conductors that coincide with each other, we obtain the self-inductance. The closed form formulas for self-inductance in [2, 6, 8] are derived in this fashion. However, the formulas in [6, 8] are numerically more stable than the formula given in [2].

To overcome the numerical instability of Eqn. (3) In this paper, we reveal in this paper the *inverse* relation between mutual inductance and self-inductance, that is, the mutual inductance can be expressed in terms of self-inductance. To be more specific, the mutual inductance of on-chip interconnects is a weighted sum of self-inductances. Just like Eqn. (3), we do not impose any restrictions on the alignment of the two parallel rectangular conductors. Moreover, the formula applies to co-planar wires or wires residing on different layers. Most important, it is exact and numerically stable for practical cases of modern on-chip interconnects. We also derive for special cases of parallel conductors that are commonly encountered among on-chip interconnects closed form formulas that are even more compact. Experimental results in Section 6 show that our formula is numerically more stable than Eqn. (3) derived in [2] for long parallel on-chip interconnects.

2 Preliminaries

The mutual inductance between two conductors with uniform cross sections is

$$M = \frac{1}{I_0 I_1}\int_{A_0}\int_{A_1} M_{01} J_0 J_1 \, dA_0 dA_1, \qquad (4)$$

where A_0 and A_1 are the cross-sectional areas of the two conductors. I_0, I_1, J_0 and J_1 are the current and the current densities of the conductors. M_{01} is the mutual inductance between two filaments dA_0 and dA_1, and the current is assumed to be constant along the the length of each filament.

At relatively low frequency, the current distribution varies very little in the cross sections and can be assumed to be constant throughout the conductors. Hence, the mutual inductance can be reduced to the following equation:

$$M = \frac{1}{A_0 A_1}\int_{A_0}\int_{A_1} M_{01} \, dA_0 dA_1. \qquad (5)$$

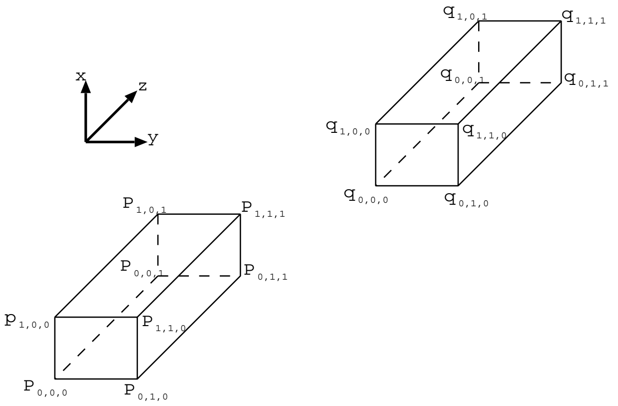


Figure 1. Two parallel wires.

As indicated by the preceding equation, the mutual inductance is determined only by the geometries of the two conductors. Under magneto-quasistatic condition, the mutual inductance between two filaments L_0 and L_1 can be calculated by Neumann's formula:

$$M_f = \frac{\mu}{4\pi}\int_{L_0}\int_{L_1} \frac{dl_0 \cdot dl_1}{r}, \qquad (6)$$

where r is the distance between dl_0 and dl_1 and μ is the permeability.

Consider two parallel rectangular wires as illustrated in Figure 1. Here, we assume that the current flows in the z direction. As can be seen in the figure, the displacements of the two wires along the x and y directions are non-zero. Let the cross-sectional dimensions (in the $x-y$ plane) of the two wires be $T_0 \times W_0$ and $T_1 \times W_1$. We use $p_{i,j,k}$ and $q_{i,j,k}$, $i,j,k \in \{0,1\}$, to denote the corners of the two wires, as illustrated in Fig 1. All corner points of the first wire has two z coordinate values. We use z_{p_k}, $k \in \{0,1\}$, to denote the z coordinate value shared by the corners $p_{*,*,k}$. Similarly, we use z_{q_k}, $k \in \{0,1\}$, to denote the z coordinate value shared by the corners $q_{*,*,k}$ of the second wire. Similarly defined are the x and y-coordinate values of the corners of the two wires: x_{p_i} and x_{q_i}, $i \in \{0,1\}$, and y_{p_j} and y_{q_j}, $j \in \{0,1\}$.

Now, substituting Eqn. (6) into Eqn. (5), we obtain

$$\begin{aligned} M &= \frac{1}{W_0 T_0 W_1 T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} M_f \, dy_0 dy_1 dx_0 dx_1 \\ &= \frac{1}{W_0 T_0 W_1 T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} \\ &\quad \left[\frac{\mu}{4\pi}\int_{z_{p_0}}^{z_{p_1}}\int_{z_{q_0}}^{z_{q_1}} \frac{1}{r} dz_0 dz_1\right] dy_0 dy_1 dx_0 dx_1. \end{aligned} \qquad (7)$$

If the two conductors coincide with each other, then the preceding mutual inductance equation gives the equation for the self-inductance of one conductor:

$$L = \frac{1}{A^2}\frac{\mu}{4\pi}\int\int\int\int\int\int\frac{1}{r}dz_0dz_1dy_0dy_1dx_0dx_1. \tag{8}$$

3 Formula for Mutual Inductance

In the following, we reveal the relation between mutual inductance and self-inductance, and then derive a closed form formula for the mutual inductance as a weighted sum of self-inductances.

It is trivial to show that for any function $f(x)$,

$$\begin{aligned}
&\int_{p_0}^{p_1}\int_{q_0}^{q_1} f(|x_0-x_1|)dx_0dx_1 \\
=\;& \frac{1}{2}\left(\int_{p_0}^{q_1}\int_{p_0}^{q_1} f(|x_0-x_1|)dx_0dx_1\right. \\
&+\int_{p_1}^{q_0}\int_{p_1}^{q_0} f(|x_0-x_1|)dx_0dx_1 \\
&-\int_{p_0}^{q_0}\int_{p_0}^{q_0} f(|x_0-x_1|)dx_0dx_1 \\
&\left.-\int_{p_1}^{q_1}\int_{p_1}^{q_1} f(|x_0-x_1|)dx_0dx_1\right) \\
=\;& \frac{1}{2}\sum_{i,j=0}^{1}(-1)^{i+j+1}\int_{p_i}^{q_j}\int_{p_i}^{q_j} f(|x_0-x_1|)dx_0dx_1.
\end{aligned} \tag{9}$$

Making use of Eqn. (9), we can rewrite Eqn. (7) as follows:

$$\begin{aligned}
&M \\
=\;& \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} \\
&\left[\frac{\mu}{4\pi}\int_{z_{p_0}}^{z_{p_1}}\int_{z_{q_0}}^{z_{q_1}}\frac{1}{r}dz_0dz_1\right]dy_0dy_1dx_0dx_1 \\
=\;& \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} \\
&\left[\frac{1}{2}\sum_{k_0,k_1=0}^{1}(-1)^{k_0+k_1+1}\frac{\mu}{4\pi}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\frac{1}{r}dz_0dz_1\right] \\
&dy_0dy_1dx_0dx_1 \\
=\;& \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}} \\
&\left[\frac{1}{4}\sum_{j_0,j_1,k_0,k_1=0}^{1}(-1)^{j_0+j_1+k_0+k_1}\frac{\mu}{4\pi}\right. \\
&\left.\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\frac{1}{r}dz_0dz_1dy_0dy_1\right]dx_0dx_1
\end{aligned}$$

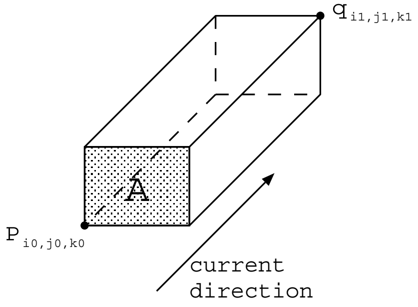


Figure 2. A virtual conductor defined by two corner points.

$$\begin{aligned}
=\;& \frac{1}{W_0T_0W_1T_1}\times \\
&\left[\frac{1}{8}\sum_{i_0,i_1,j_0,j_1,k_0,k_1=0}^{1}(-1)^{i_0+i_1+j_0+j_1+k_0+k_1+1}\right. \\
&\frac{\mu}{4\pi}\int_{x_{p_{i_0}}}^{x_{q_{i_1}}}\int_{x_{p_{i_0}}}^{x_{q_{i_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}} \\
&\left.\frac{1}{r}dz_0dz_1dy_0dy_1dx_0dx_1\right].
\end{aligned} \tag{10}$$

Note that the six-fold integration in Eqn. (10) is simply Eqn. (8), the formula for the self-inductance of a rectangular conductor defined by its two diagonal corner points p_{i_0,j_0,k_0} and q_{i_1,j_1,k_1}, $i_0, i_1, j_0, j_1, k_0, k_1 \in \{0, 1\}$, as illustrated in Figure 2. The indices i_0, j_0, and k_0 of p_{i_0,j_0,k_0} identify a corner of the first wire (see Figure 1). Similarly, q_{i_1,j_1,k_1} is one of the eight corner points of the second wire. Altogether, the corner points of the first wire and second wire defines 64 virtual wires, each defined by a corner point from the first wire and a corner point from the second wire.

Let $L_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}$ refer to the self-inductance of a rectangular conductor with two points p_{i_0,j_0,k_0} and q_{i_1,j_1,k_1} on the diagonal ends, and $A_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}$ denote the cross-sectional area of the conductor. Substituting Eqn. (8) into Eqn. (10) yields

$$\begin{aligned}
&M \\
=\;& \frac{1}{W_0T_0W_1T_1}\frac{1}{8}\sum_{i_0,i_1,j_0,j_1,k_0,k_1=0}^{1}(-1)^{i_0+i_1+j_0+j_1+k_0+k_1+1} \\
&A^2_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}L_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}.
\end{aligned} \tag{11}$$

In other words, the mutual inductance of two parallel wires is a weighted sum of the self-inductances of the 64 virtual wires defined by the two wires, the weight of each self-inductance being $+A^2$ or $-A^2$. In some cases, p_{i_0,j_0,k_0} and q_{i_1,j_1,k_1} may share one or more coordinate values, resulting in one or more dimensions in the defined virtual conductor being zero. The self-inductance of such a virtual conductor is infinite. However, the cross-sectional area A of such a virtual conductor is zero. As A^2 of such a virtual conductor approaches zero faster than the inductance approaches infinity, the multiplication in Eqn. (11) is zero. Therefore, the equation is still valid in this special case. In fact, this equation is valid for any two parallel conductors that have rectangular cross sections.

The remaining issue is the computation of the self-inductances. In [2, 6, 8], closed form formulas for the self-inductance of a rectangular conductor are derived. Although the formulas are symbolically equivalent, the closed-form formulas from [6, 8] are numerically more stable. The closed-form formula for the self-inductance of a rectangular conductor of length l, thickness T, and width W is as follows [8]:

$$\begin{aligned}\frac{L}{l} = {} & \frac{2\mu}{\pi}\Big(\frac{1}{4}\Big[\frac{1}{w}S\Big(\frac{w}{\alpha_t}\Big)+\frac{1}{t}S\Big(\frac{t}{\alpha_w}\Big)+S\Big(\frac{1}{r}\Big)\Big] \\ & +\frac{1}{24}\Big[\frac{t^2}{w}S\Big(\frac{w}{t\alpha_t(r+\alpha_r)}\Big)+\frac{w^2}{t}S\Big(\frac{t}{w\alpha_w(r+\alpha_r)}\Big) \\ & +\frac{t^2}{w^2}S\Big(\frac{w^2}{tr(\alpha_t+\alpha_r)}\Big)+\frac{w^2}{t^2}S\Big(\frac{t^2}{wr(\alpha_w+\alpha_r)}\Big) \\ & +\frac{1}{wt^2}S\Big(\frac{wt^2}{\alpha_t(\alpha_w+\alpha_r)}\Big)+\frac{1}{tw^2}S\Big(\frac{tw^2}{\alpha_w(\alpha_t+\alpha_r)}\Big)\Big] \\ & -\frac{1}{6}\Big[\frac{1}{wt}T\Big(\frac{wt}{\alpha_r}\Big)+\frac{t}{w}T\Big(\frac{w}{t\alpha_r}\Big)+\frac{w}{t}T\Big(\frac{t}{w\alpha_r}\Big)\Big] \\ & -\frac{1}{60}\Big[\frac{(\alpha_r+r+t+\alpha_t)t^2}{(\alpha_r+r)(r+t)(t+\alpha_t)(\alpha_t+\alpha_r)}+ \\ & \frac{(\alpha_r+r+w+\alpha_w)w^2}{(\alpha_r+r)(r+w)(w+\alpha_w)(\alpha_w+\alpha_r)}+ \\ & \frac{(\alpha_r+\alpha_w+1+\alpha_t)}{(\alpha_r+\alpha_w)(\alpha_w+1)(\alpha_t+1)(\alpha_t+\alpha_r)}\Big] \\ & -\frac{1}{20}\Big[\frac{1}{r+\alpha_r}+\frac{1}{\alpha_w+\alpha_r}+\frac{1}{\alpha_t+\alpha_r}\Big]\Big) \qquad (12)\end{aligned}$$

where $w = W/l$, $t = T/l$, $r = \sqrt{w^2+t^2}$, $\alpha_w = \sqrt{w^2+1}$, $\alpha_t = \sqrt{t^2+1}$, $\alpha_r = \sqrt{w^2+t^2+1}$, $S(x) = \sinh^{-1}(x) = \ln(x+\sqrt{1+x^2})$, $T(x) = \tan^{-1}(x)$. In this work, we compute self-inductances using Eqn. (12).

4 Special Cases

Eqn. (11) is valid for any two conductors that are rectangular and parallel. For the general case, we need to calculate 64 self-inductances to obtain the mutual inductance between the two conductors. For special cases (such as aligned interconnects in the same layer), the computation complexity becomes much less. In Figure 3, for example, we consider two identical conductors that are parallel and properly aligned. The width of the wires is w and the spacing between the two wires is s. In such a case, only three distinct integrations (out of the 64 in Eqn. (11)) remains after eliminating those that are equal to zero. The formula for mutual inductance can be simplified as

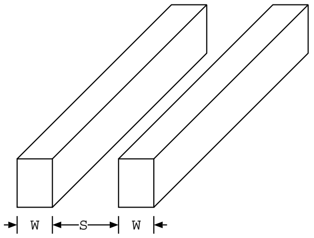


Figure 3. Two parallel conductors that are aligned.

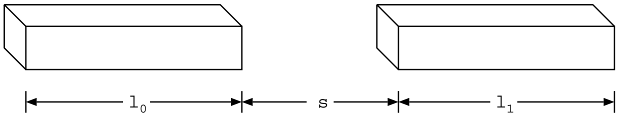


Figure 4. Two coaxial conductors.

$$M = \frac{1}{2w^2}[(2w+s)^2L_{2w+s} + s^2L_s - 2(w+s)^2L_{w+s}], \qquad (13)$$

where L_W represents the self-inductance of a rectangular conductor with width W.

Consider another special case where two conductors with identical cross section are coaxial, as shown in Figure 4. Let the lengths of the two conductors be l_0 and l_1. The distance between the two closest end points of the conductors is s. The exact formula for the mutual inductance between the two conductors can be simplified as

$$M = \frac{1}{2}[L_{l_0+s+l_1} - L_{l_0+s} - L_{l_1+s} + L_s], \qquad (14)$$

where L_l is the self-inductance of a conductor with length l.

Now, consider the special case where the two conductors are identical and they coincide with each other. In this case, only eight of the 64 integrations deal with virtual conductors with non-zero cross-sectional areas and lengths. Moreover, these virtual conductors are identical to the conductor of interest. Therefore, the mutual inductance between the two conductors is

$$M = \frac{1}{8WT}(8AL) = L. \qquad (15)$$

That coincides with the definition of self-inductance of the conductor.

5 Skin Effect and Other Considerations

In the preceding derivations, we assume that the current distribution is uniform in the cross section of the conductor. In other words, we ignore the skin effect. Theoretically, the current in the a conductor is not uniformly distributed due to skin effect. However, for relatively low frequency, it is reasonable to ignore the skin effect on current distribution and assume that the current distribution is purely determined by the resistive effect and is thus uniform. For cases where the skin effect cannot be ignored, we can divide the cross section of a conductor into a mesh and then apply the formula to each pair of filaments in those meshes. The frequency-dependent inductances (and resistances) of conductors can then be calculated based on the resistances and partial inductances of the meshes based on a nodal analysis formulation. In such an approach, even the computation of the self-inductance of a conductor requires the mutual inductances of filaments in the mesh that discretizes the conductor.

It is worthy of note that the result of mutual inductance is a weighted sum of the self-inductance of 64 rectangular virtual wires. Some virtual wires may be too large to be realistic or so large we should consider skin effect. However, it is important to realize that they constitute only the intermediate results. The validity of the final result depends on the structures of the two real wires, not these virtual wires.

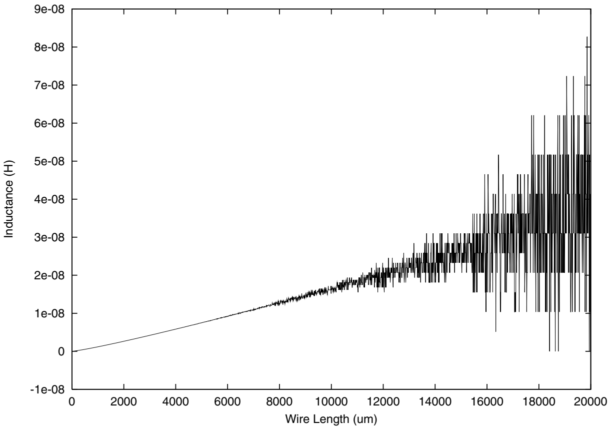
The formula we derived applies only to parallel rectangular conductors. If the structures of the wires are complex, the formula may not apply directly. However, if they can be decomposed into rectangular conductors that are parallel or orthogonal to each other, we can still apply this formula to each pair of conductors and obtain the total inductance value according to the PEEC model.

6 Numerical Results

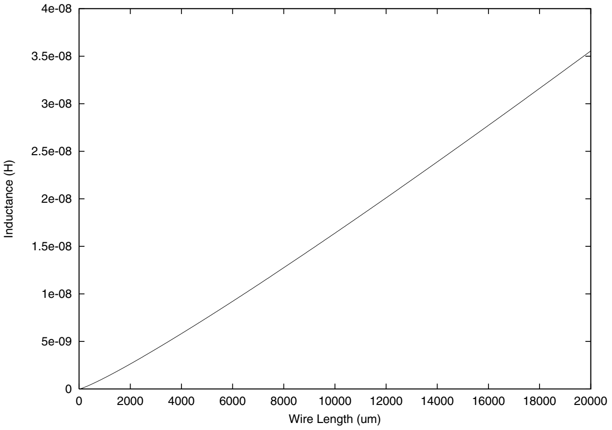
In this section, we compare the numerical results obtained by our formula with those obtained by the formula in [2] and FastHenry [3]. Eqn. (3) is also a result of 64 expressions. However, the function $f(X, Y, Z)$ (see [2] for details) in this expression is different from the product of the square of cross-sectional area and self-inductance in our formula.

In Figure 5, we obtain the mutual inductance between two wires with cross-sectional dimensions of $0.5\mu\text{m} \times 1\mu\text{m}$. They are 1.5μm apart. The plots in Figure 5(a), Figure 5(b) and Figure 5(c) are respectively obtained with the formula from [2], our formula and FastHenry. It is evident that the formula from [2], and FastHenry are numerically less stable than our formula; the mutual inductance should increase smoothly as wire length increases.

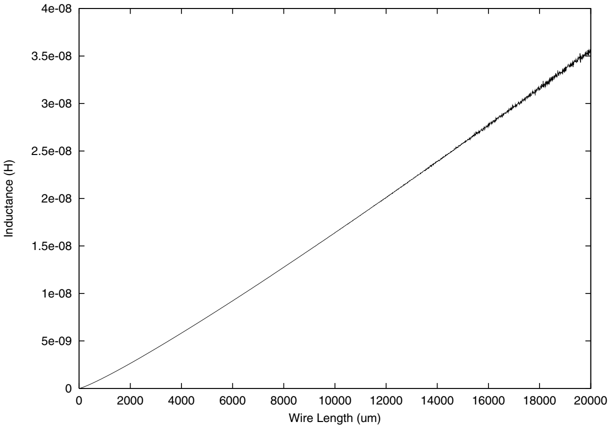
FastHenry uses the formula from [2] not only in the mutual inductance calculation, but also in the self induc-



(a) Formula from [2]



(b) Our formula



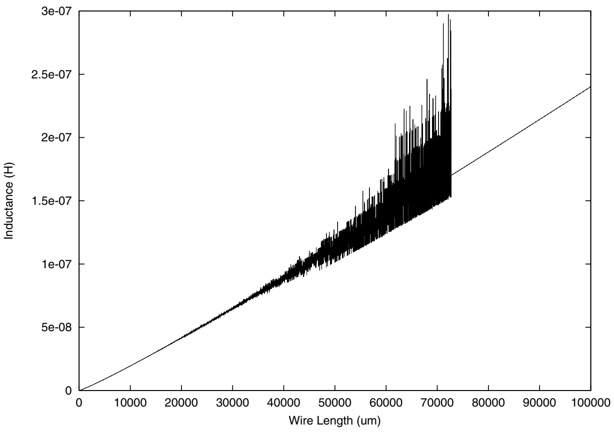
(c) FastHenry [3]

Figure 5. Mutual inductance extracted with (a) the formula from [2], (b) our formula, and (c) FastHenry [3] for two wires with a fixed spacing of 1.5μm at varying lengths.

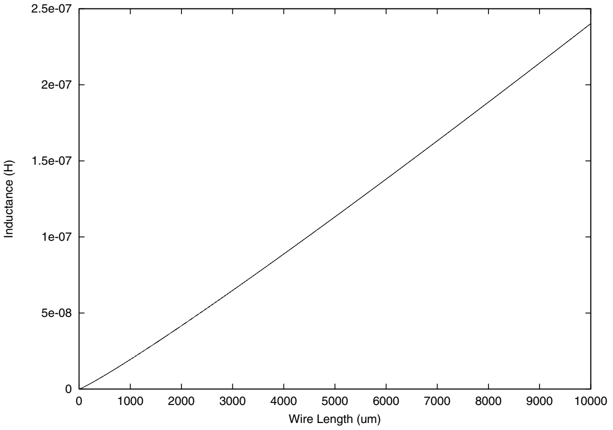
tance. Figure 6(a) plots the self-inductance values obtained by FastHenry [3] for a rectangular bar with a cross section of 1μm$\times 1\mu$m at different lengths. We expect the self-inductance to increase smoothly as the wire length increases. However, Figure 6(a) does not show such a trend. The behavior of FastHenry can be explained as follows: When calculating the self-inductance, FastHenry first divides a wire into several segments along the length, then obtains the self-inductance of the wire by summing up the self-inductances of the segments and the mutual inductances between the segments. When the wire length is small, the formula from [2] is sufficiently accurate to get the mutual inductances between the segments and the plot shows a smooth increase in self-inductance when the wire length increases. However, as the wire length becomes longer, numerical errors cause wide-range fluctuations in the inductance values. When the wire length is sufficiently long, FastHenry uses some other approximate methods, which are numerically stable, to calculate the mutual inductance. As a result, the inductance plot becomes smooth again. When we replace the mutual inductance formula from [2] with our formula, the results of FastHenry improve significantly (Figure 6(b)).

7 Conclusion

In this paper, we proposed a new closed form formula for on-chip mutual inductance. It is an exact formula that is practical and convenient for on-chip inductance extraction. Most important, it is numerically stable for long parallel on-chip interconnects.



(a) FastHenry [3]



(b) Modified FastHenry

Figure 6. Self-inductance extracted with (a) FastHenry (DC analysis) [3] and (b) modified FastHenry for a wire at varying lengths.

References

[1] F. Grover. *Inductance Calculations: Working formulas and Tables*. Dover, New York, 1962.

[2] C. Hoer and C. Love. Exact inductance equations for rectangular conductors with applications to more complicated geometries. *Journal of Research of the National Bureau of Standards*, 69C(2):127–137, April-June 1965.

[3] M. Kamon, M. J. Tsuk, and J. K. White. FASTHENRY: A multipole-accelerated 3-D inductance extraction program. *IEEE Journal on Microwave Theory and Techniques*, 42(9):1750–1758, September 1994.

[4] X. Qi, G. Wang, Z. Yu, R. W. Dutton, T. Yong, and N. Chang. On-chip inductance modeling and RLC extraction of VLSI interconnects for circuit simulation. In *IEEE custom integrated circuits conference*, pages 487–490, 2000.

[5] A. J. Rainal. Computing inductive noise of chip packages. *ATT Bell Lab. Tech. J.*, 63(1):177–195, January 1984.

[6] A. E. Ruehli. Inductance calculation in a complex integrated circuit environment. *IBM Journal of Research and Development*, pages 470–481, September 1972.

[7] K. L. Shepard and Z. Tian. Return-limited inductances:a practical approach to on-chip inductance extraction. *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, 19:425–436, April 2000.

[8] Ruey-Beei Wu, Chien-Nan Kuo, and Kwei K. Chang. Inductance and resistance computations for three-dimensional multiconductor interconnect structures. *IEEE Trans. on Microwave Theory and Techniques*, 40(2):263–270, February 1992.