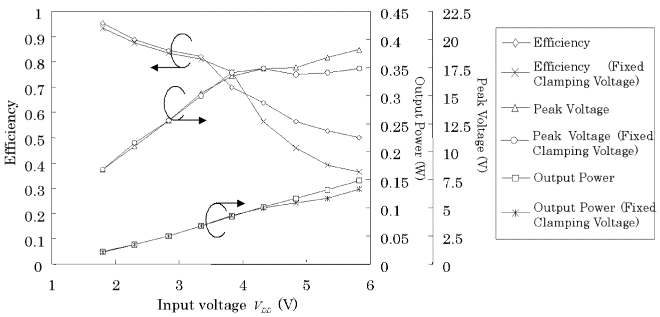

Fig. 4. Efficiency, peak switch voltage, and output power versus $V_{\rm DD}$.

cuit was $\eta = 1.33/(1.33 + 0.37) = 78\%$ and the efficiency of the fixed clamping voltage circuit was $\eta = 1.33/(1.33 + 0.56) = 70\%$. In other words, the power loss was reduced from 30% to 22%. (It is not reduced to 20% because the denominator is different from that for the fixed clamping voltage circuit.)

Plots of the measured peak switch voltage, output power, and efficiency versus dc input voltage $V_{\rm DD}$ are shown in Fig. 4. When $V_{\rm DD}$ was equal to 4 V, the switch peak voltage reached the diode breakdown voltage. The rate of increase of the peak switch voltage decreased for $V_{\rm DD} > 4$ V. In the fixed clamping voltage circuit, the peak voltage did not increase above the breakdown voltage of the Zener diode. In the proposed circuit, the Zener diode is connected between $V_{\rm DD}$ and the switch, and therefore, the switch voltage can increase above the designed maximum voltage. For $V_{\rm DD} > 4$ V, the efficiency dropped due to the power loss in the Zener diode. The efficiency of the fixed clamping voltage circuit was lower than that of the proposed circuit. For example, the efficiency of the fixed clamping voltage circuit was 40% and 52% for the proposed circuit at $V_{\rm DD} = 5$ V. In this case, the power loss was reduced from 60% to 48%. The power loss consisted of the Zener diode loss and power loss in the remaining class-E amplifier. The power loss in the Zener diode was $0.36P_o$ for the fixed clamping voltage circuit and $0.24P_o$ in the Zener diode for the proposed circuit. The power loss in the remaining class-E amplifier was calculated to be $0.24P_o$ for both circuits. Hence, the efficiency of the remaining class-E amplifier was calculated $1/(1+0.24) = 80\%$. Even if the peak switch voltage was clamped for $V_{\rm DD} > 4$ V, the output power still increased. The output power was almost the same in both circuits.

IV. Conclusion

In this brief, the class-E amplifier that clamps peak switch voltage by a Zener diode across the choke was proposed, analyzed, and tested. In this circuit, the peak voltage can be reduced to a required value and the output power is increased, but a power loss occurs in the Zener diode. Compared to the voltage-clamped class-E amplifier with a Zener diode across the switch, the power loss in the Zener diode is reduced by factor of $(k-1)/k$. The peak switch current and voltage as well as the output power are identical in both circuits.

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Exact Closed-Form Formula for Partial Mutual Inductances of Rectangular Conductors

Guoan Zhong and Cheng-Kok Koh

***Abstract*—In this brief, we propose a new exact closed-form mutual inductance equation for rectangular conductors. We express the mutual inductance between two parallel rectangular conductors as a weighted sum of self inductances. We do not place any restrictions on the alignment of the two parallel rectangular conductors. Moreover, they could be coplanar or reside on different layers. Most important, experimental results show that our formula is numerically more stable than that derived by Hoer and Love.**

***Index Terms*—Inductance extraction, interconnect, partial element equivalent circuit (PEEC).**

I. Introduction

In modern very large-scale integration (VLSI) design, it is prudent to consider the parasitic inductance in the on-chip global interconnects and the packages for the timing and noise analysis. The concept of inductance is defined based on the magnetic fields caused by currents flowing through closed conductor loops. For general three-dimensional interconnects, however, the return paths of currents are distributed and not known *a priori*. An approach that obviates the need for prior knowledge of return paths in circuit simulation is the use of the partial element equivalent circuit (PEEC) model [2]. In this model, partial inductances are defined to represent the loop interactions among conductors, each forming its own return loop with infinity. In the following discourse, we use mutual inductance to refer strictly to partial mutual inductance and self inductance to refer strictly to partial self inductance.

In [1], the authors derived a closed-form formula for the mutual inductance of any pair of parallel rectangular conductors even if they

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are not aligned. Unfortunately, the computation of this formula is numerically unstable (see Section IV for the numerical results). Consider the special case when we calculate the mutual inductance between two identical conductors that coincide with each other, we obtain the self inductance. The closed-form formulas for self inductance in [1]–[3] are derived in this fashion.

In this brief, we derive the exact closed-form formula for the mutual inductance of two parallel conductors. We reveal the *inverse* relation between mutual inductance and self inductance, that is, the mutual inductance can be expressed in terms of self inductance. To be more specific, the mutual inductance of two conductors is a weighted sum of self inductances. As with the formula in [1], we do not impose any restrictions on the alignment of the two parallel rectangular conductors. Moreover, the formula applies to coplanar wires or wires residing on different layers. Experimental results in Section IV show that our formula is numerically more stable than the formula in [1].

II. Preliminaries

The mutual inductance between two conductors with uniform cross sections is

$$M = \frac{1}{I_0 I_1}\int_{A_0}\int_{A_1} M_{01} J_0 J_1 dA_0 dA_1 \tag{1}$$

where A_0 and A_1 are the cross-sectional areas of the two conductors. I_0, I_1, J_0, and J_1 are the current and the current densities of the conductors. M_{01} is the mutual inductance between two filaments dA_0 and dA_1, and the current is assumed to be constant along the the length of each filament.

At relatively low frequency, the current distribution varies very little in the cross sections and can be assumed to be constant throughout the conductors. Hence, the mutual inductance can be reduced to

$$M = \frac{1}{A_0 A_1}\int_{A_0}\int_{A_1} M_{01}\, dA_0\, dA_1. \tag{2}$$

As indicated by the preceding equation, the mutual inductance is determined only by the geometries of the two conductors. If there are more than two conductors, the mutual inductances among them can be computed pairwisely [2], [3].

Under magneto-quasi-static condition, the mutual inductance between two filaments L_0 and L_1 can be calculated by Neumann's formula

$$M_f = \frac{\mu}{4\pi}\int_{L_0}\int_{L_1}\frac{dl_0\cdot dl_1}{r} \tag{3}$$

where r is the distance between dl_0 and dl_1 and μ is the permeability.

Consider two parallel rectangular wires as illustrated in Fig. 1. Here, we assume that the current flows in the z direction. As can be seen in the figure, the displacements of the two wires along the x and y directions are nonzero. Let the cross-sectional dimensions (in the x–y plane) of the two wires be $T_0 \times W_0$ and $T_1 \times W_1$. We use $p_{i,j,k}$ and $q_{i,j,k}, i, j, k \in \{0,1\}$, to denote the corners of the two wires, as illustrated in Fig. 1. All corner points of the first wire has two z coordinate values. We use $z_{p_k}, k \in \{0,1\}$, to denote the z coordinate value shared by the corners $p_{*,*,k}$. Similarly, we use $z_{q_k}, k \in \{0,1\}$, to denote the z coordinate value shared by the corners $q_{*,*,k}$ of the second wire. Similarly defined are the x-coordinate and y-coordinate values of the corners of the two wires: x_{p_i} and $x_{q_i}, i \in \{0,1\}$, and y_{p_j} and $y_{q_j}, j \in \{0,1\}$.

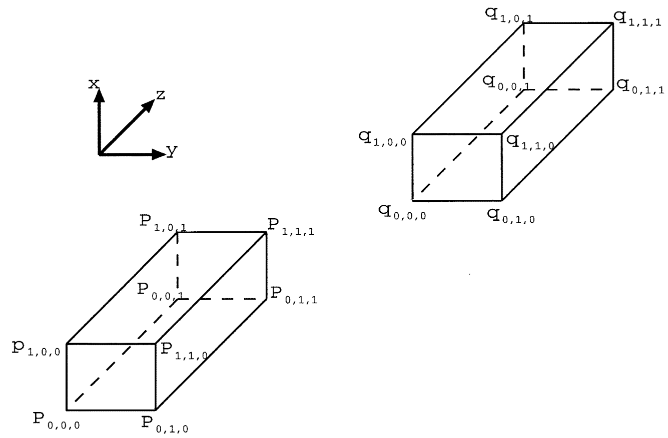


Fig. 1. Two parallel wires.

Now, substituting (3) into (2), we obtain

$$\begin{aligned} M &= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} M_f dy_0 dy_1 dx_0 dx_1 \\ &= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} \left[\frac{\mu}{4\pi}\right. \\ &\quad \times \left.\int_{z_{p_0}}^{z_{p_1}}\int_{z_{q_0}}^{z_{q_1}}\frac{1}{r}\,dz_0\,dz_1\right] dy_0\,dy_1\,dx_0\,dx_1. \end{aligned} \tag{4}$$

If the two conductors coincide with each other, the preceding mutual inductance equation gives the equation for the self inductance of one conductor

$$L = \frac{1}{A^2}\frac{\mu}{4\pi}\int\int\int\int\int\int\frac{1}{r}\,dz_0\,dz_1\,dy_0\,dy_1\,dx_0\,dx_1. \tag{5}$$

III. Formula for Mutual Inductance

In the following, we reveal the relation between mutual inductance and self inductance and then derive a closed-form formula for the mutual inductance as a weighted sum of self inductances.

It is trivial to show that for any function $f(x)$

$$\begin{aligned} &\int_{p_0}^{p_1}\int_{q_0}^{q_1} f(|x_0-x_1|)\,dx_0\,dx_1 \\ &\quad= \frac{1}{2}\left(\int_{p_0}^{q_1}\int_{p_0}^{q_1} f(|x_0-x_1|)\,dx_0\,dx_1\right. \\ &\qquad+ \int_{p_1}^{q_0}\int_{p_1}^{q_0} f(|x_0-x_1|)\,dx_0\,dx_1 \\ &\qquad- \int_{p_0}^{q_0}\int_{p_0}^{q_0} f(|x_0-x_1|)\,dx_0\,dx_1 \\ &\qquad- \left.\int_{p_1}^{q_1}\int_{p_1}^{q_1} f(|x_0-x_1|)\,dx_0\,dx_1\right) \\ &\quad= \frac{1}{2}\sum_{i,j=0}^{1}(-1)^{i+j+1}\int_{p_i}^{q_j}\int_{p_i}^{q_j} f(|x_0-x_1|)\,dx_0\,dx_1. \end{aligned} \tag{6}$$

Making use of (6), we can rewrite (4) as shown in (7), shown at the bottom of the next page.

Note that the six-fold integration in (7) is very similar to (5). The only difference is that integration in (5) is scaled by $1/A^2$, where A is the cross-sectional area. If scaled by $1/A^2$, the six-fold integration in (7) becomes the self inductance of a rectangular

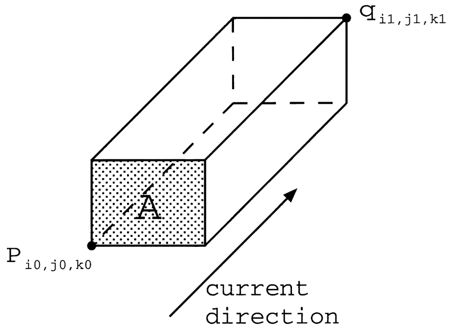


Fig. 2. Virtual conductor defined by two corner points.

conductor defined by its two diagonal corner points p_{i_0,j_0,k_0} and $q_{i_1,j_1,k_1}, i_0, i_1, j_0, j_1, k_0, k_1 \in \{0,1\}$, as illustrated in Fig. 2. The indices i_0, j_0, and k_0 of p_{i_0,j_0,k_0} identify a corner of the first wire (see Fig. 1). Similarly, q_{i_1,j_1,k_1} is one of the eight corner points of the second wire. Altogether, the corner points of the first wire and second wire defines 64 virtual wires, each defined by a corner point from the first wire and a corner point from the second wire.

Let $L_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}$ refer to the self inductance of a rectangular conductor with two points p_{i_0,j_0,k_0} and q_{i_1,j_1,k_1} on the diagonal ends, and $A_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}$ denote the cross-sectional area of the conductor. Substituting (5) into (7) yields

$$M = \frac{1}{W_0T_0W_1T_1}\frac{1}{8}\sum_{i_0,i_1,j_0,j_1,k_0,k_1=0}^{1}(-1)^{i_0+i_1+j_0+j_1+k_0+k_1+1} \times A^2_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}} L_{p_{i_0,j_0,k_0},q_{i_1,j_1,k_1}}. \tag{8}$$

In other words, the mutual inductance of two parallel wires is a weighted sum of the self inductances of the 64 virtual wires defined by the two wires, the weight of each self inductance being $+A^2$ or $-A^2$. In some cases, p_{i_0,j_0,k_0} and q_{i_1,j_1,k_1} may share one or more coordinate values, resulting in one or more dimensions in the defined virtual conductor being zero. If the z dimension is zero, the self inductance of such a virtual conductor is zero. If x or y dimension is zero, the self inductance is infinite. However, the cross-sectional area A of such a virtual conductor is zero. As A^2 of such a virtual conductor approaches zero faster than the inductance approaches infinity, the multiplication in (8) is zero. Therefore, the equation is still valid in this special case. In fact, this equation is valid for any two parallel conductors that have rectangular cross sections.

The remaining issue is the computation of the self inductances. In [1]–[3], closed-form formulas for the self inductance of a rectangular conductor are derived. Although the formulas are symbolically equivalent, the closed-form formulas from [2], [3] are numerically more stable. The closed-form formula for the self inductance of a rectangular conductor of length l, thickness T, and width W is as follows [3]:

$$\begin{aligned}\frac{L}{l} = \frac{2\mu}{\pi}\Bigg(&\frac{1}{4}\left[\frac{1}{w}S\left(\frac{w}{\alpha_t}\right)+\frac{1}{t}S\left(\frac{t}{\alpha_w}\right)+S\left(\frac{1}{r}\right)\right]\\
&+\frac{1}{24}\left[\frac{t^2}{w}S\left(\frac{w}{t\alpha_t(r+\alpha_r)}\right)+\frac{w^2}{t}S\left(\frac{t}{w\alpha_w(r+\alpha_r)}\right)\right.\\
&+\frac{t^2}{w^2}S\left(\frac{w^2}{tr(\alpha_t+\alpha_r)}\right)+\frac{w^2}{t^2}S\left(\frac{t^2}{wr(\alpha_w+\alpha_r)}\right)\\
&\left.+\frac{1}{wt^2}S\left(\frac{wt^2}{\alpha_t(\alpha_w+\alpha_r)}\right)+\frac{1}{tw^2}S\left(\frac{tw^2}{\alpha_w(\alpha_t+\alpha_r)}\right)\right]\\
&-\frac{1}{6}\left[\frac{1}{wt}T\left(\frac{wt}{\alpha_r}\right)+\frac{t}{w}T\left(\frac{w}{t\alpha_r}\right)+\frac{w}{t}T\left(\frac{t}{w\alpha_r}\right)\right]\\
&-\frac{1}{60}\left[\frac{(\alpha_r+r+t+\alpha_t)t^2}{(\alpha_r+r)(r+t)(t+\alpha_t)(\alpha_t+\alpha_r)}\right.\\
&+\frac{(\alpha_r+r+w+\alpha_w)w^2}{(\alpha_r+r)(r+w)(w+\alpha_w)(\alpha_w+\alpha_r)}\\
&\left.+\frac{(\alpha_r+\alpha_w+1+\alpha_t)}{(\alpha_r+\alpha_w)(\alpha_w+1)(\alpha_t+1)(\alpha_t+\alpha_r)}\right]\\
&-\frac{1}{20}\left[\frac{1}{r+\alpha_r}+\frac{1}{\alpha_w+\alpha_r}+\frac{1}{\alpha_t+\alpha_r}\right]\Bigg)\end{aligned} \tag{9}$$

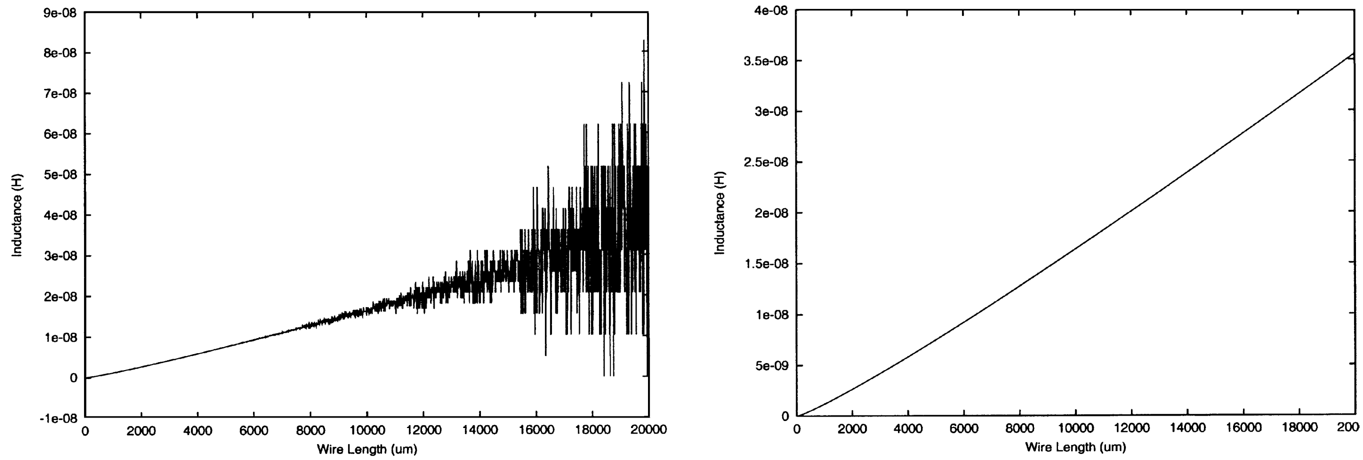
where $w = W/l, t = T/l, r = \sqrt{w^2+t^2}, \alpha_w = \sqrt{w^2+1}, \alpha_t = \sqrt{t^2+1}, \alpha_r = \sqrt{w^2+t^2+1}, S(x) = \sinh^{-1}(x) = \ln(x+\sqrt{1+x^2}), T(x) = \tan^{-1}(x)$. In this brief, we use (9) to compute the self inductance in (8).

IV. Numerical Results

In this section, we compare the numerical stability of the proposed formula and the formula from [1]. Proper numerical manipulation does improve the stability of these formulas, just as what FastHenry [4] does when it uses the mutual inductance formula in [1]. However, for a fair comparison, no such numerical techniques are applied to either of the formulas. Several cases are studied. In all cases, there are two parallel wires with the same dimensions. The cross-sectional dimensions of the wires are fixed at 0.5 μm × 1 μm.

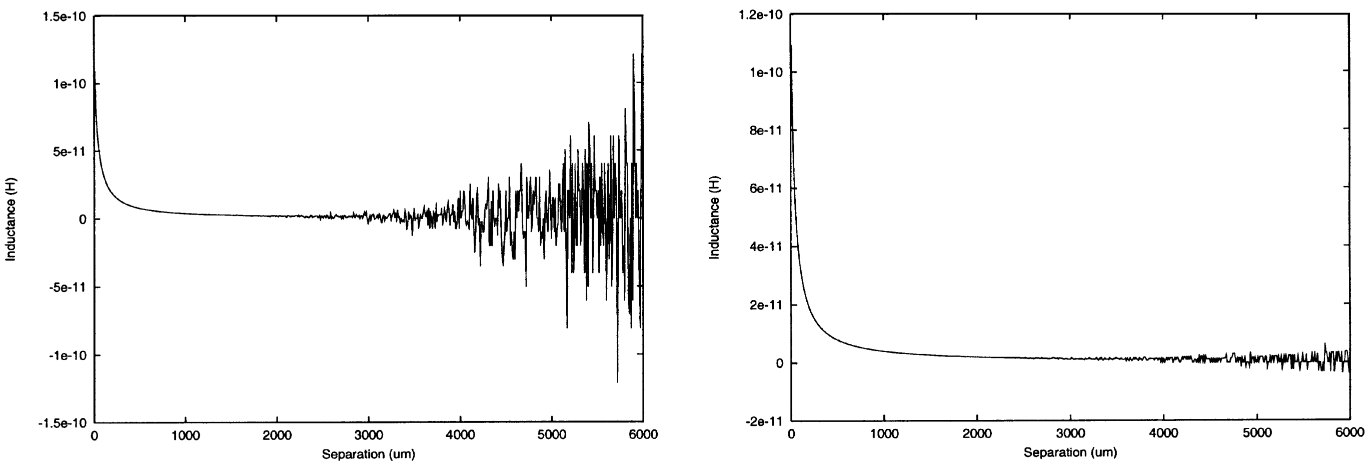
- The two wires are aligned and 1.5 μm apart. Mutual inductance values are extracted for varying lengths.

$$\begin{aligned}M &= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}} M_f\, dy_0 dy_1 dx_0 dx_1\\
&= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}}\left[\frac{\mu}{4\pi}\int_{z_{p_0}}^{z_{p_1}}\int_{z_{q_0}}^{z_{q_1}}\frac{1}{r}\,dz_0\,dz_1\right]dy_0\,dy_1\,dx_0\,dx_1\\
&= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\int_{y_{p_0}}^{y_{p_1}}\int_{y_{q_0}}^{y_{q_1}}\frac{1}{2}\sum_{k_0,k_1=0}^{1}(-1)^{k_0+k_1+1}\frac{\mu}{4\pi}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\frac{1}{r}\,dz_0\,dz_1\,dy_0\,dy_1\,dx_0\,dx_1\\
&= \frac{1}{W_0T_0W_1T_1}\int_{x_{p_0}}^{x_{p_1}}\int_{x_{q_0}}^{x_{q_1}}\frac{1}{4}\sum_{j_0,j_1,k_0,k_1=0}^{1}(-1)^{j_0+j_1+k_0+k_1}\frac{\mu}{4\pi}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\frac{1}{r}\,dz_0\,dz_1\,dy_0\,dy_1\,dx_0\,dx_1\\
&= \frac{1}{W_0T_0W_1T_1}\frac{1}{8}\sum_{i_0,i_1,j_0,j_1,k_0,k_1=0}^{1}(-1)^{i_0+i_1+j_0+j_1+k_0+k_1+1}\frac{\mu}{4\pi}\int_{x_{p_{i_0}}}^{x_{q_{i_1}}}\int_{x_{p_{i_0}}}^{x_{q_{i_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{y_{p_{j_0}}}^{y_{q_{j_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\int_{z_{p_{k_0}}}^{z_{q_{k_1}}}\frac{1}{r}\,dz_0\,dz_1\,dy_0\,dy_1\,dx_0\,dx_1\end{aligned} \tag{7}$$



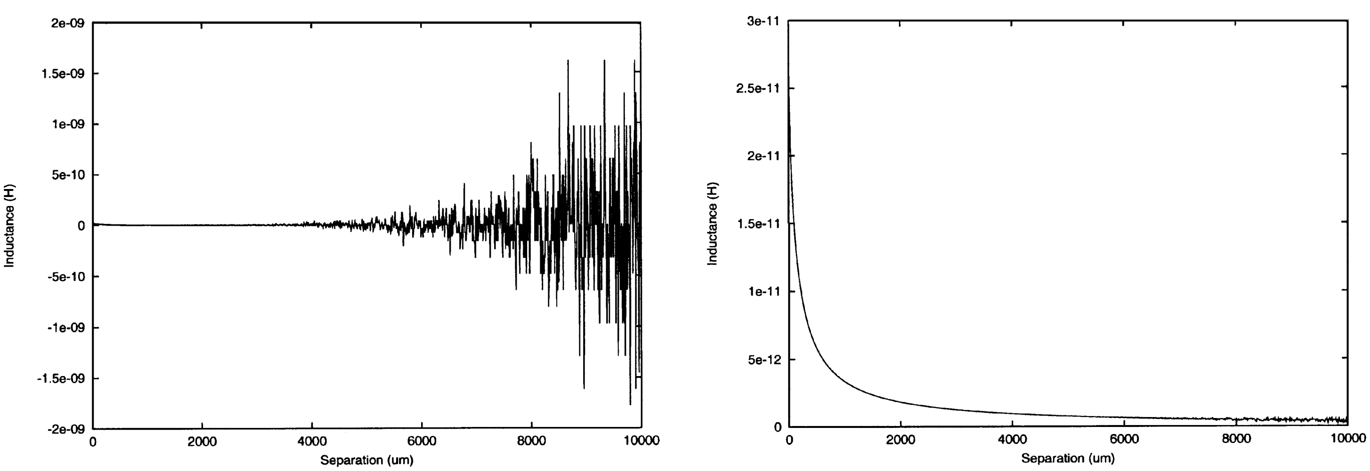
(a) (b)

Fig. 3. (a) Mutual inductance extracted with the formula from [1]. (b) Our formula for two wires with a fixed spacing of 1.5 μm but varying lengths.



(a) (b)

Fig. 4. (a) Mutual inductance extracted with the formula from [1]. (b) Our formula for two aligned wires varying spacing.

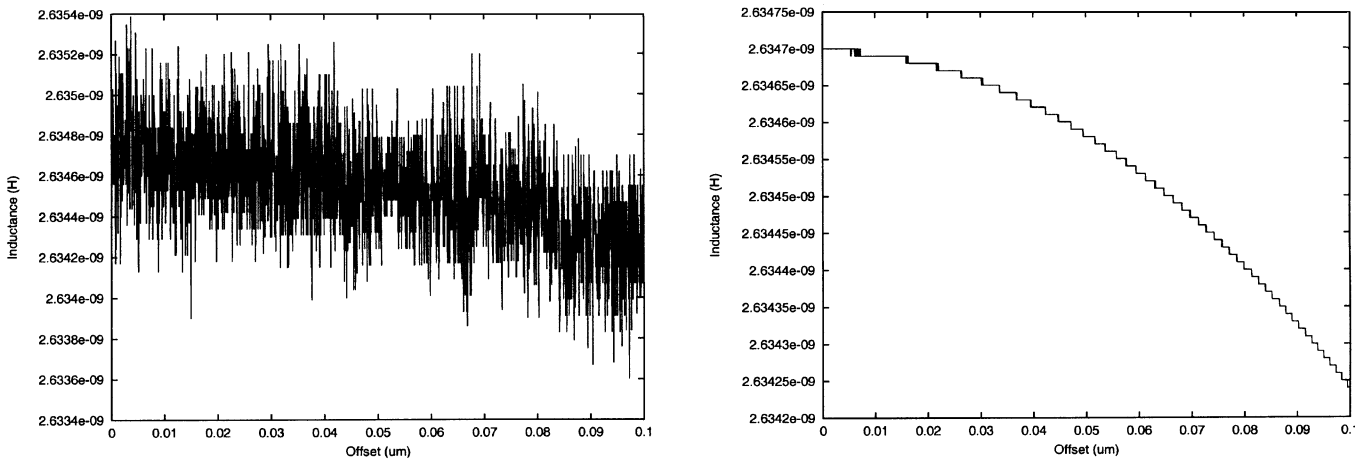


(a) (b)

Fig. 5. (a) Mutual inductance extracted with the formula from [1]. (b) Our formula for two coaxial wires varying separation.

- The two wires are aligned and their lengths are 200 μm. Mutual inductance values are extracted for varying lateral separations.
- The two wires are coaxial; their lengths are 200 μm. Mutual inductance values are extracted for varying longitudinal separations (the separation between two nearest endpoints).
- The two wires are aligned at two ends; their lengths are 2000 μm. The lateral separation is 1.5 μm. Mutual inductance values are extracted for varying vertical offset, which is the vertical distance between the tops of the wires.

The results are shown in Figs. 3–6, respectively. The results from [1] are shown at left and the results from our formula are at right. It is ev-



(a) (b)

Fig. 6. (a) Mutual inductance extracted with the formula from [1]. (b) Our formula for two wires with varying vertical offset.

ident that the formula from [1] is numerically less stable than our formula; the mutual inductance should change smoothly in all the cases. Although our formula also suffers from some numerical stability problems in some cases (such as Figs. 4–6), the numerical problems are minuscule compared to those in the counterparts. Double precision is used in the previous results. At long double precision, the results from both formulas will prove. However, our results will still be much better. The reason that our formula is more stable is that it expresses the mutual inductance in terms of self inductance and makes use of the stable self inductance formula.

V. Conclusion

In this brief, we proposed a new exact closed-form formula for the mutual inductance between parallel rectangular conductors. It reveals the inverse relation between mutual inductance and self inductance. Moreover, it is numerically more stable than the formula in [1].

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Global Exponential Stability of a General Class of Recurrent Neural Networks With Time-Varying Delays

Zhigang Zeng, Jun Wang, and Xiaoxin Liao

***Abstract*—This brief presents new theoretical results on the global exponential stability of neural networks with time-varying delays and Lipschitz continuous activation functions. These results include several sufficient conditions for the global exponential stability of general neural networks with time-varying delays and without monotone, bounded, or continuously differentiable activation function. In addition to providing new criteria for neural networks with time-varying delays, these stability conditions also improve upon the existing ones with constant time delays and without time delays. Furthermore, it is convenient to estimate the exponential convergence rates of the neural networks by using the results.**

***Index Terms*—Global exponential stability, neural networks, rate of exponential convergence, time-varying delays.**

I. Introduction

In recent years, recurrent neural networks are widely studied, because of their immense potentials of application prospective. The Hopfield neural networks and cellular neural networks are typical representative recurrent neural networks among others.

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