On Characterizing the Throughput Degradation for Inter-session Network Coding

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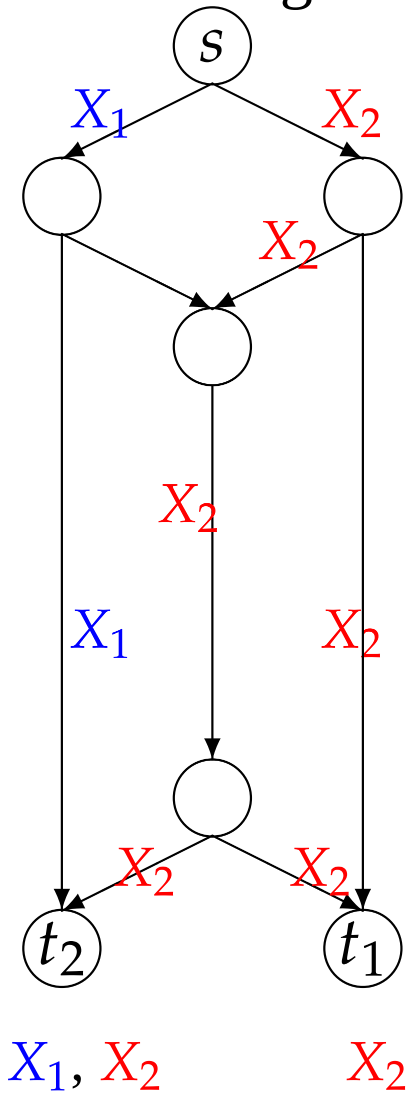
Single Session

The Routing Solution



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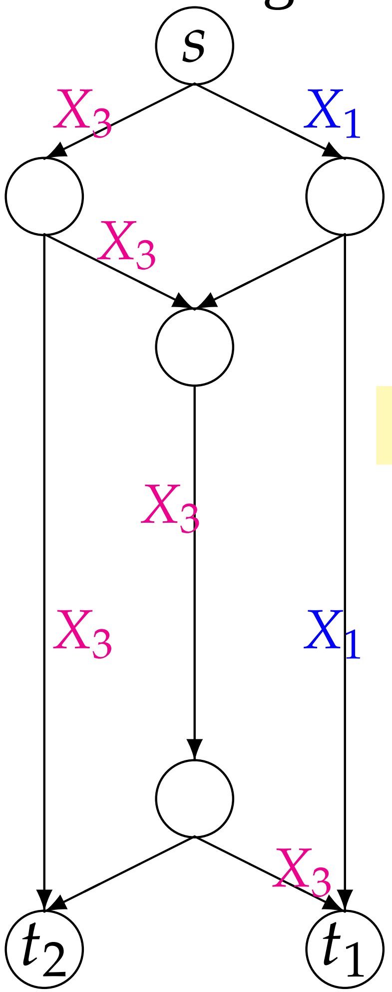

Rate: 3/2

X_1, X_2, X_3 X_2, X_1, X_3

Single Session

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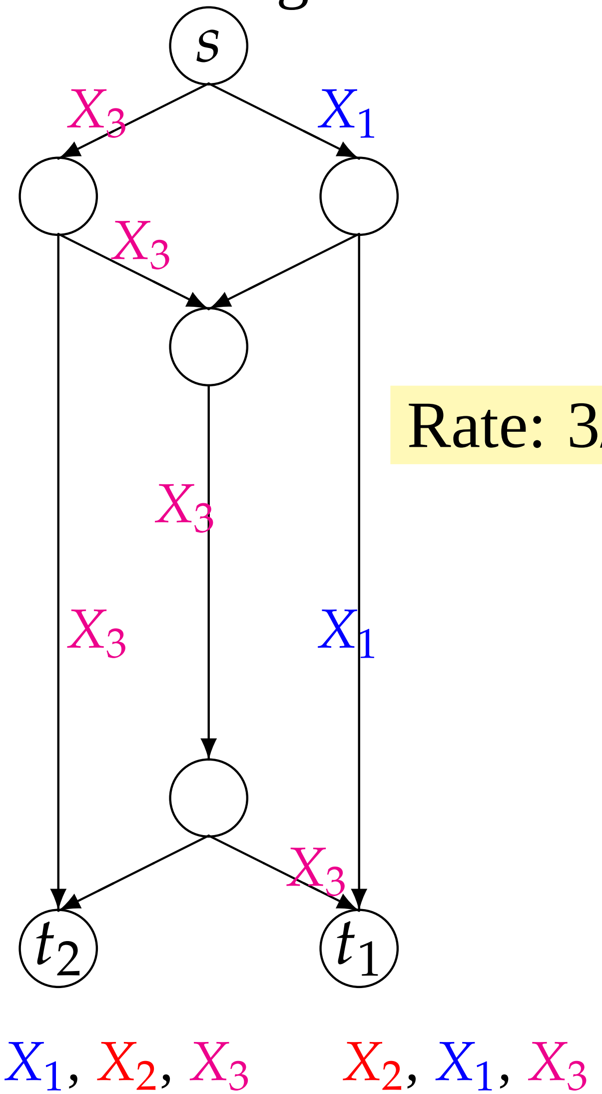
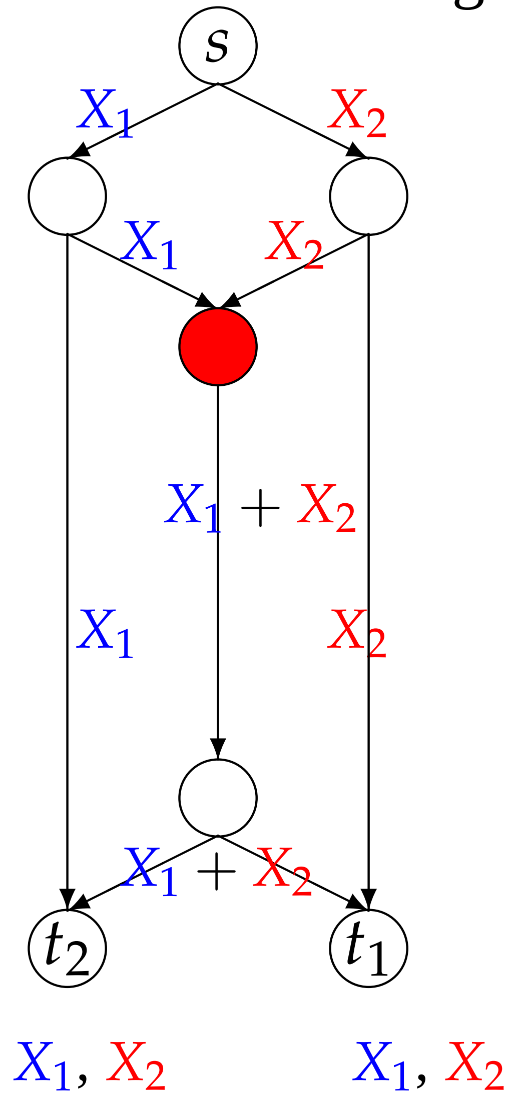


Rate: 3/2

The Network Coding Solution



Single Session

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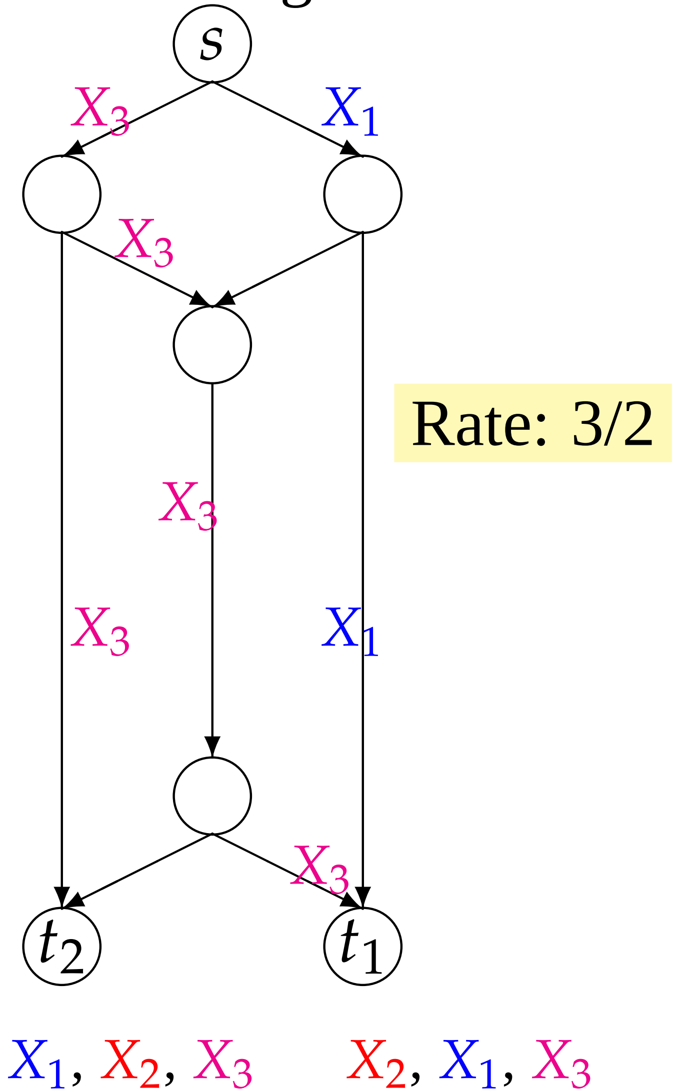

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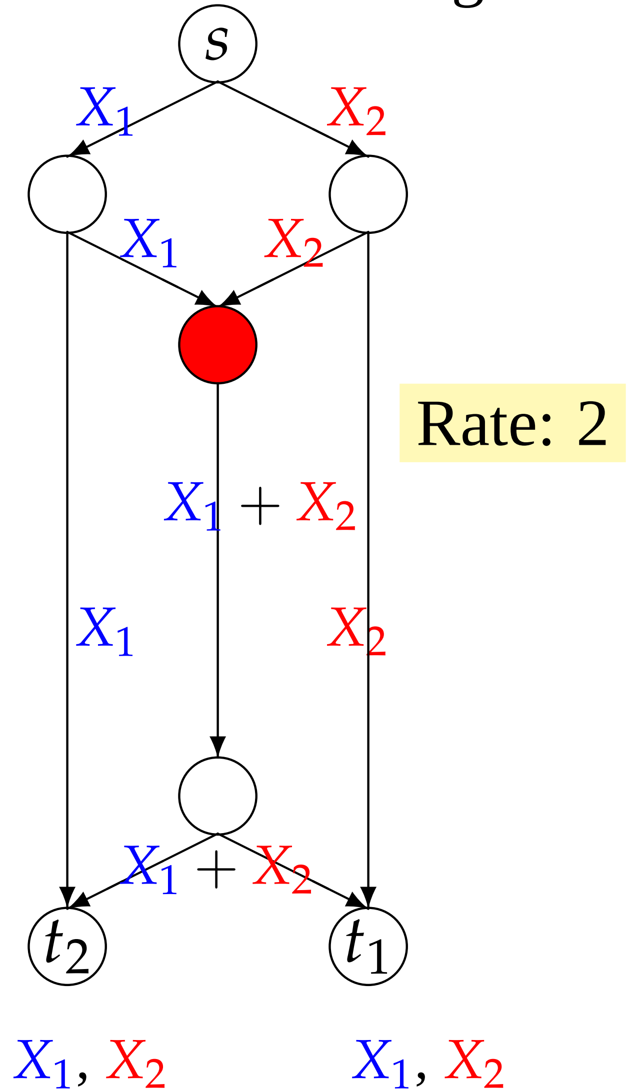


Achievable Rate Characterization

Theorem 1 *[Ahlswede et al. 00] For a single multicast session, rate r is achievable if for all dest. t_i, the min-cut/max-flow $\rho_G(s, t_i)$ between s and t_i satisfies*

$$r \leq \rho_G(s, t_i), \ \forall i.$$

Achievable Rate Characterization

Theorem 2 *[Ahlswede et al. 00] For a single multicast session, rate r is achievable if for all dest. t_i, the min-cut/max-flow $\rho_G(s, t_i)$ between s and t_i satisfies*

$$r \leq \rho_G(s, t_i), \ \forall i.$$

Utility optimization & cost minimization [Wu *et al.* 06]:

- Directed acyclic graph: $G = (V, E, \{c_e\}_{e \in E})$
-

$$\max_{r, \{c_e\}} \quad U(r) - \sum_{e \in E} p_e(c_e)$$

$$\text{subject to} \quad r \leq \rho_G(s, t_i), \ \forall i$$

$$0 \leq c_e \leq \mathsf{ub}_e, \ \forall e \in E.$$

Multiple Sessions

- Routing solution $\Longleftrightarrow$ Each session i takes an exclusive share of the network.

Multiple Sessions

- Routing solution $\Longleftrightarrow$ Each session i takes an exclusive share of the network.
- Intra-session network coding only [Chen *et al.* 07]

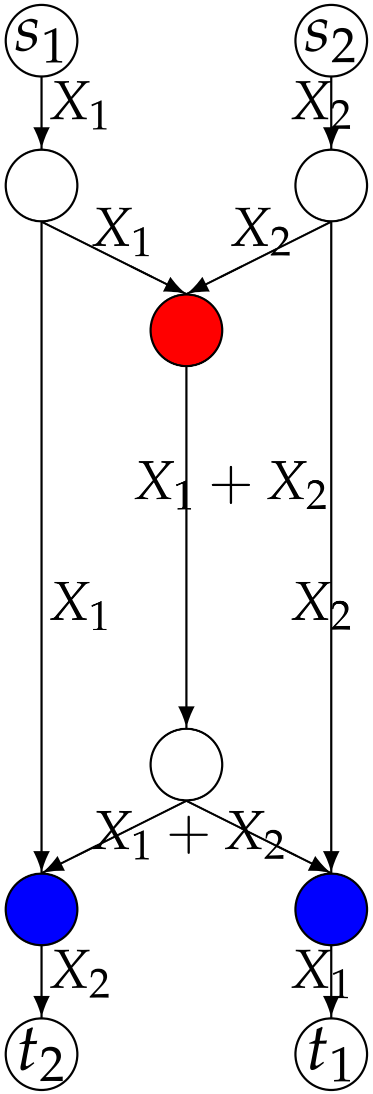
$$\max_{r_i} \quad \sum_i U(r_i)$$

$$\text{subject to} \quad \sum_i f_{i,e} \le c_e, \ \forall e \in E$$

$$\forall i, \ \{f_{i,e}\}_{e \in E} \text{ and } r_i \text{ satisfy the flow conditions.}$$

Multiple Sessions (Cont'd)

- Inter-session network coding: The benefit is apparent.



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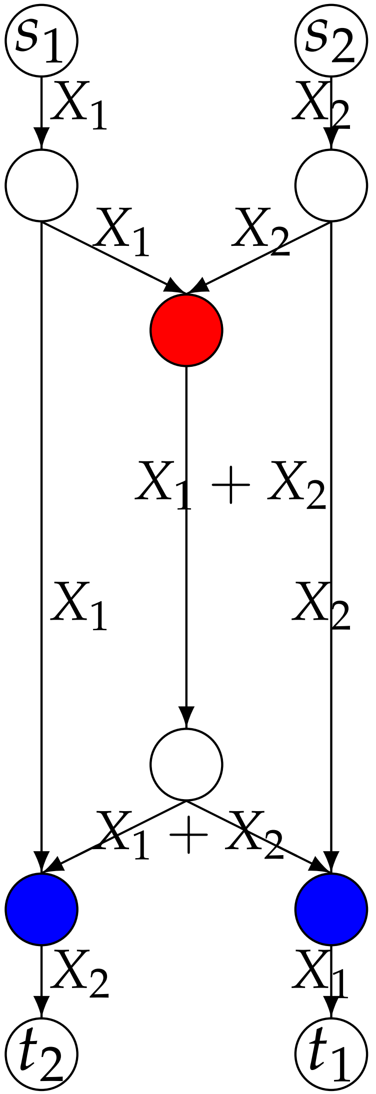

- Characterization??

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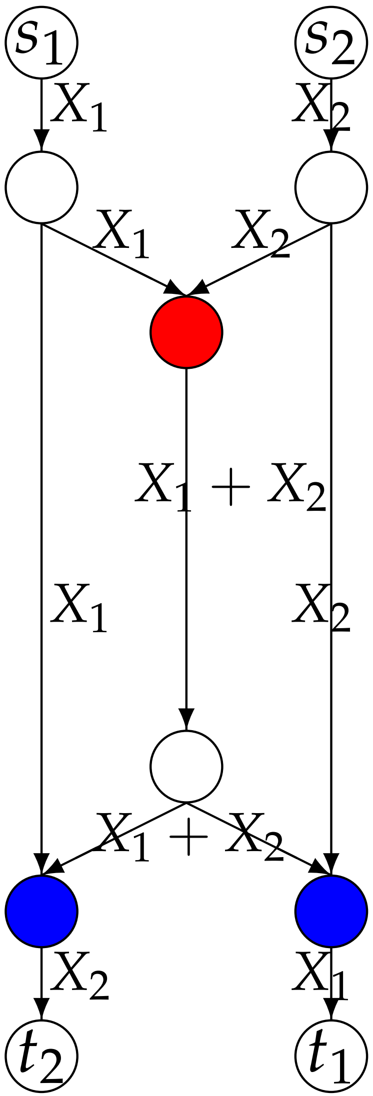

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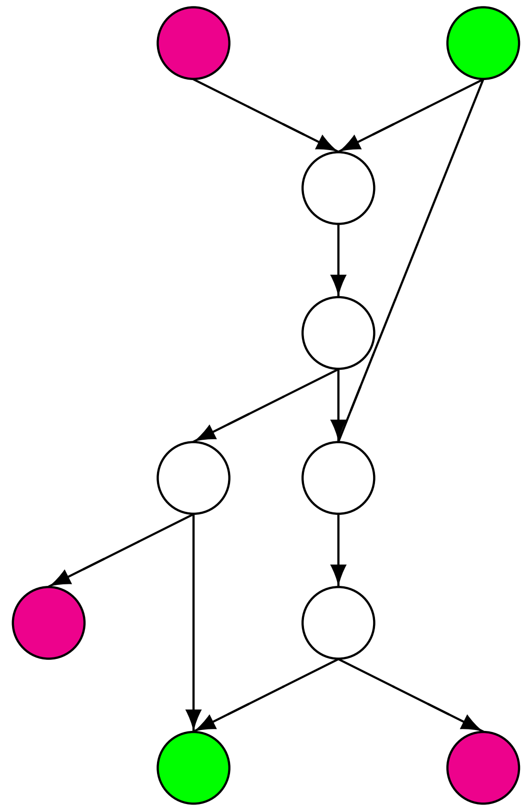

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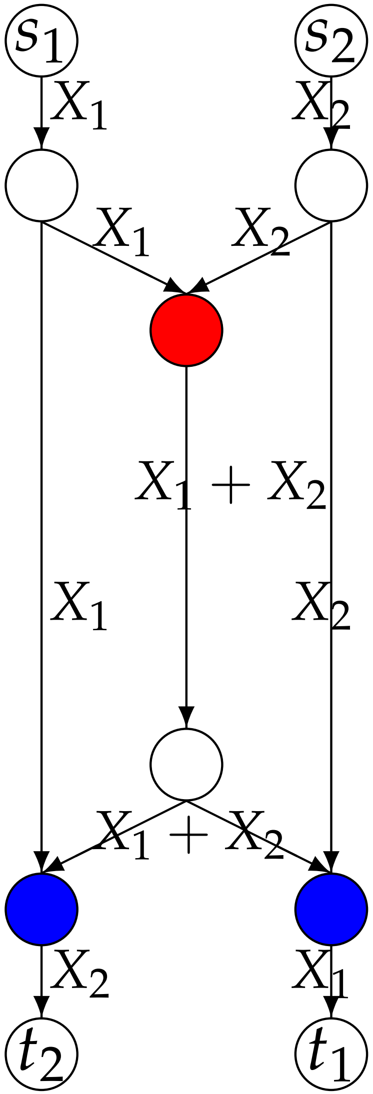
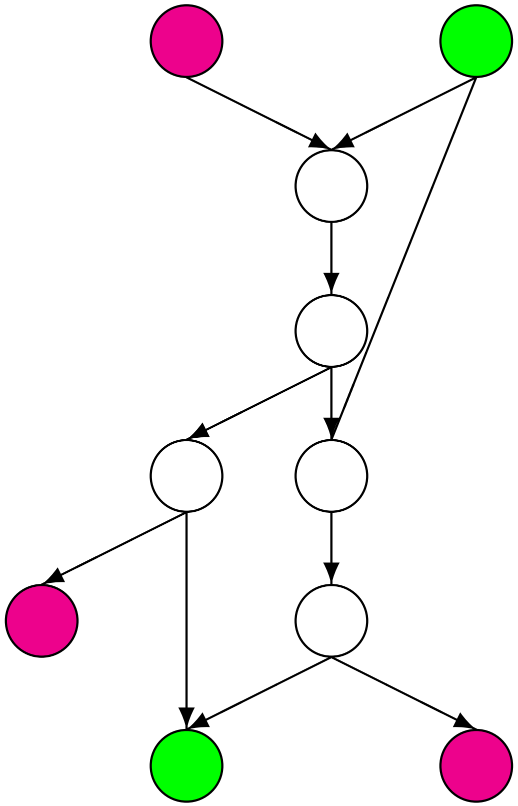


- Characterization??
- Utility Optimization?



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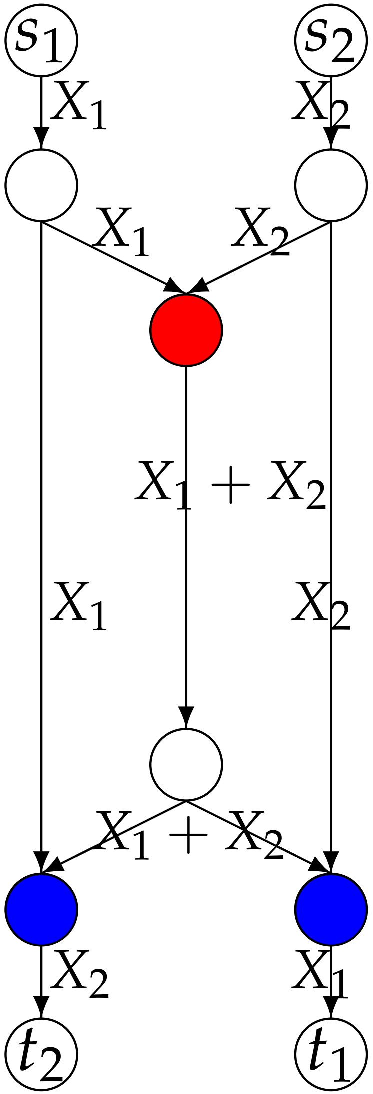
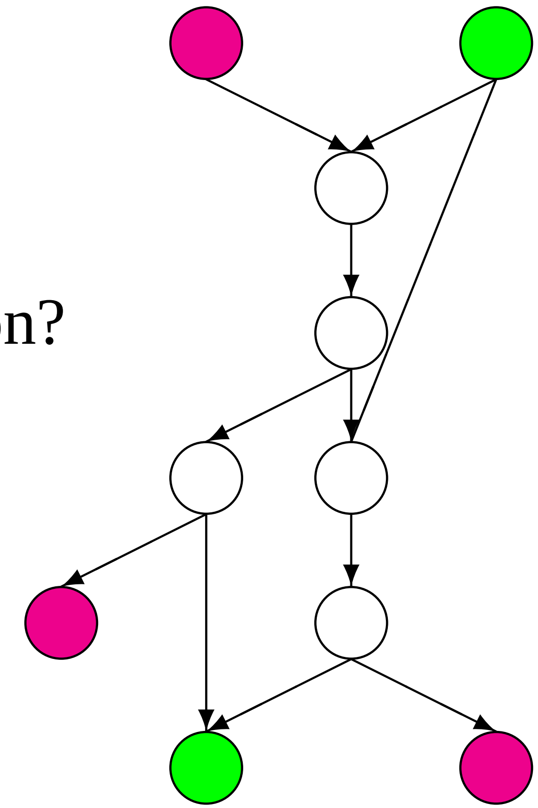


- Characterization??
- Utility Optimization?
- Distributed Implementation?

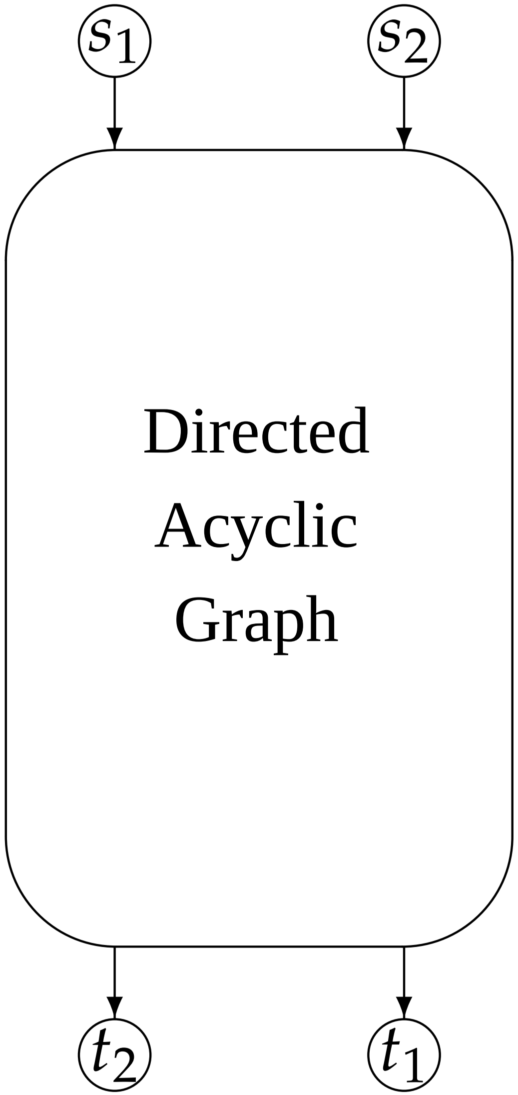


Content

- Existing results for inter-session network coding.
  - A structure-based approach and the backlog algorithm.
- New characterization theorems.
  - Two simple unicast/multicast sessions.
- A utility optimization problem.
  - A path-based approach.
- Distributed algorithms.
- Implementation issues.
- Experimental results

Two Simple Unicast Sessions

When can we send X_1 and X_2 simultaneously?

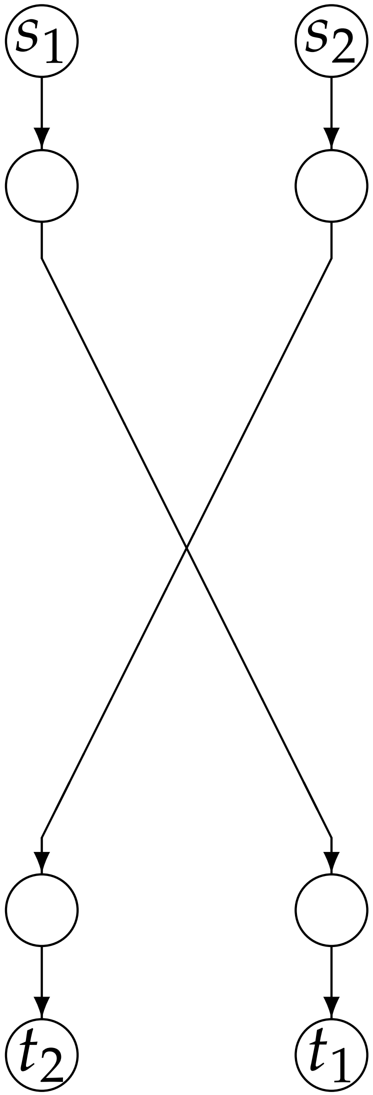


Two Simple Unicast Sessions

When can we send X_1 and X_2 simultaneously?

Routing solutions

$\Longleftrightarrow$ Edge disjoint paths



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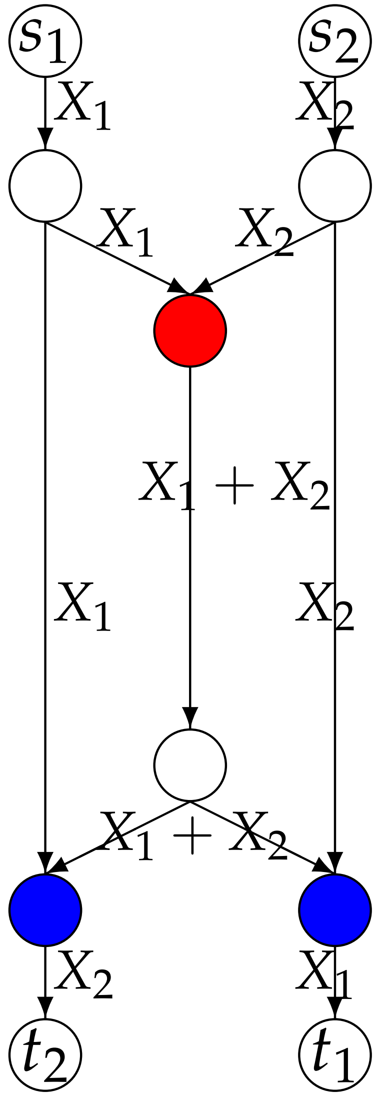
The existence of a butterfly

$\Longrightarrow$ Network coding solutions



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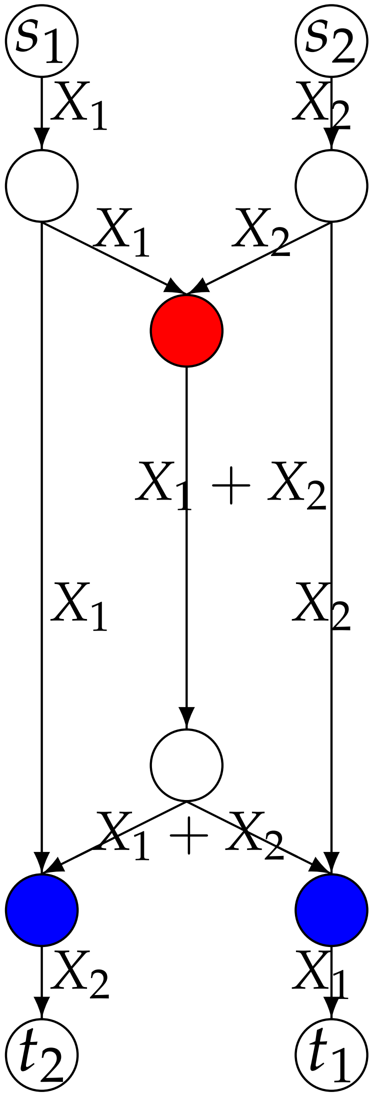
Vice versa?



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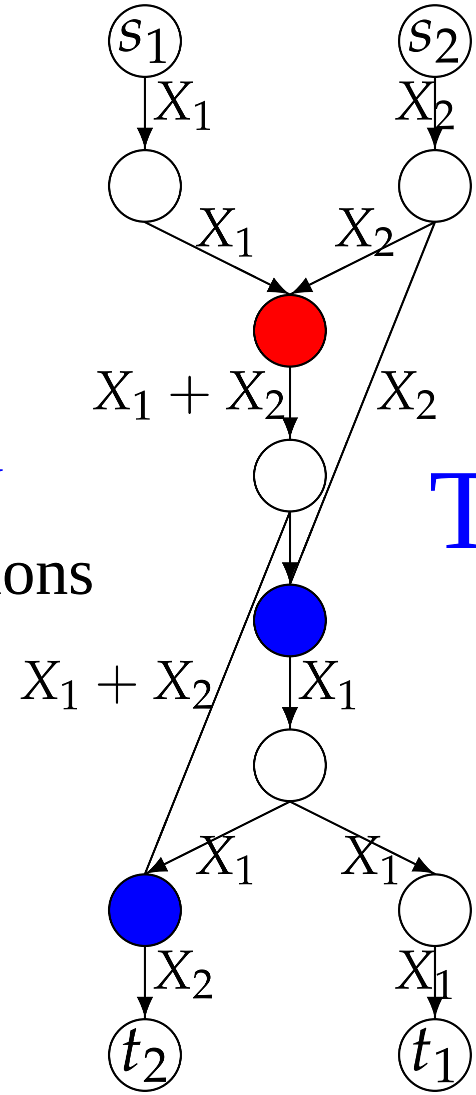

The grail structure

Two Simple Unicast Sessions

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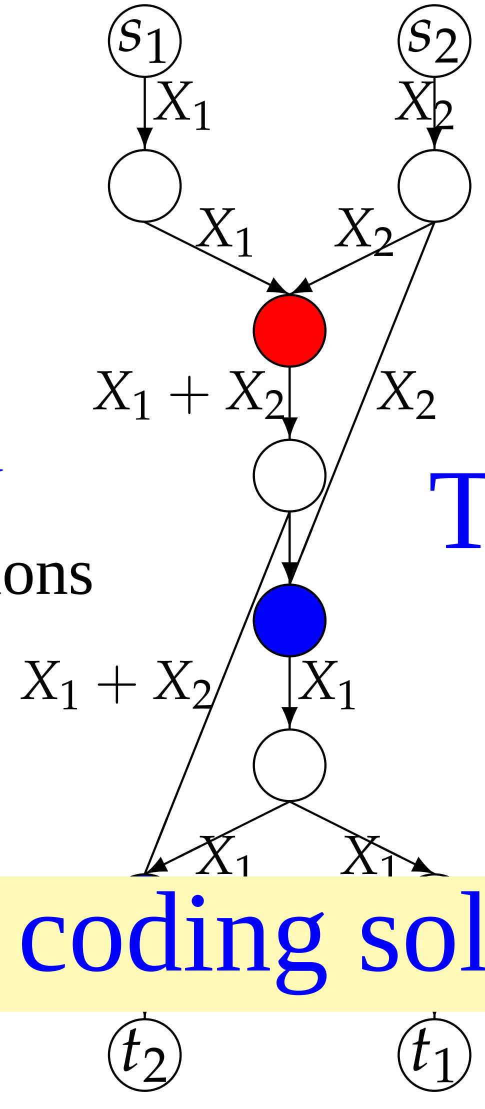
Vice versa?



The grail structure

Q: Network coding solutions $\Leftrightarrow$???

Two Simple Multicast Sessions

When can we send X_1 and X_2 simultaneously?

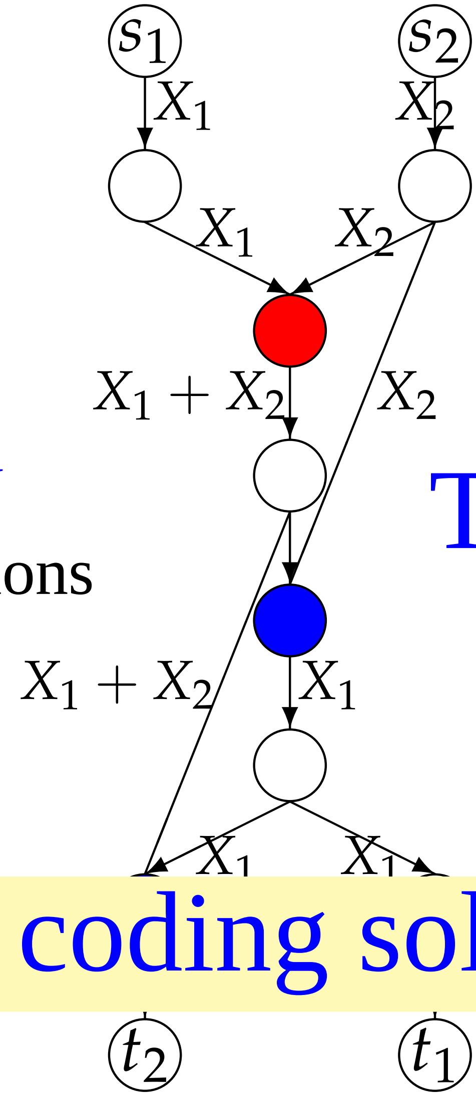
Routing solutions

$\Longleftrightarrow$ Edge disjoint paths

The existence of a butterfly

$\Longrightarrow$ Network coding solutions

Vice versa?



The grail structure

Q: Network coding solutions $\Leftrightarrow$???

Existing Multiple Session Results — Searching for Butterflies

- [Traskov *et al.* 06], [Ho *et al.* 06], [Eryilmaz *et al.* 07].

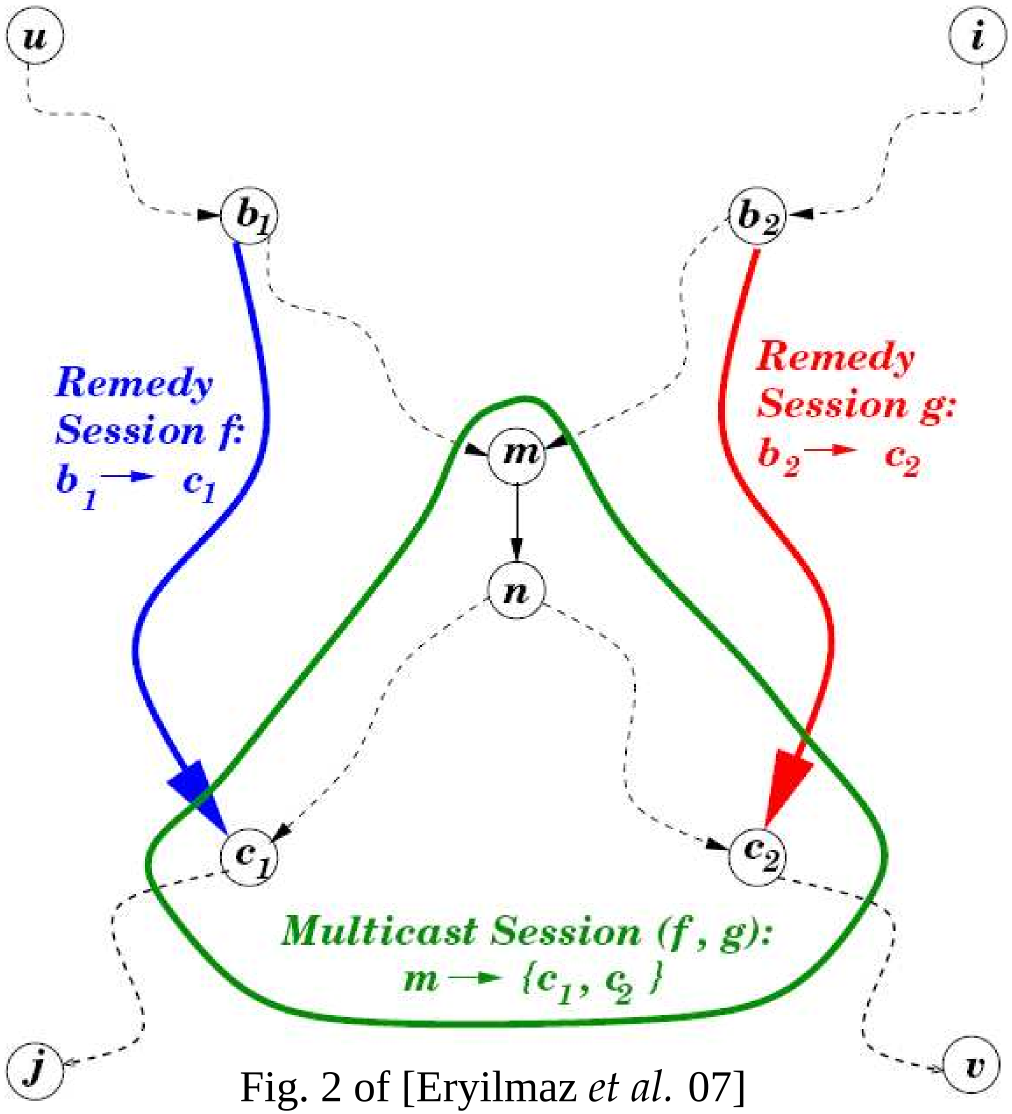
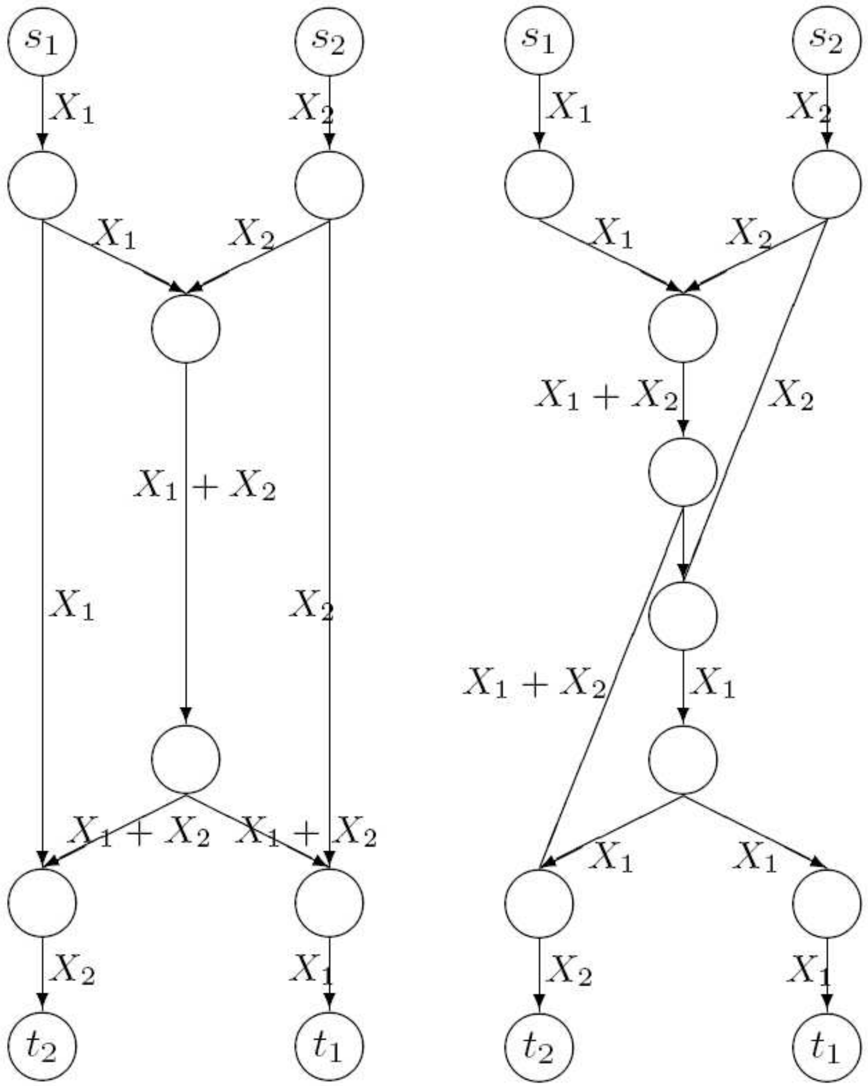


Fig. 2 of [Eryilmaz *et al.* 07]

- Create artificial flows p, q, w for each butterfly‡.
- $$\max_{r_i} \quad \sum_i U(r_i)$$
  subject to p, q, w, r_i
  satisfy the butterfly flow cond.
- With queues for each (artificial) flow, a backlog algorithm can distributively stabilize any rates in the above region.

The First Generalization

- Since the grail structure also admits coding benefit, we have



- Create artificial flows p, q, w for each butterfly and grail.
- $\max_{r_i} \quad \sum_i U(r_i)$

  subject to p, q, w, r_i

  satisfy the butterfly+grail cond.

- With queues for each (artificial) flow, a backlog algorithm can distributively stabilize any rates in the above region.

The Main Theorem — 2 Unicasts

- Setting: General finite directed acyclic graphs, unit edge capacity, (s_1, t_1) & (s_2, t_2), two integer symbols X_1 and X_2.
- Number of Coinciding Paths of edge e: $\mathcal{P} = \{P_1, \cdots, P_k\}$, and $\mathsf{ncp}_{\mathcal{P}}(e) = |\{P \in \mathcal{P} : e \in P\}|$.

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Theorem 2 *Network coding* $\Longleftrightarrow$ *one of the following two holds.*

1. $\exists \mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}\}$, *such that*

$$\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 1.$$

2. $\exists \mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}, P_{s_2,t_1}\}$ *and* $\mathcal{Q} = \{Q_{s_1,t_1}, Q_{s_2,t_2}, Q_{s_1,t_2}\}$ *s.t.*

$$\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 2 \text{ and } \max_{e \in E} \mathsf{ncp}_{\mathcal{Q}}(e) \leq 2.$$

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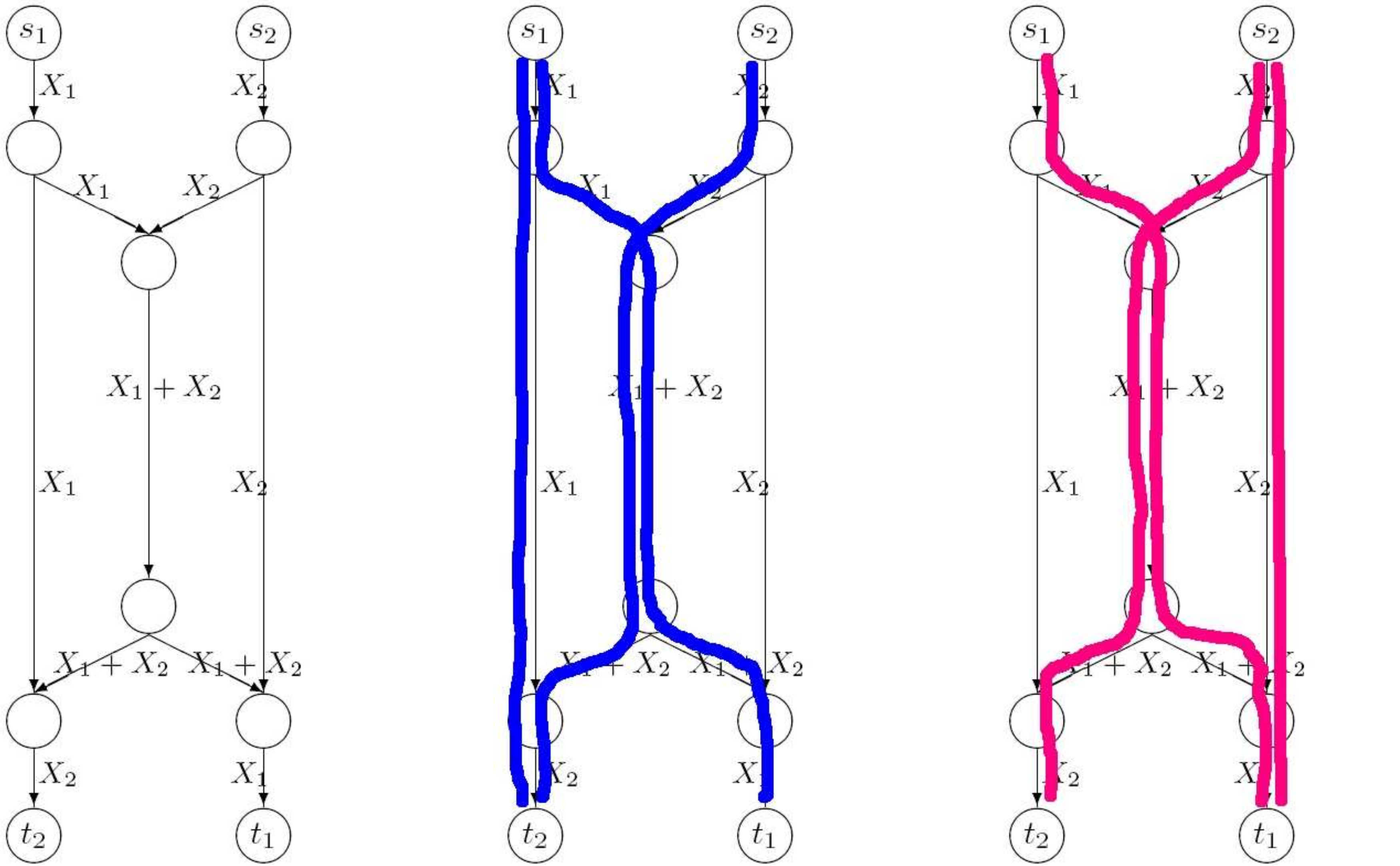
$$\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 2 \text{ and } \max_{e \in E} \mathsf{ncp}_{\mathcal{Q}}(e) \leq 2.$$

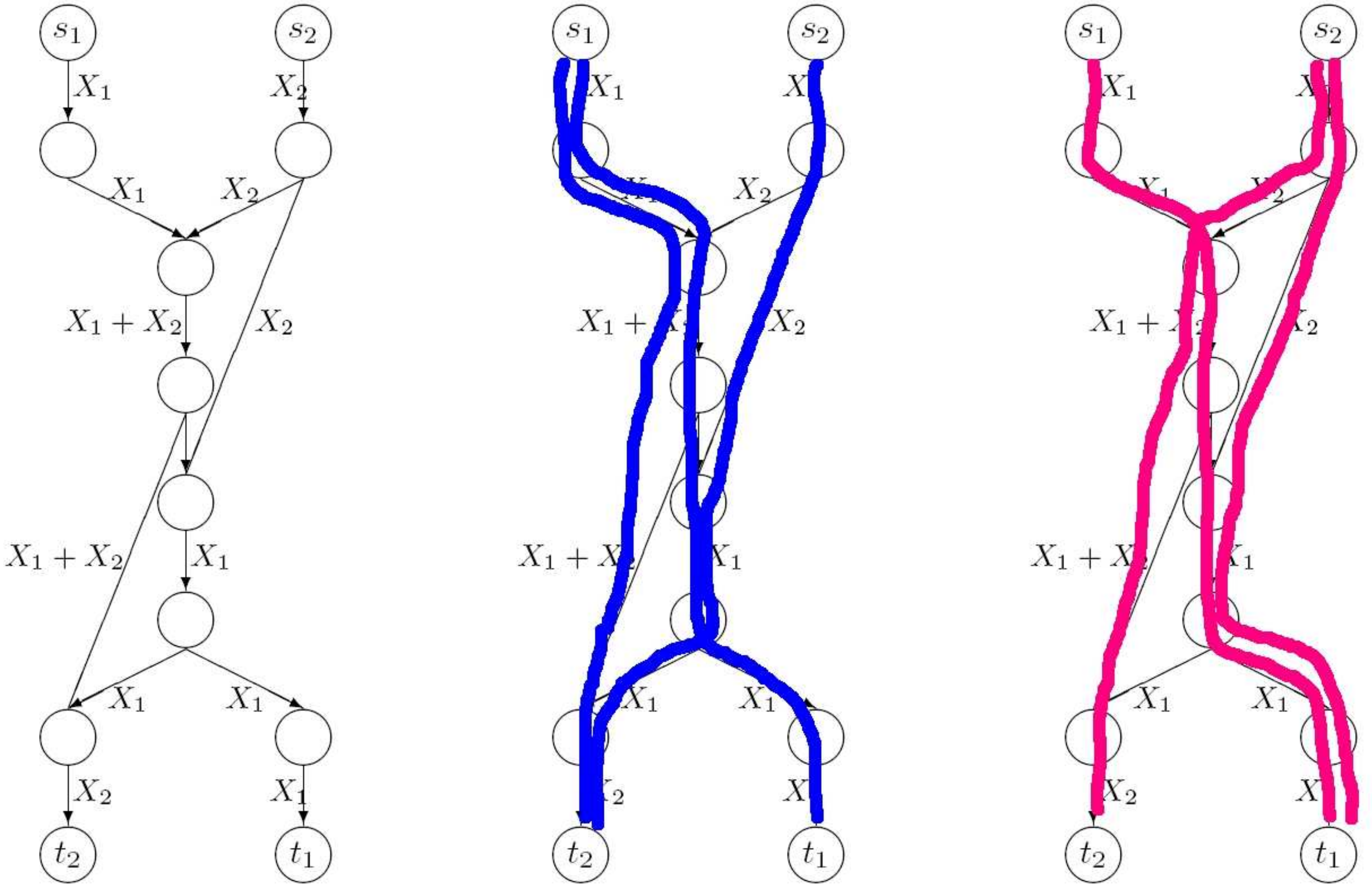
Routing: edge disjointness vs. Network coding: controlled overlaps.

Feasible Example: The Butterfly



$\mathcal{Q} = \{Q_{s_1,t_1}, Q_{s_2,t_2}, Q_{s_1,t_2}\}$ $\mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}, P_{s_2,t_1}\}$

Feasible Example 2: The Grail



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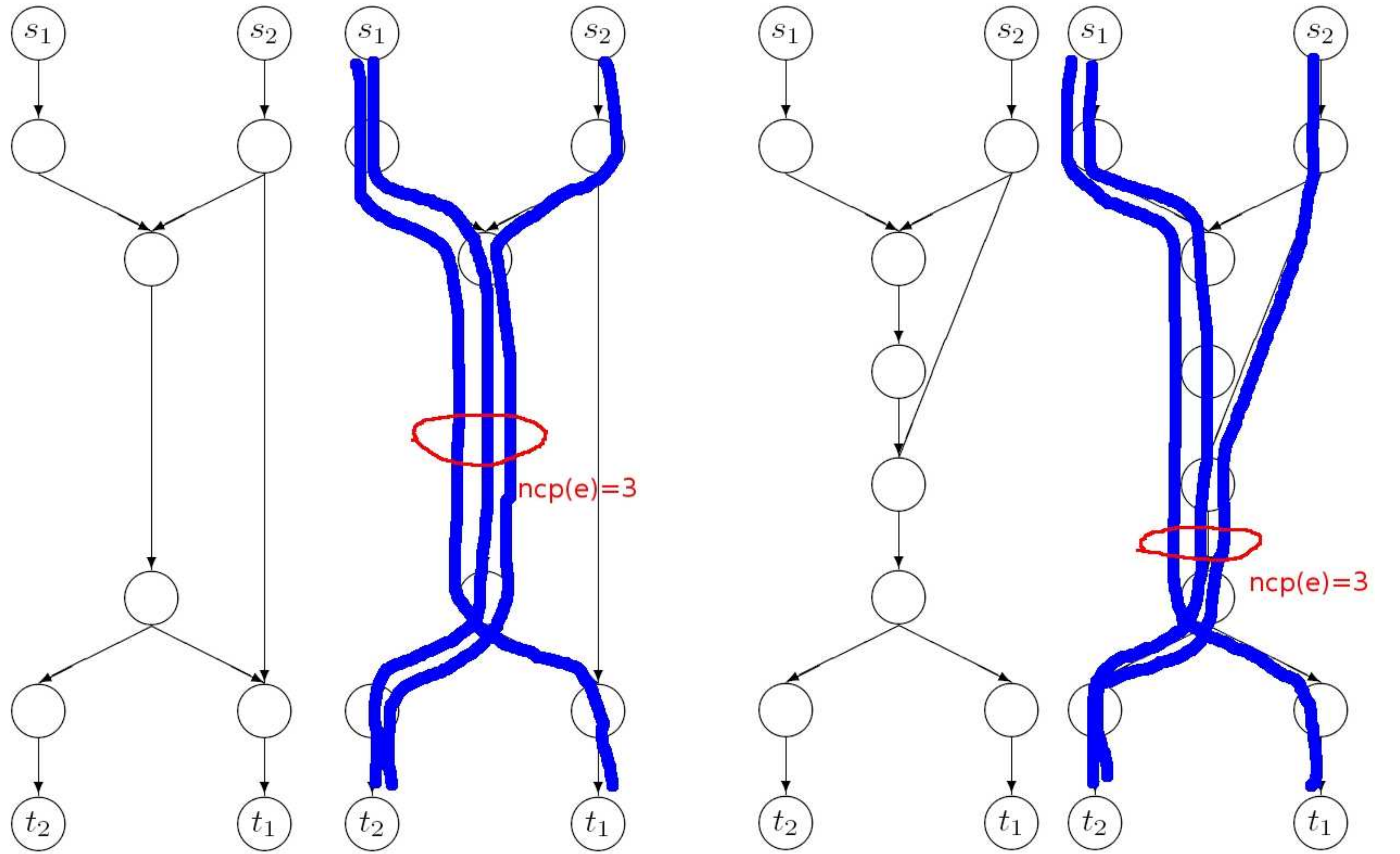
Infeasible Examples



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The Main Theorem — Multicasts

- Setting: General finite directed acyclic graphs, unit edge capacity, $(s_1, \{t_{1,i}\}_i)$ & $(s_2, \{t_{2,j}\}_j)$, two integer symbols X_1 and X_2.

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Theorem 3 *The existence of intersession network coding* $\Leftrightarrow$

$$\exists \mathcal{P} = \{P_{s_1,t_{1,i}}, P_{s_2,t_{1,i}} : \forall i\} \cup \{P_{s_2,t_{2,j}} : \forall j\},$$
$$\exists \mathcal{Q} = \{Q_{s_2,t_{2,j}}, Q_{s_1,t_{2,j}} : \forall j\} \cup \{Q_{s_1,t_{1,i}} : \forall i\},$$

such that

$$\max_{e \in E} \mathsf{ncp}_{\{P_{s_1,t_{1,i}}, P_{s_2,t_{1,i}}, P_{s_2,t_{2,j}}\}}(e) \leq 2, \ \ \forall i, j,$$

and

$$\max_{e \in E} \mathsf{ncp}_{\{Q_{s_2,t_{2,j}}, Q_{s_1,t_{2,j}}, Q_{s_1,t_{1,i}}\}}(e) \leq 2, \ \ \forall i, j.$$

The Main Theorem — Multicasts

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Theorem 3 *The existence of intersession network coding* $\Leftrightarrow$

$$\exists \mathcal{P} = \{\overset{1\longrightarrow 1}{P_{s_1,t_{1,i}}}, \overset{2\longrightarrow 1}{P_{s_2,t_{1,i}}} : \forall i\} \cup \{\overset{2\longrightarrow 2}{P_{s_2,t_{2,j}}} : \forall j\},$$

$$\exists \mathcal{Q} = \{\underset{2\longrightarrow 2}{Q_{s_2,t_{2,j}}}, \underset{1\longrightarrow 2}{Q_{s_1,t_{2,j}}} : \forall j\} \cup \{\underset{1\longrightarrow 1}{Q_{s_1,t_{1,i}}} : \forall i\},$$

such that

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$$\exists \mathcal{Q} = \{\underset{2\longrightarrow 2}{Q_{s_2,t_{2,j}}}, \underset{1\longrightarrow 2}{Q_{s_1,t_{2,j}}} : \forall j\} \cup \{\underset{1\longrightarrow 1}{Q_{s_1,t_{1,i}}} : \forall i\},$$

such that

Choose paths for i and j separately.

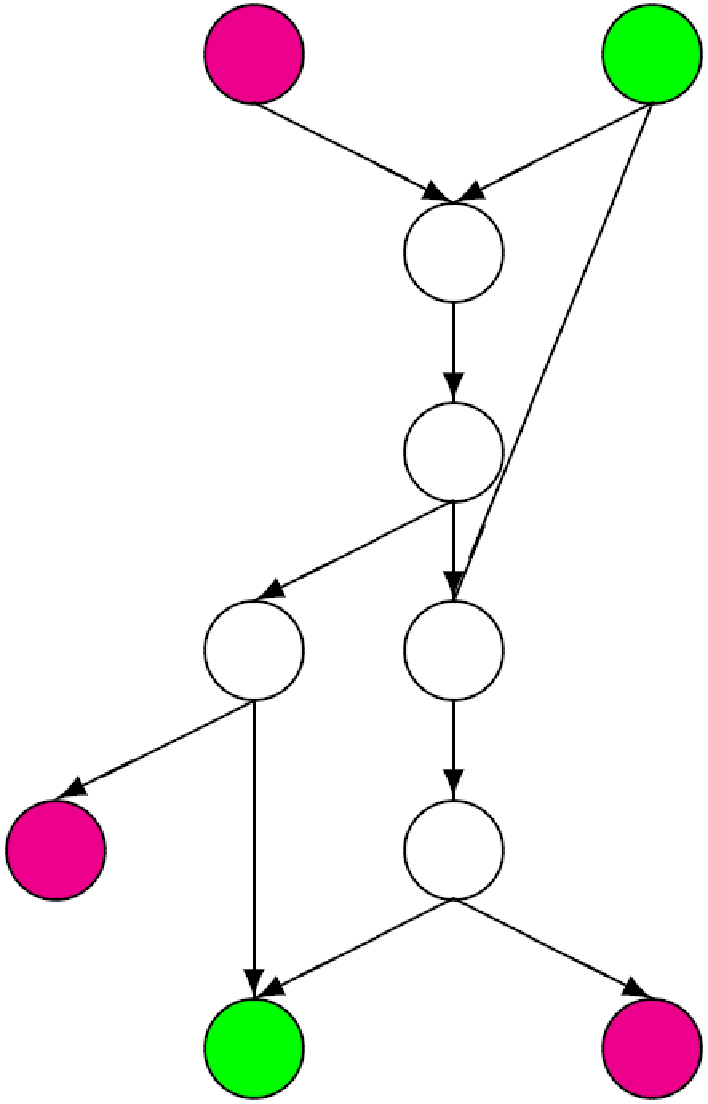
$$\max_{e\in E} \mathsf{ncp}_{\{P_{s_1,t_{1,i}}, P_{s_2,t_{1,i}}, P_{s_2,t_{2,j}}\}}(e) \leq 2, \quad \forall i, j,$$

and

$$\max_{e\in E} \mathsf{ncp}_{\{Q_{s_2,t_{2,j}}, Q_{s_1,t_{2,j}}, Q_{s_1,t_{1,i}}\}}(e) \leq 2, \quad \forall i, j.$$

Then the conditions have to be satisfied for all (i, j) combinations.

An Infeasible Example



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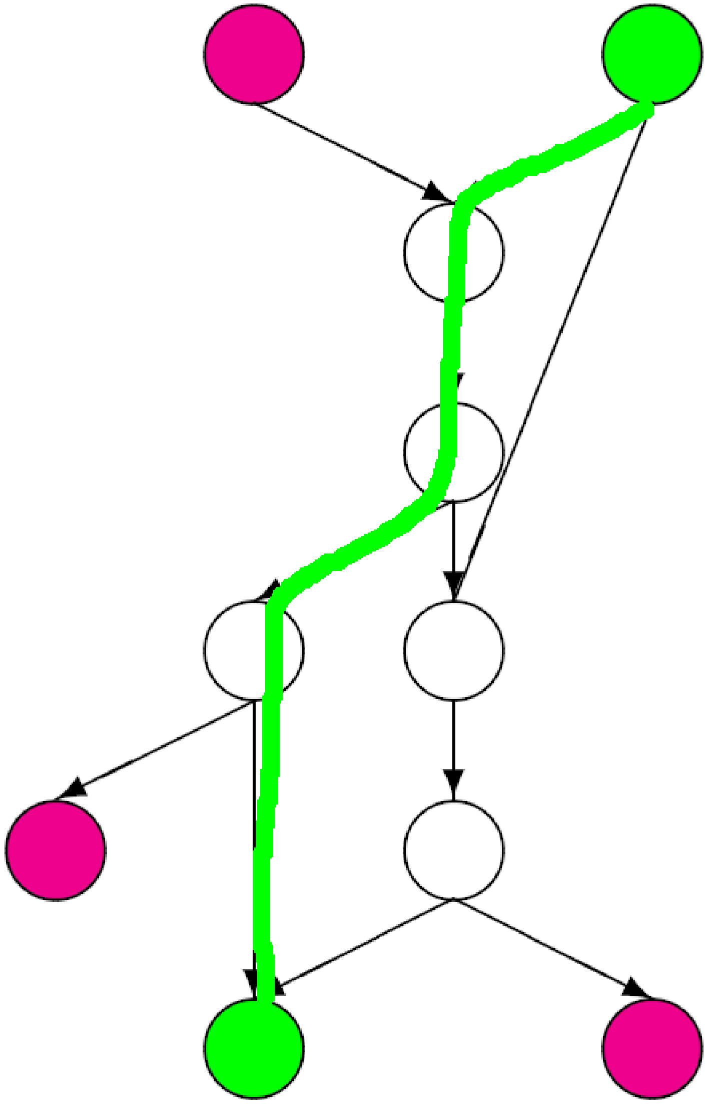

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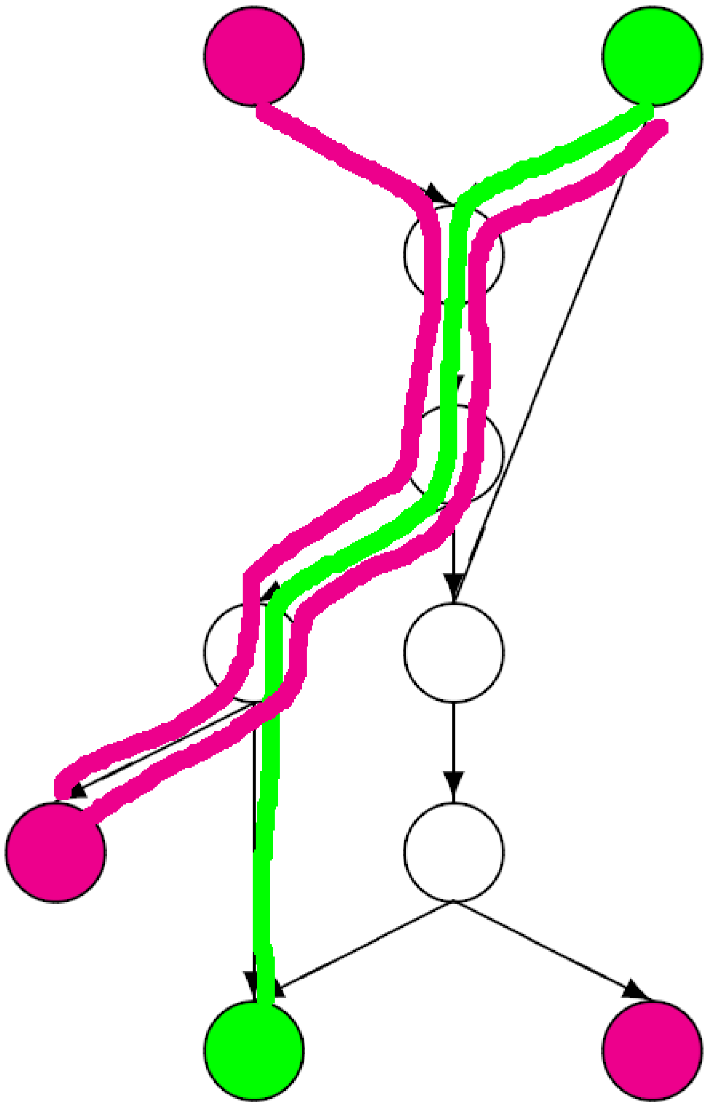

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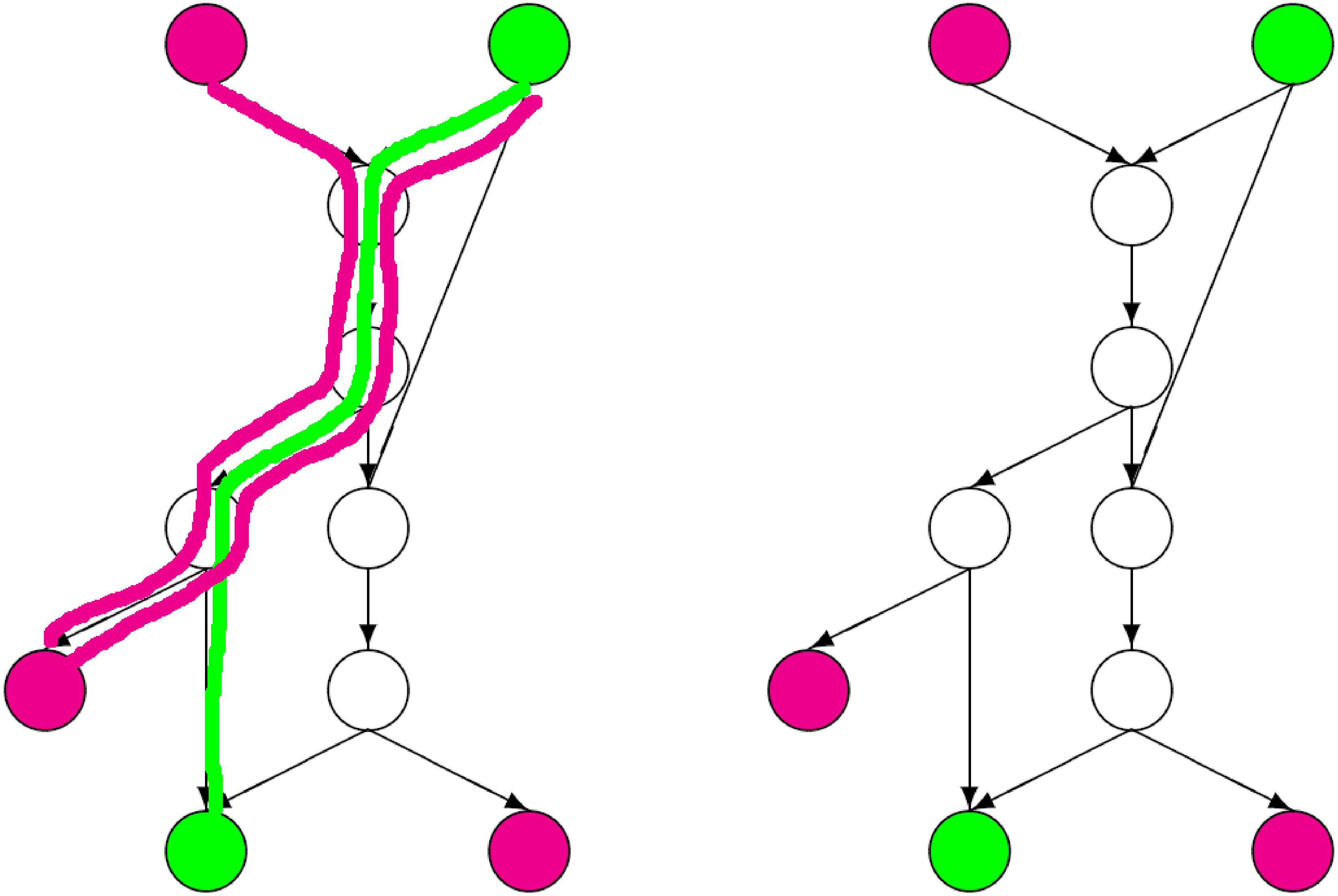

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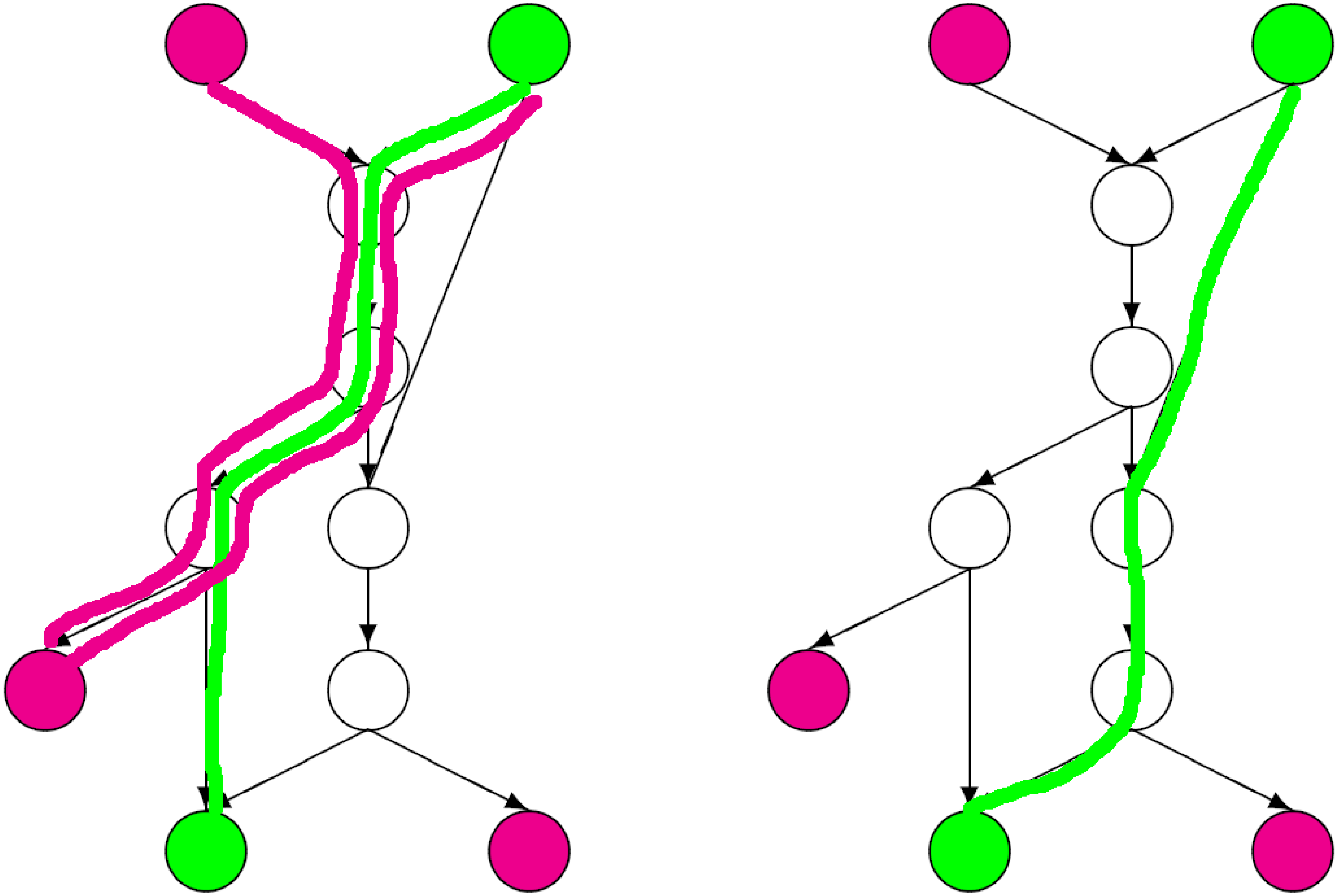

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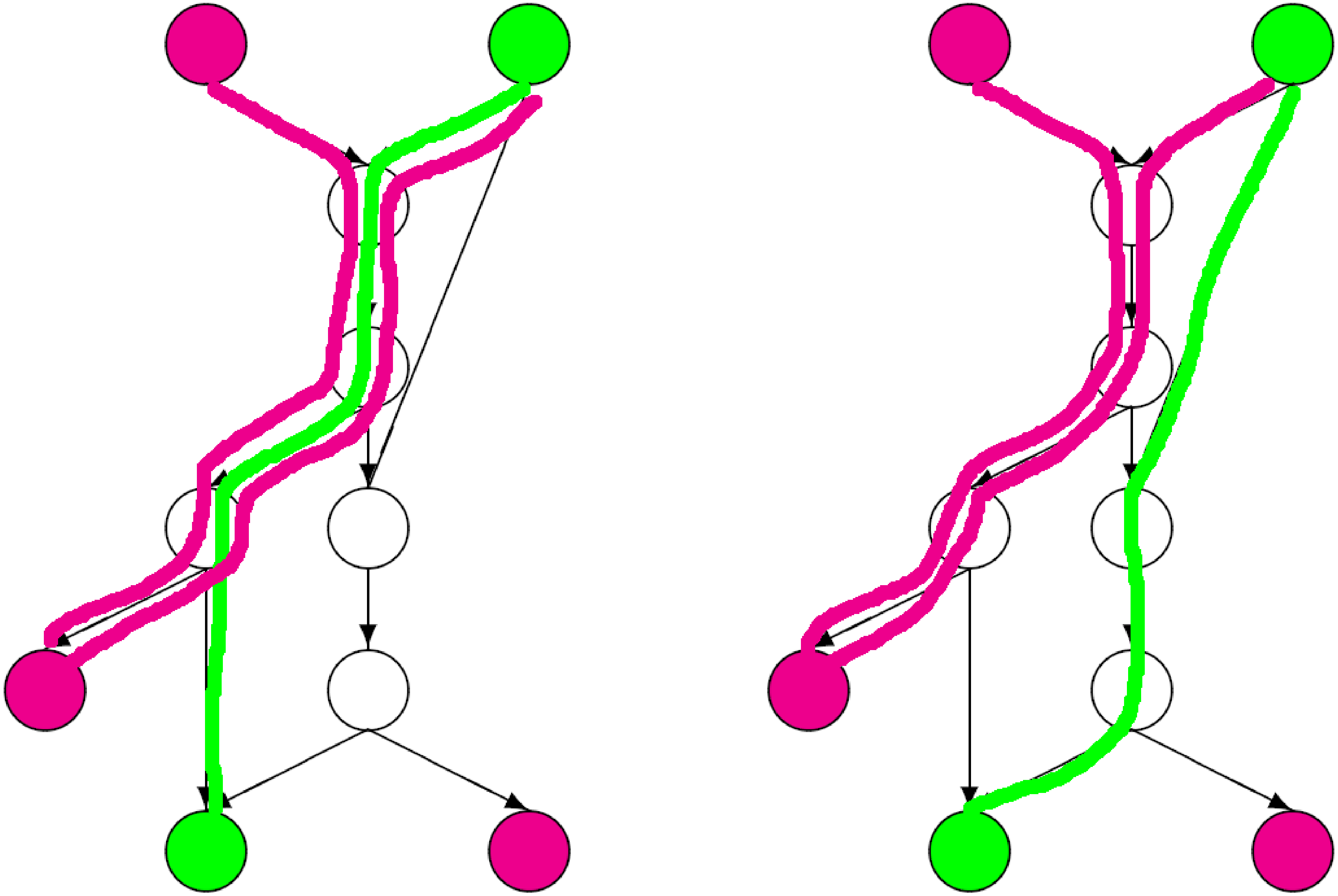

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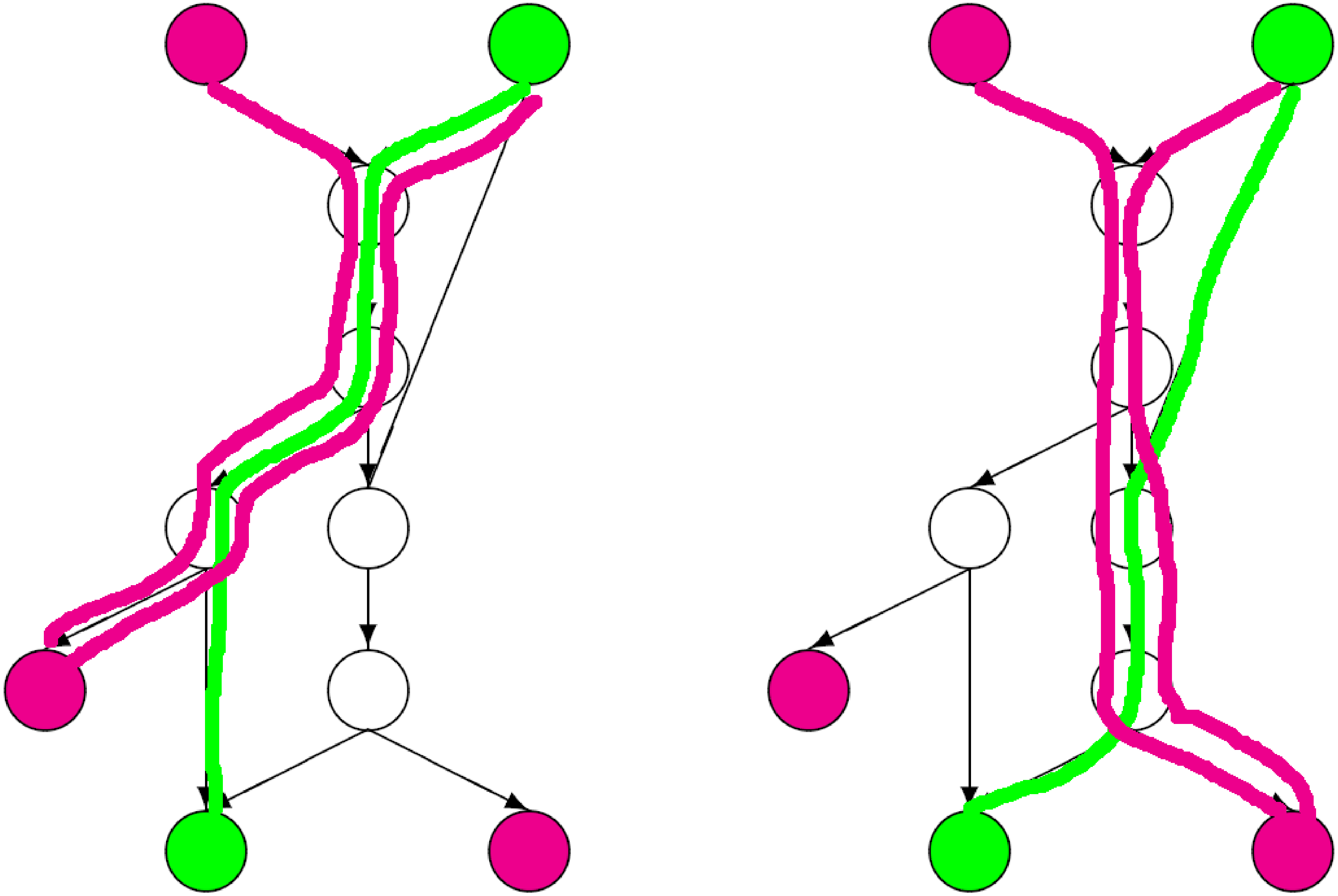

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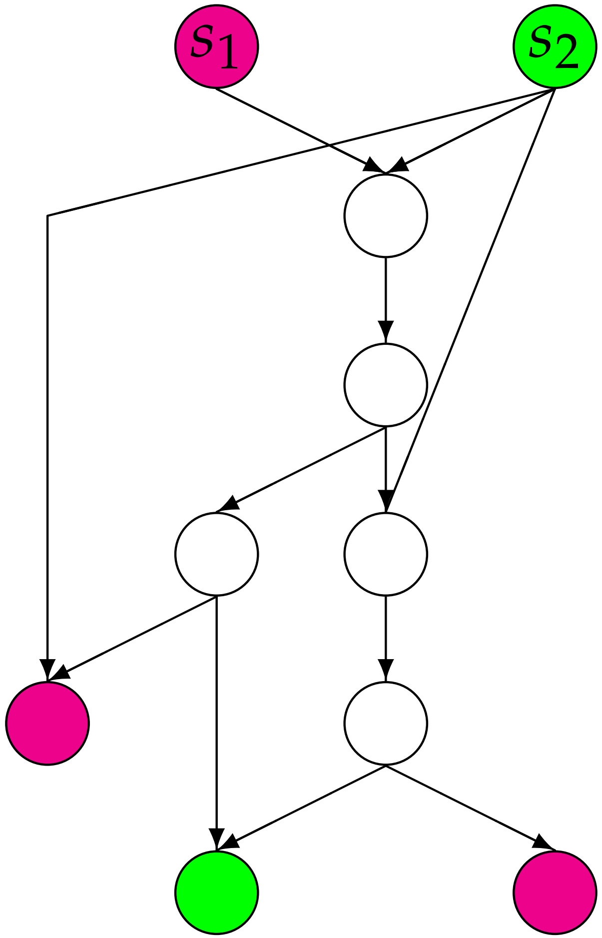

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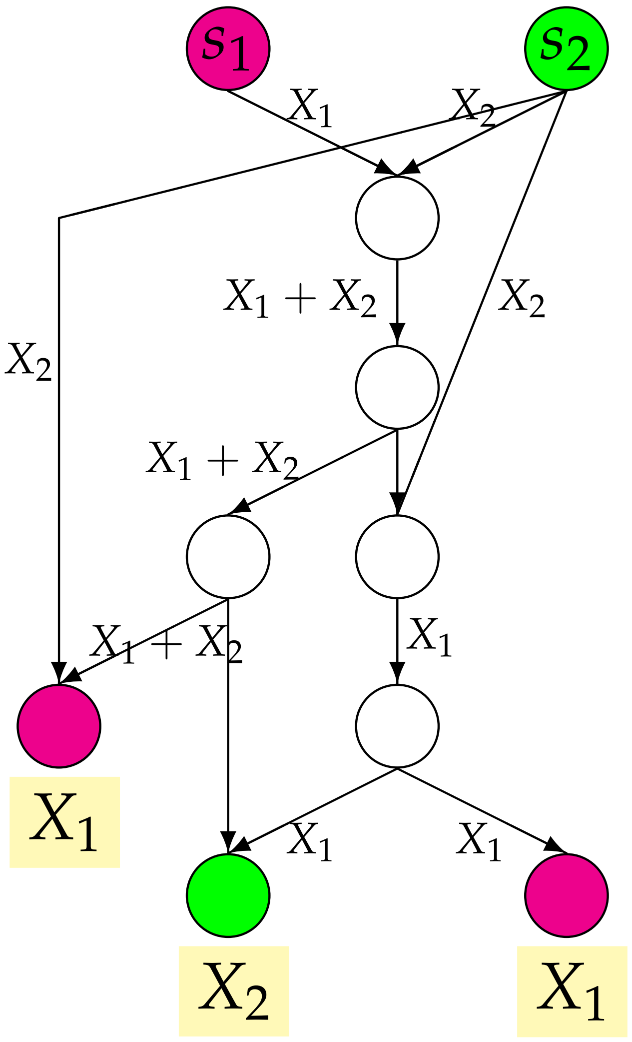

Utility Optimization

I: the no. coexisting unicast sessions (s_i, t_i)

$\mathcal{P}(i)$: the set of all (s_i, t_i) paths

$\mathbb{P}(i,j)$: the set of all $(P_{s_i,t_i}, P_{s_j,t_i}, P_{s_j,t_j})$ tuples

$$E^k_{e,i}: \quad = 1, \text{ if link } e \text{ uses the } k\text{-th path in } \mathcal{P}(i)$$
$$= 0, \text{ otherwise}$$

$$H^l_{e,ij}: \quad = 2, \text{ if for the } l\text{-th tuple in } \mathbb{P}(i,j), \mathsf{ncp}(e) = 3$$
$$= 1, \text{ if for the } l\text{-th tuple in } \mathbb{P}(i,j), \mathsf{ncp}(e) = 1, 2$$
$$= 0, \text{ if for the } l\text{-th tuple in } \mathbb{P}(i,j), \mathsf{ncp}(e) = 0$$

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$E^k_{e,i}$: $= 1$, if link e uses the k-th path in $\mathcal{P}(i)$

$= 0$, otherwise

$H^l_{e,ij}$: $= 2$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 3$

$= 1$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 1, 2$

$= 0$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 0$

$$\max_{\vec{x}, \vec{g}} \sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$$

$$\text{s.t.} \sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E^k_{e,i} x^k_i + \sum_{i=1}^{I} \sum_{i<j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \max(H^l_{e,ij}, H^m_{e,ji}) g^{lm}_{ij} \leq C_e, \forall e$$

$$x_i^k \geq 0, \quad g_{ij}^{lm} = g_{ji}^{ml} \geq 0, \quad \forall i \neq j, l, m$$

Incorporating the Proximal Meth.

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- The Slater condition holds.
- Solve the dual of the intermediate problem.

The Proximal Method (Cont'd)

- The Lagrangian $L_{\overrightarrow{y},\overrightarrow{h}}(\overrightarrow{x},\overrightarrow{g},\overrightarrow{\lambda},\overrightarrow{\mu})$ is

$$\sum_{i=1}^{I} U_i\left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j\neq i}\sum_{l=1}^{|\mathbb{P}(i,j)|}\sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm}\right)$$

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- Separable!

The Distributed Solver

- Repeat the following K times:
  - Solve $D_{\overrightarrow{y},\overrightarrow{h}}(\overrightarrow{\lambda},\overrightarrow{\mu}) = \max_{\overrightarrow{x},\overrightarrow{g}} L_{\overrightarrow{y},\overrightarrow{h}}(\overrightarrow{x},\overrightarrow{g},\overrightarrow{\lambda},\overrightarrow{\mu})$ via separability.
  - Solve the dual problem $\min D_{\overrightarrow{y},\overrightarrow{h}}(\overrightarrow{\lambda},\overrightarrow{\mu})$ by the gradient method with step size α.
- Update $\overrightarrow{y} \leftarrow \overrightarrow{x}^*$, $\overrightarrow{h} \leftarrow \overrightarrow{g}^*$, and go back to the beginning.

The Convergence Result

Theorem 4 *If the step size α of the gradient method (for the dual) and the proximal method coefficients c_i and d_i satisfy the following:*

$$\alpha \left(2 + \sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^{k} + \frac{1}{4} \sum_{i=1}^{I} \sum_{i \neq j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(ji)|} (\max(H_{e,ij}^{l}, H_{e,ji}^{m}))^2 \right) < 2 \min_{i} \min(c_i, d_i),$$

then as $K \rightarrow \infty$, the proximal method converges to the optimal $\overrightarrow{x}_{\text{opt}}$ and $\overrightarrow{g}_{\text{opt}}$ for the original problem.

- For bounded K, the convergence is verified by simulations.

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- Coding scheme?

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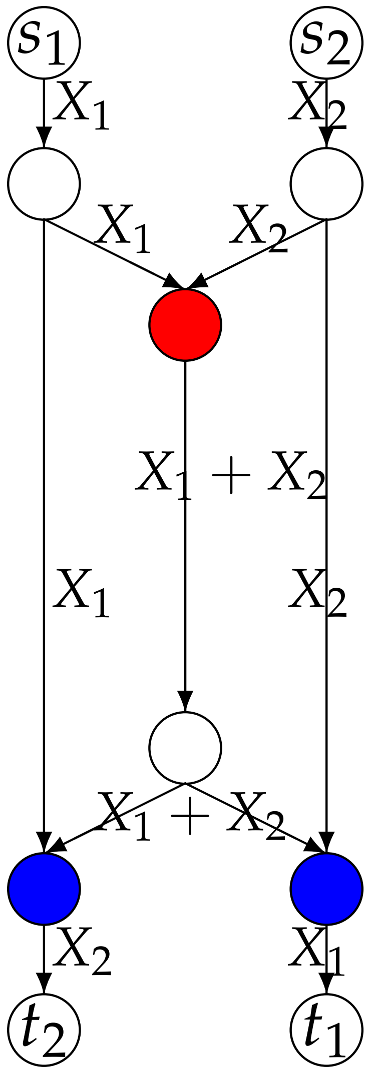

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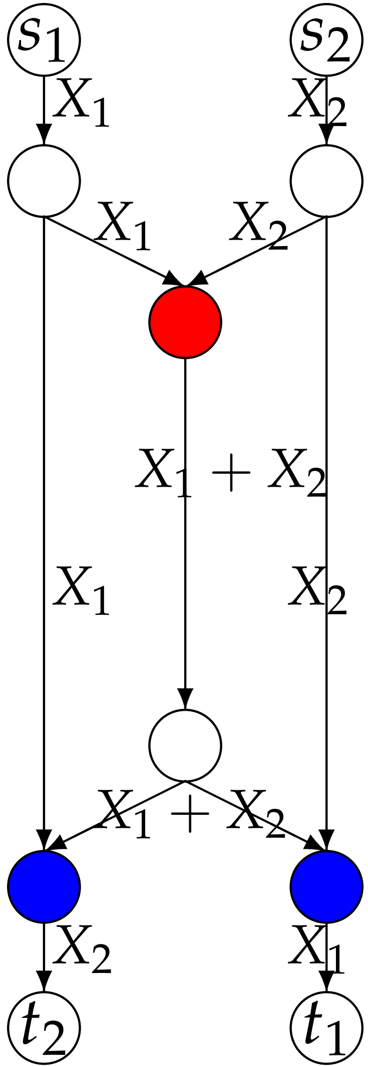

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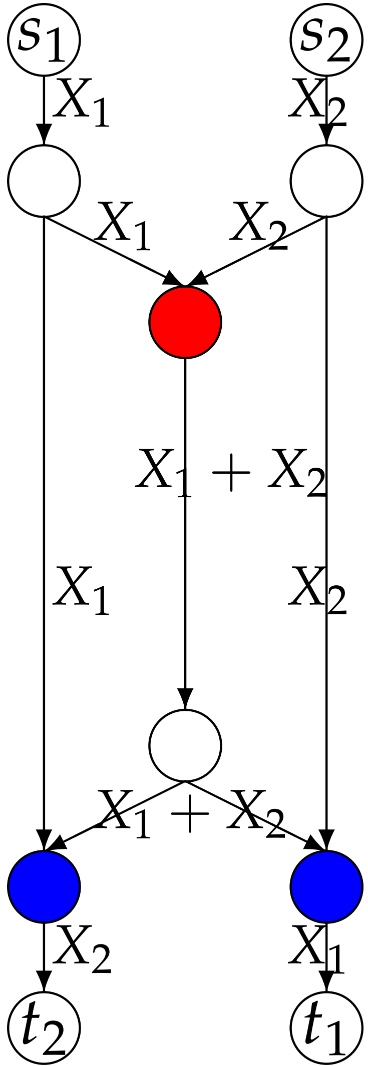

Theorem 4 *With modified random linear coding over* GF(q), *the success probability is*

$$\text{Prob}(\textit{success}) \geq \left(1 - \frac{4}{q}\right)^{6|E|}.$$

The Implementation Issues

- The control messages to collect the info. nec. for maximizing the Lagrangian.

$$
\begin{aligned}
&\sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right) \\
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&- \sum_{e} \lambda_e \left(\sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^k x_i^k + \sum_{i=1}^{I} \sum_{i<j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \max(H_{e,ij}^l, H_{e,ji}^m) g_{ij}^{lm} - C_e \right) \\
&- \sum_{i=1}^{I} \sum_{i<j} \sum_{l} \sum_{m} \mu_{ij}^{lm} \left(g_{ij}^{lm} - g_{ji}^{ml} \right)
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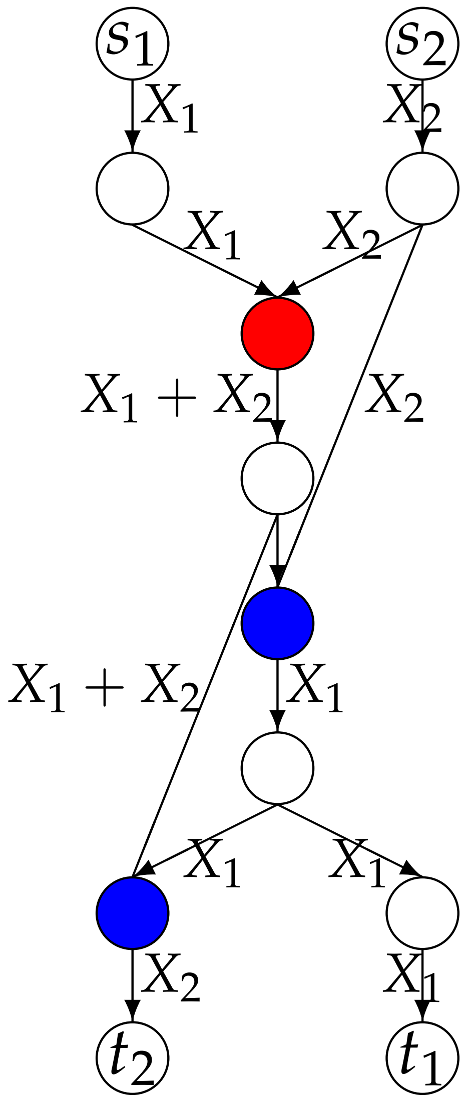
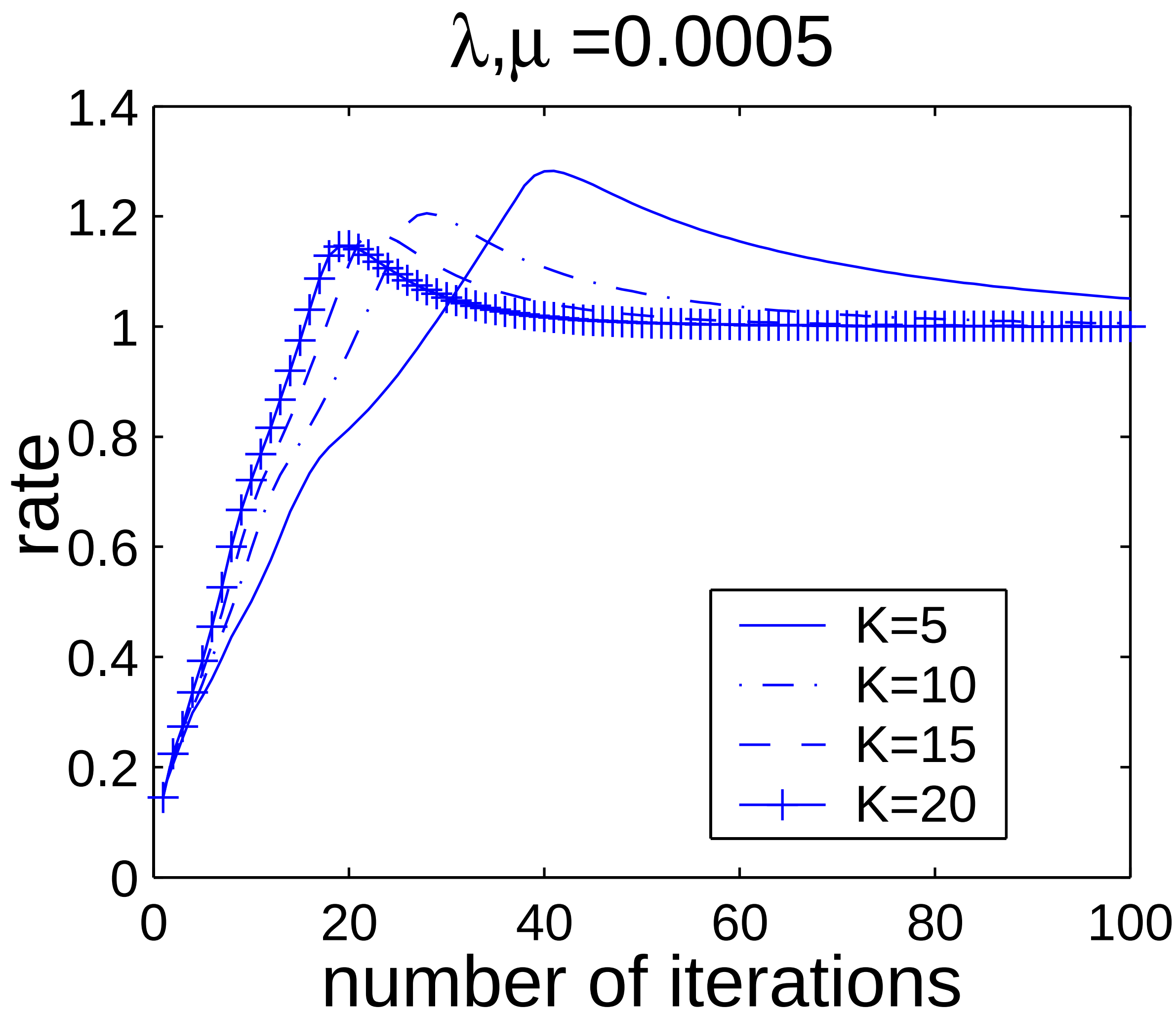
- Adaptively select $\mathcal{P}(i)$ and $\mathbb{P}(i,j)$.

Numerical Experiments





Numerical Experiments

- Utility gain $\mathcal{UG}$: $\frac{\sum U_i(\text{intersession NC.}) - \sum U_i(\text{routing})}{\sum U_i(\text{routing})}$.
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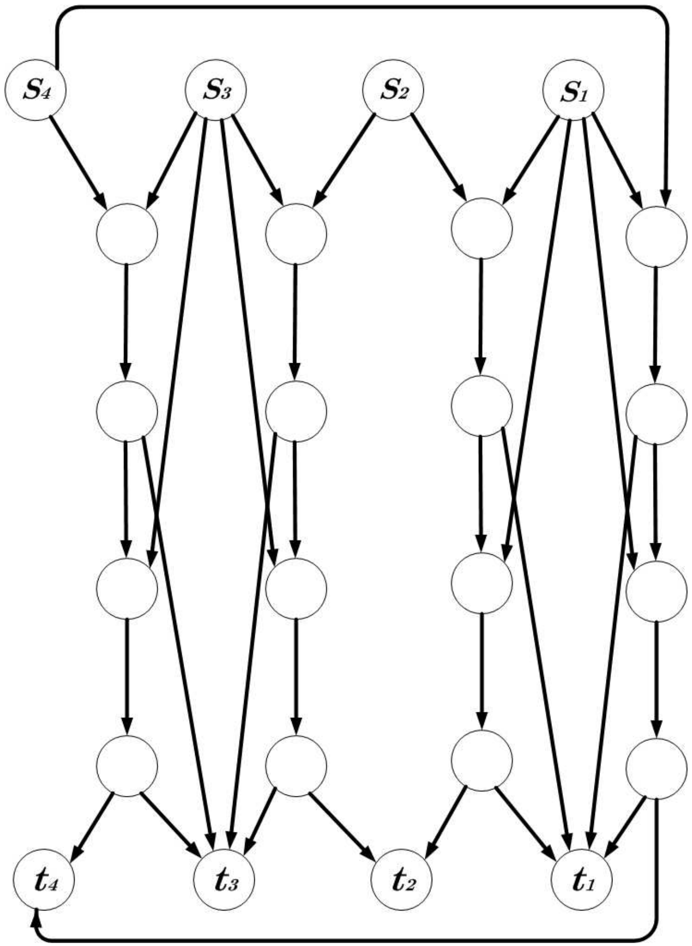

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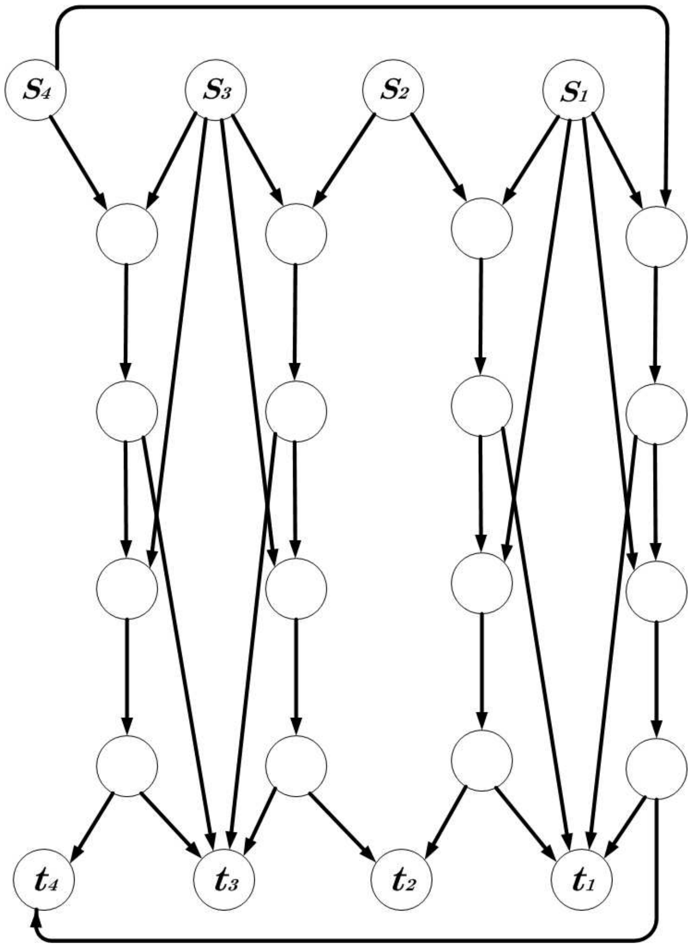

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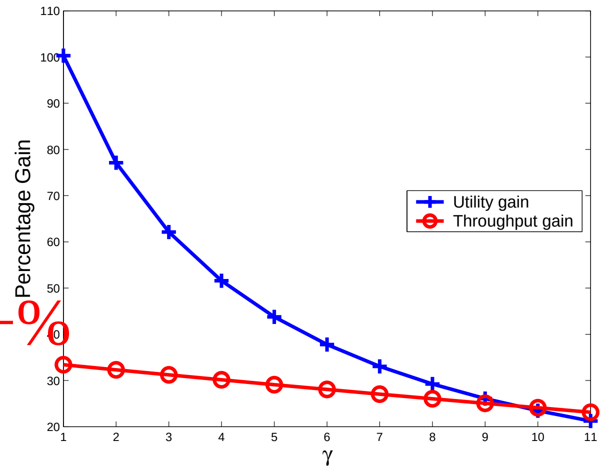

$\log(\gamma + r_i)$

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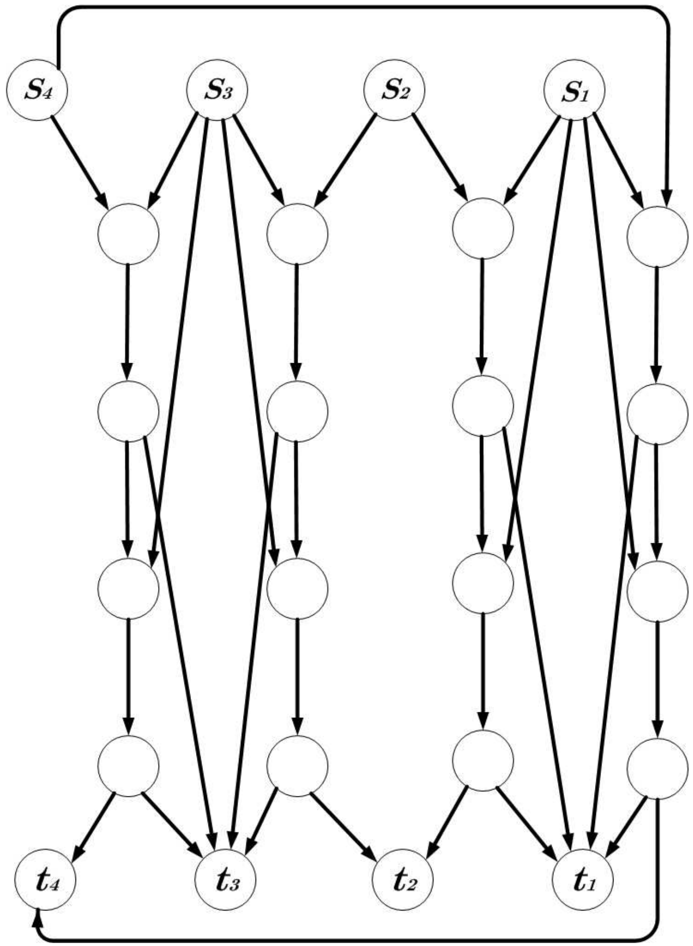

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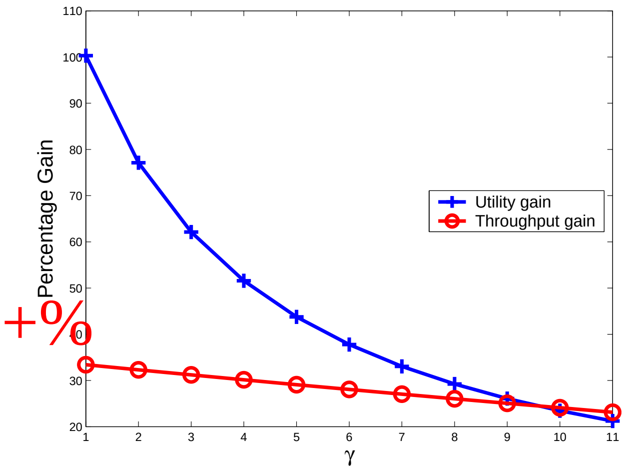

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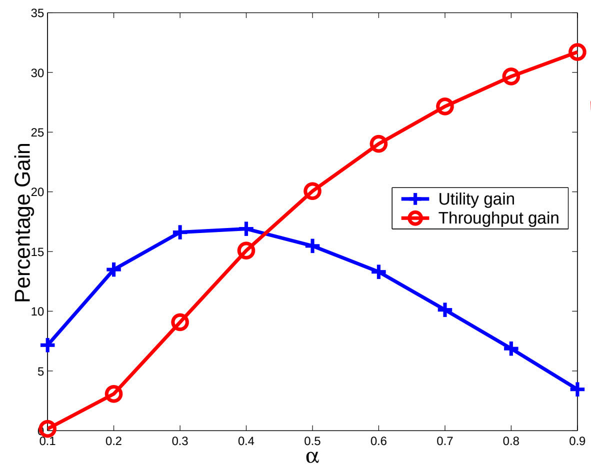

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$\frac{r_i^{1-\alpha}}{1-\alpha}$

Conclusions

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- Intersession network coding promotes further fairness.