From Stopping sets to Trapping sets

The Exhaustive Search Algorithm & The Suppressing Effect

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- The suppressing effect for cyclically lifted code ensembles.

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  - A design criteria for base code optimization.

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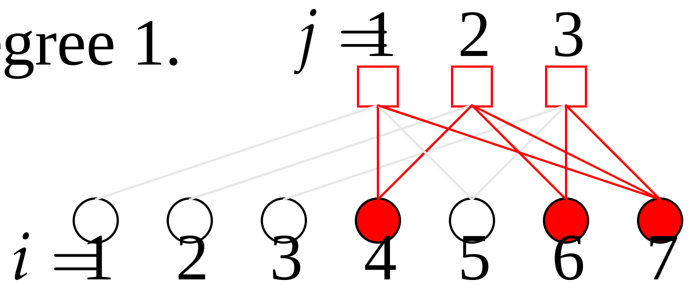
- Definition: a set of variable nodes ⇒ the induced graph contains no check node of degree 1.



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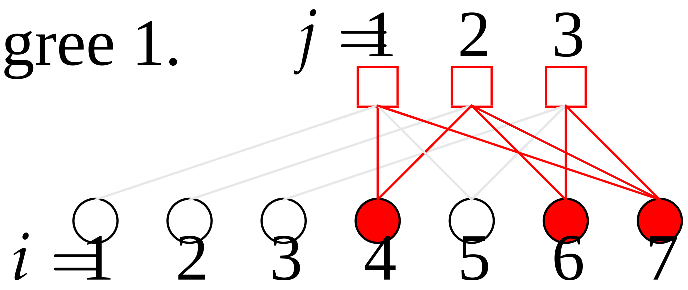

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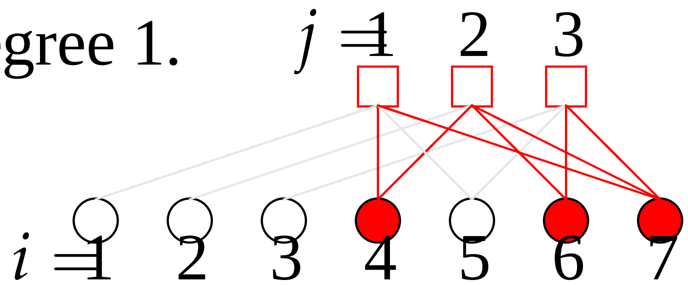

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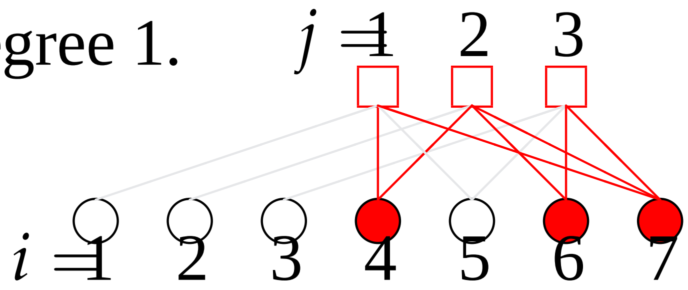

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  - Error floor optimization. BECs vs. non-erasure channels.
- Good but inexhaustive search algorithms: error floors of LDPC codes [Richardson 03], projection algebra [Yedidia *et al.* 01], the approximate minimum distance of LDPC codes [Hu *et al.* 04], [Hirotomo *et al.* 05], [Richter 06]

An NP-Hard Problem

=== The $\mathrm{SD}(H, t)$ problem ===

INPUT: A code represented by its parity-check matrix H and an integer t.

OUTPUT: Output 1 if the minimal stopping distance of H is $\leq t$. Otherwise, output 0.

The hardness results:

- [Krishnan *et al.* 06]: For arbitrary H, $\mathrm{SD}(H, t)$ is NP-complete. Proof: By reducing a VERTEX-COVER problem to $\mathrm{SD}(H, t)$.
- A byproduct of [Krishnan *et al.* 06]: With the sparsity restriction that the number of 1's in H is limited to $O(n)$ rather than $O(n^2)$, then $\mathrm{SD}(H, t)$ is still NP-complete.

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- We propose a new graph-theoretic definition:

  Definition 1 (k-out Trapping Sets) *A subset of* $\{v_1, \ldots, v_n\}$ *such that in the induced subgraph, there are exactly k check nodes of degree one.*

k-Out Trapping Sets vs. Near-Codeword

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- k-out trapping sets $\longleftrightarrow$ stopping sets
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- Better analogy to stopping sets.

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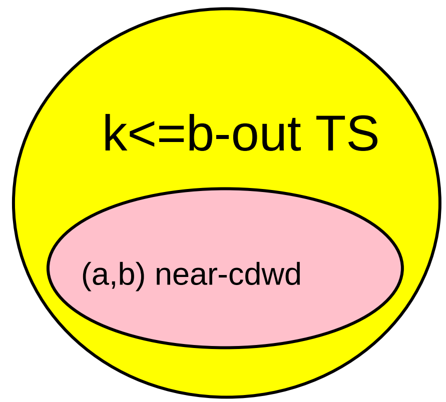

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- Our goal: With fixed b, search all min. $k \leq b$-out TSs.

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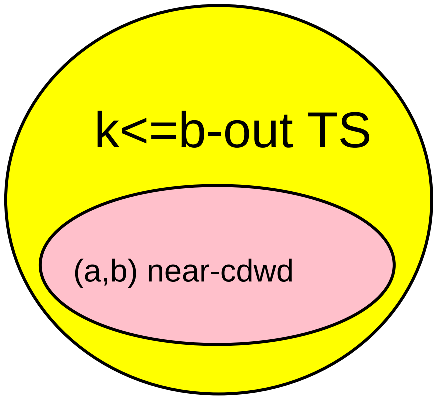

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- Our goal: With fixed b, search all min. $k \leq b$-out TSs.
- Empirically, all error bits consist of only degree 1 & 2 check nodes. (The elementary trapping set [Landner *et al.* 05].)

The Hardness of k-OTD(H, t)

=== The k-OTD(H, t) problem ===

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- When $k = 0$, then 0-OTD(H, t) = SD(H, t) is NP-complete.
- Is the hardness the same for any fixed $k > 0$ values?

Our First Result

Theorem 1 *Consider a fixed $k > 0$. For arbitrary H, k-OTD(H, t) is NP-complete.*

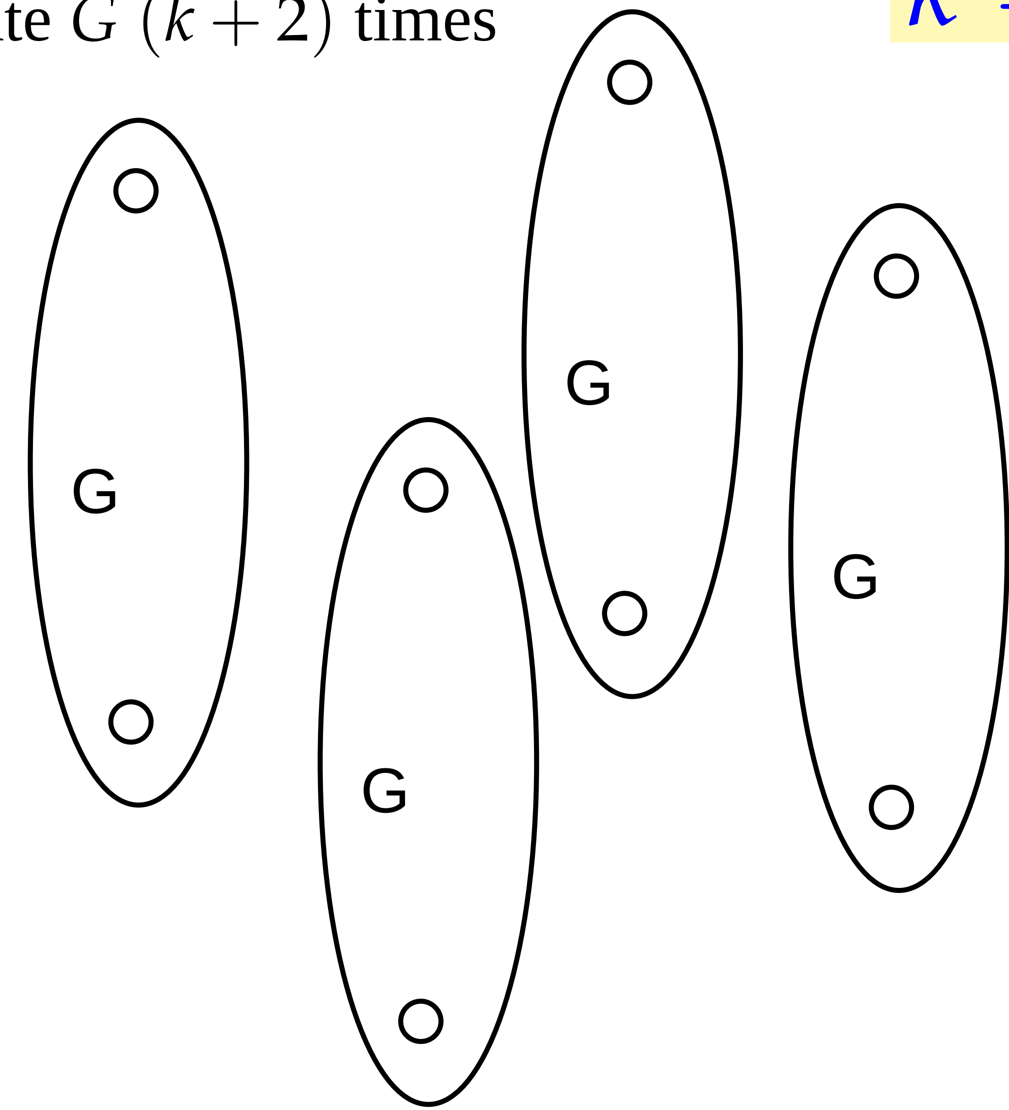
Theorem 2 *Consider a fixed $k > 0$. With the sparsity restriction that the number of 1's in H is limited to $O(n)$ rather than $O(n^2)$, then k-OTD(H, t) is still NP-complete.*

Proof: Reduction from SD(H, t).

SD(H,t) By k-OTD(H',t')

$k = 2$

Step 1: Duplicate G $(k+2)$ times



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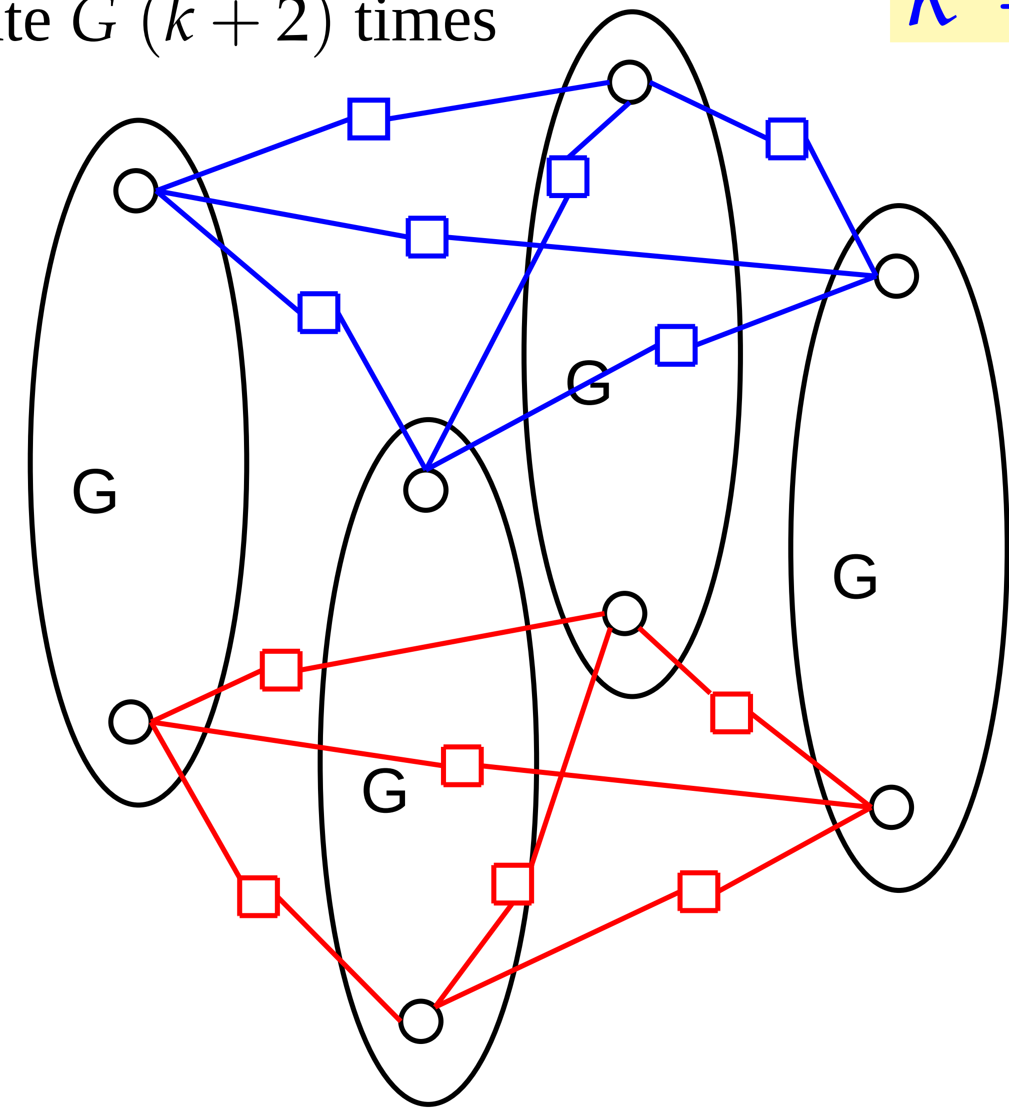

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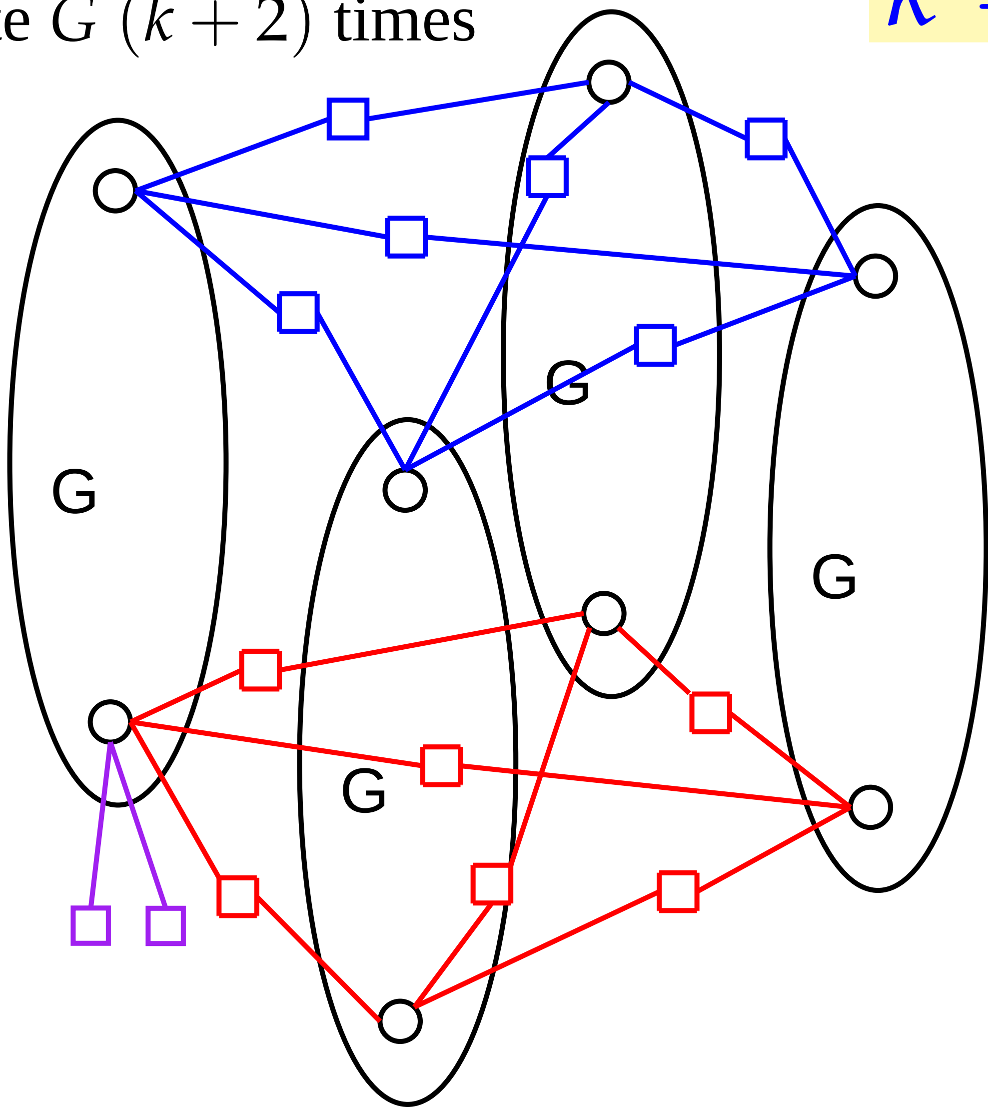

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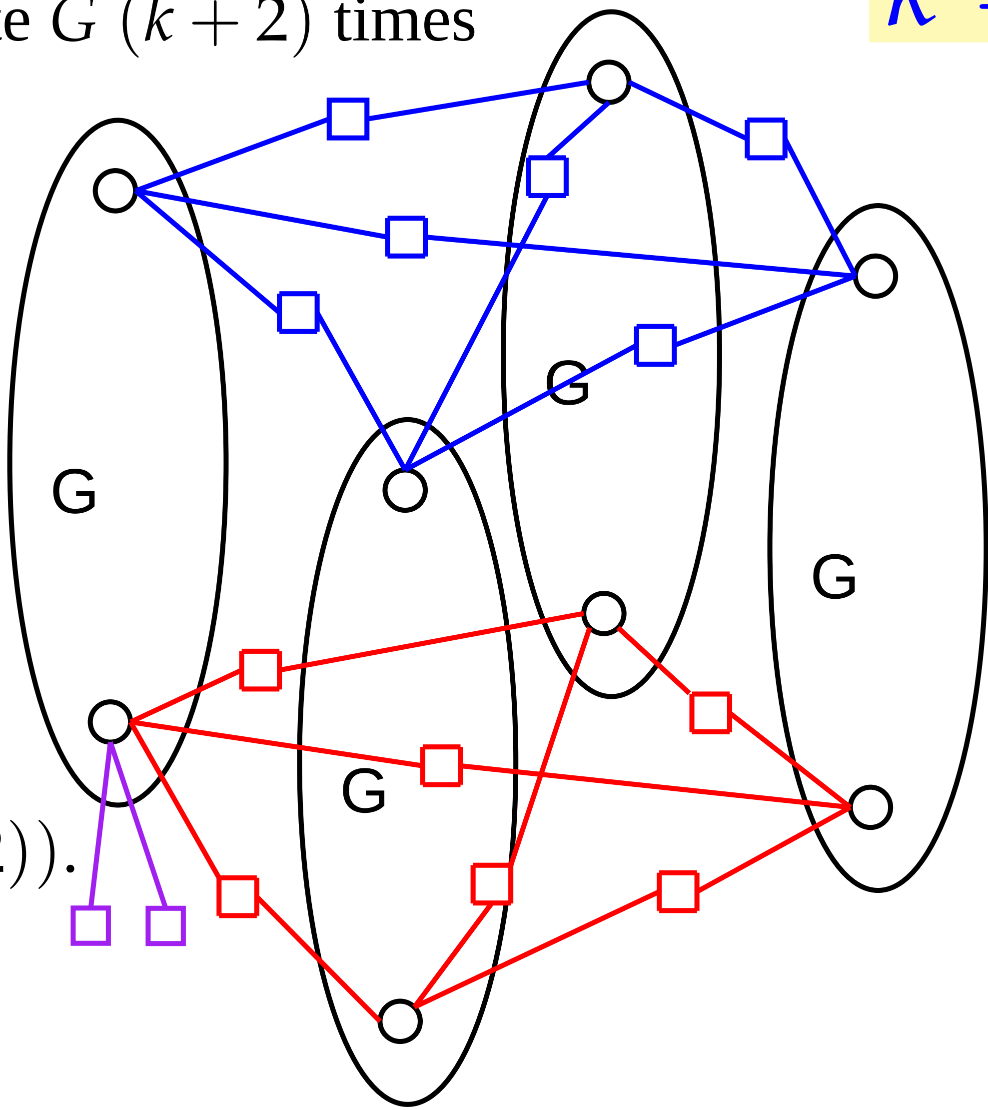

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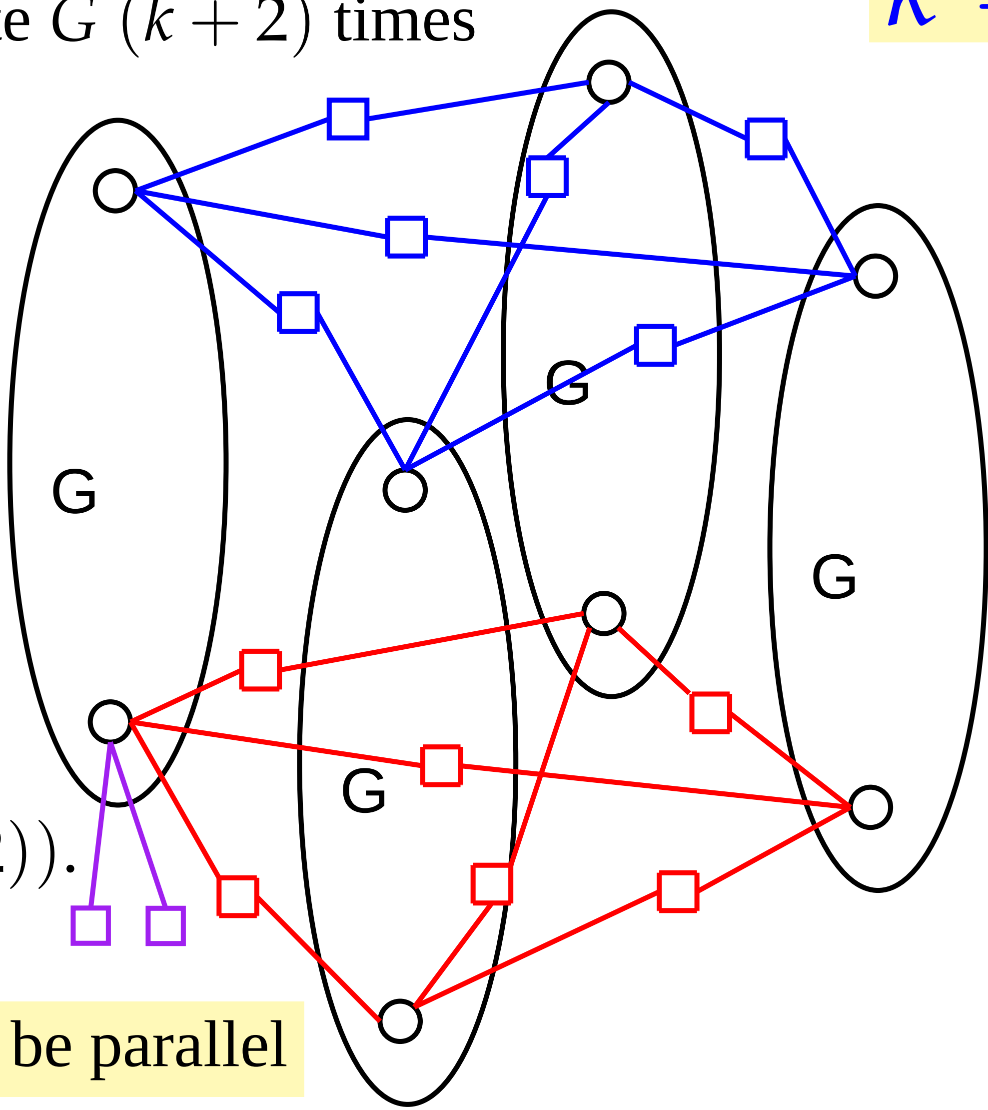

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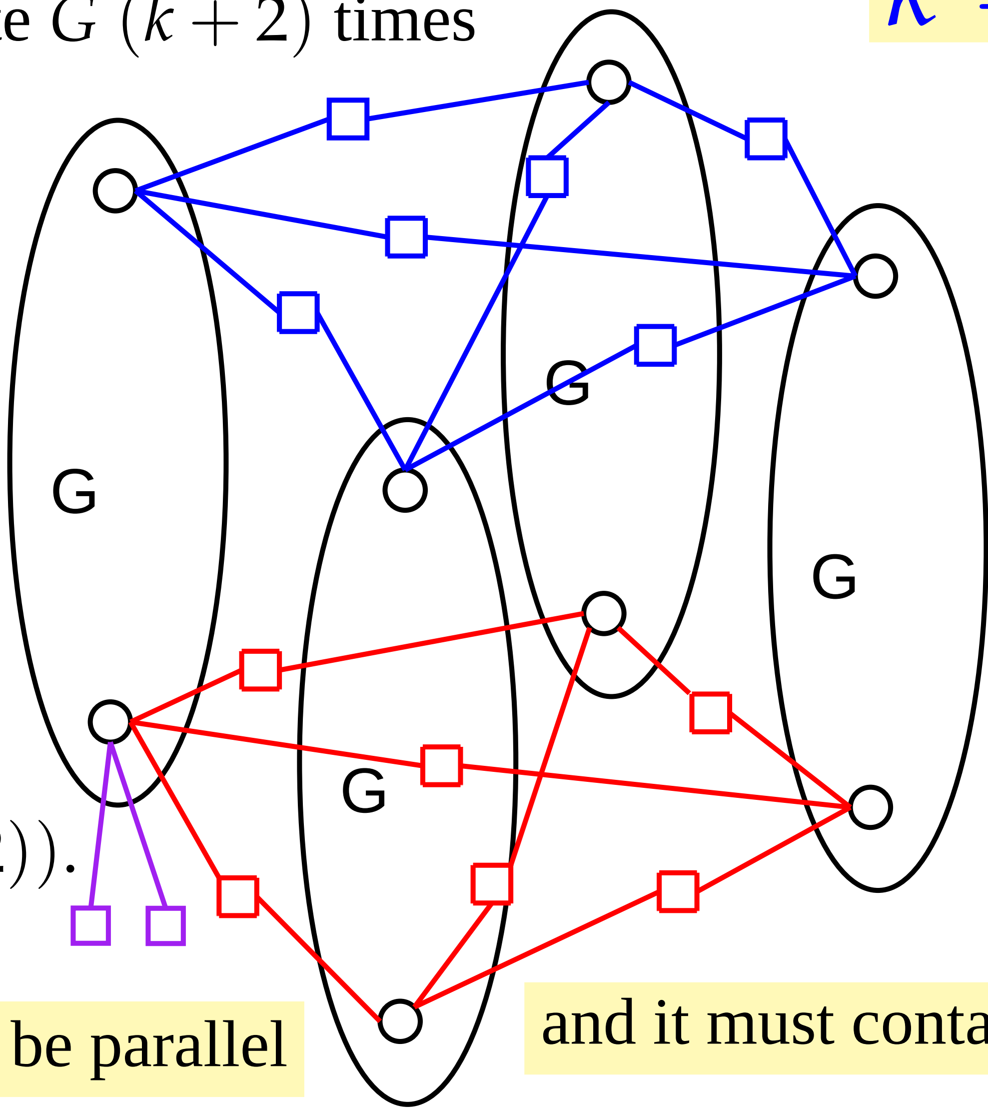

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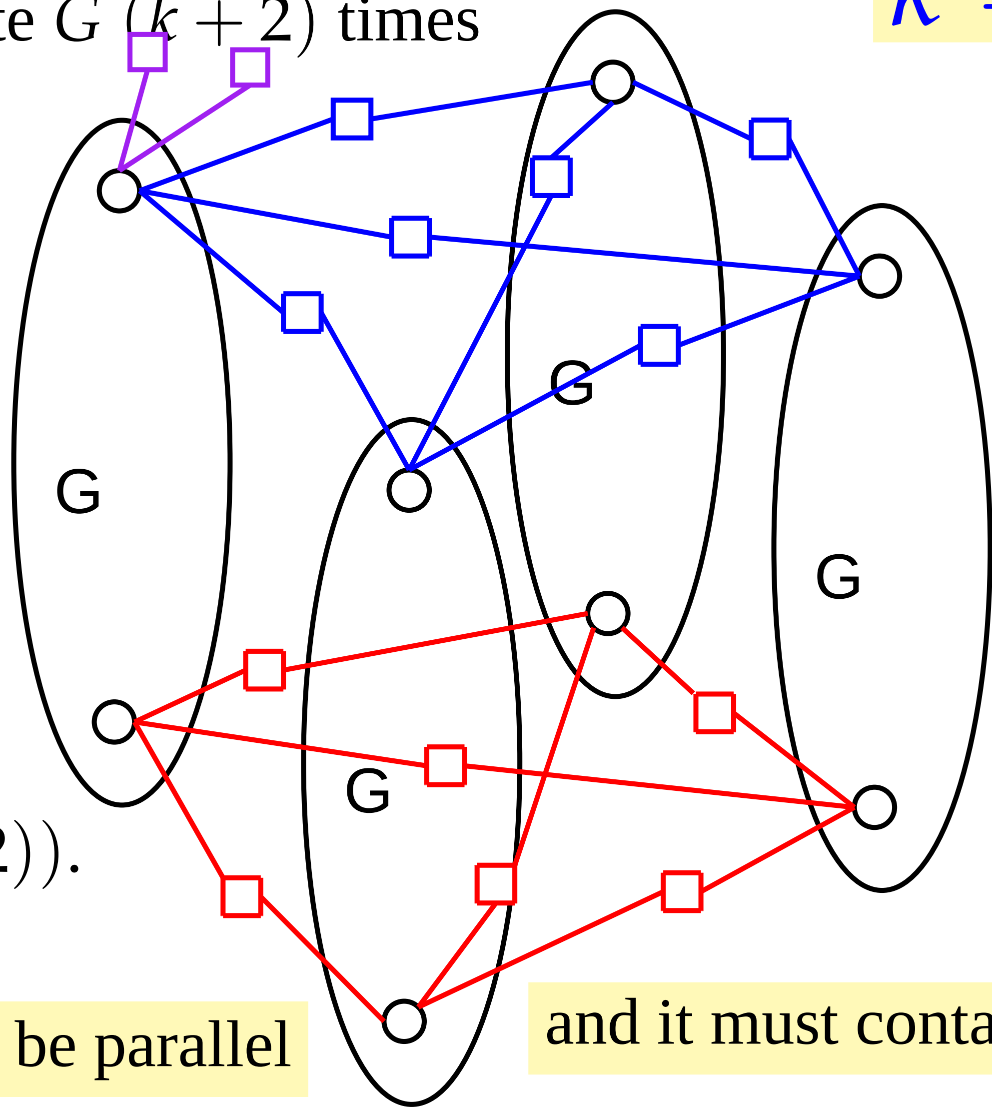

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- An encouraging example: The travelling salesman problem. Optimal solution for 24,978 cities in Sweden is found in 2004.

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=== The SD(H, t) problem ===

OUTPUT: Output an exhaustive list of minimum stopping sets if the minimal stopping distance is $\leq t$. Otherwise, output $\emptyset$.

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k-OTD(H, t') By SD(H, t)

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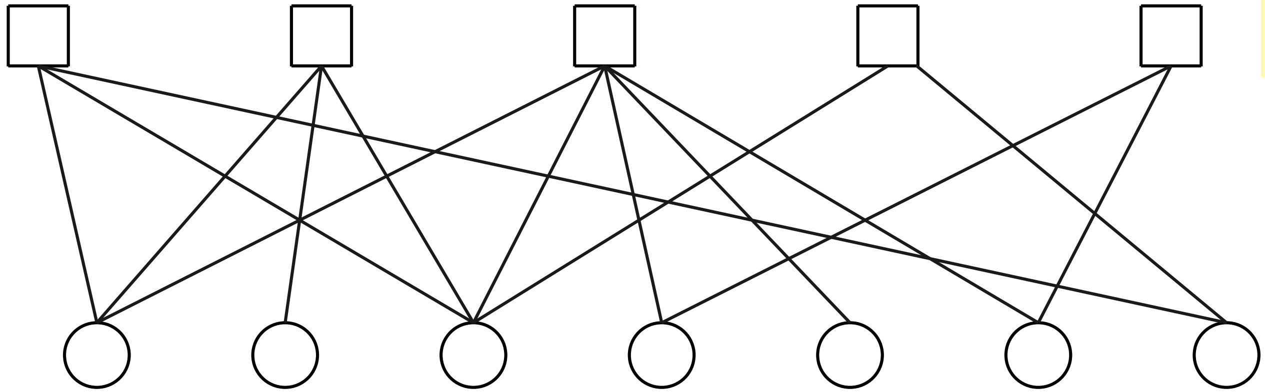

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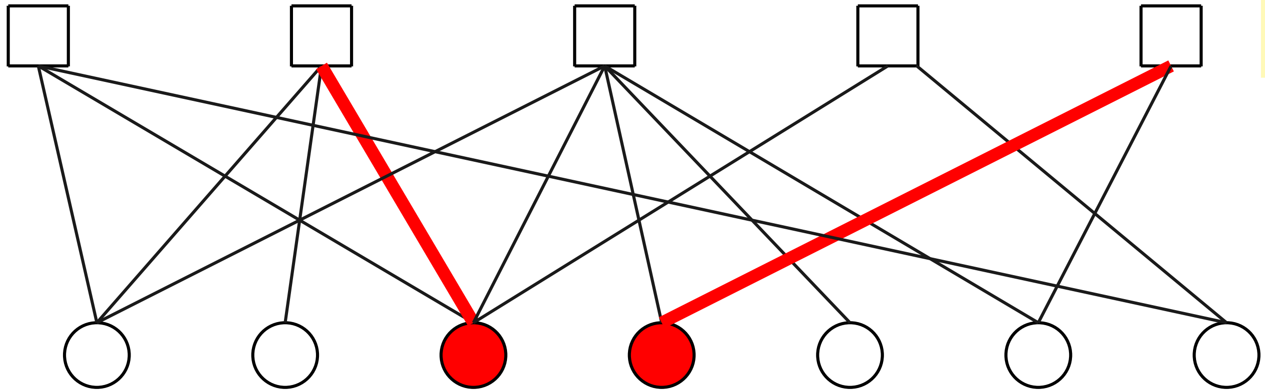

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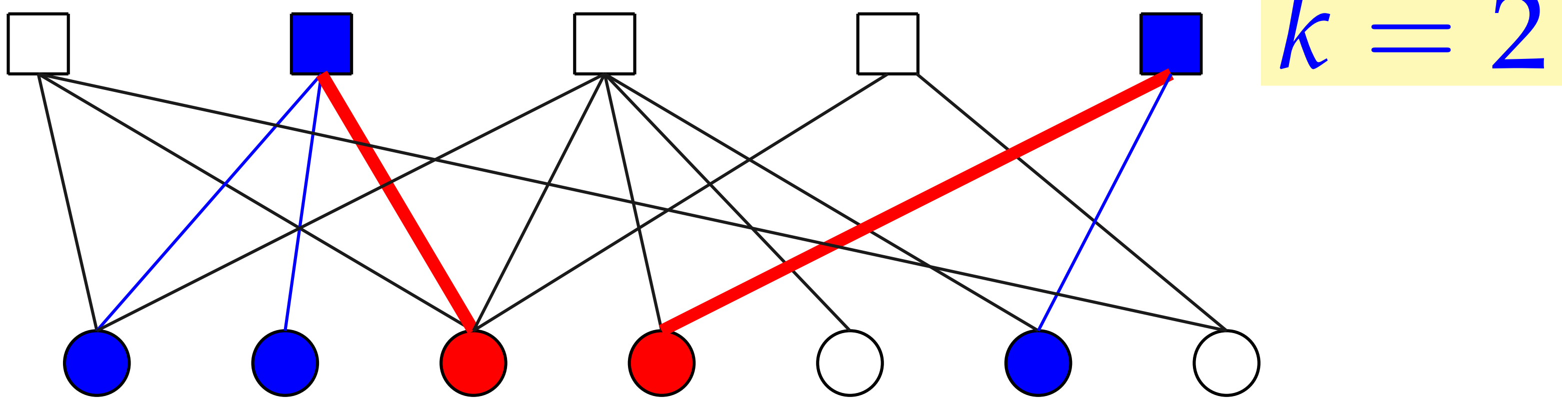

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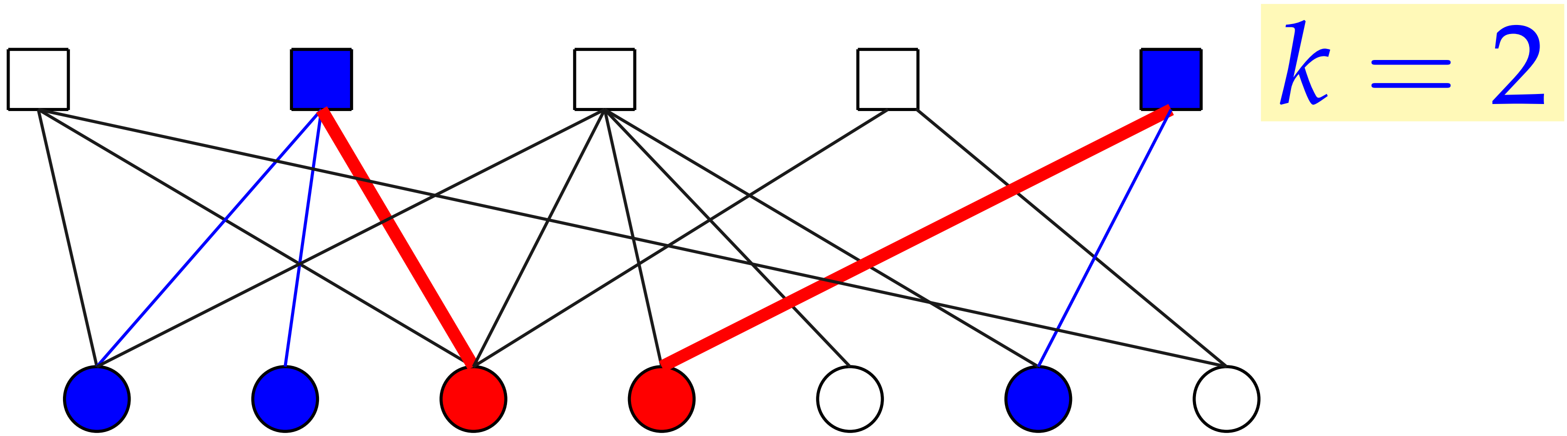

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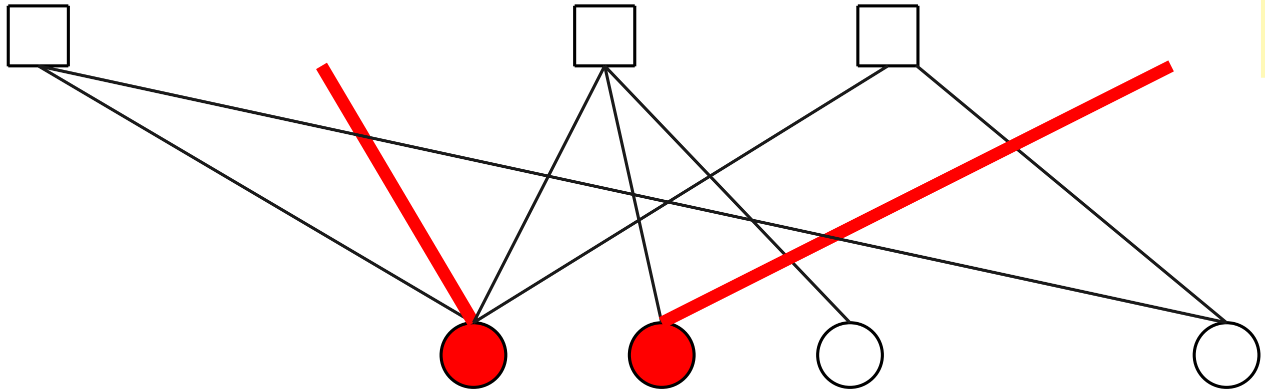

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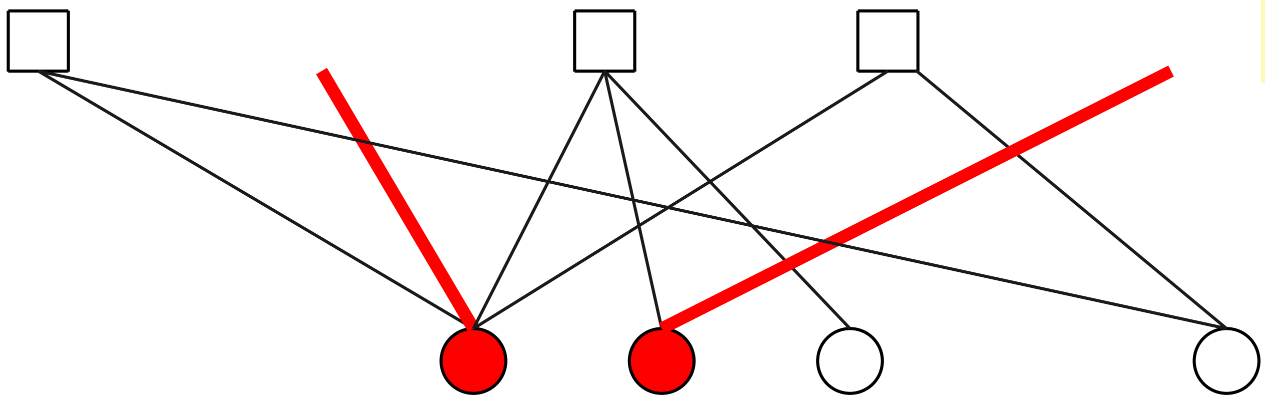

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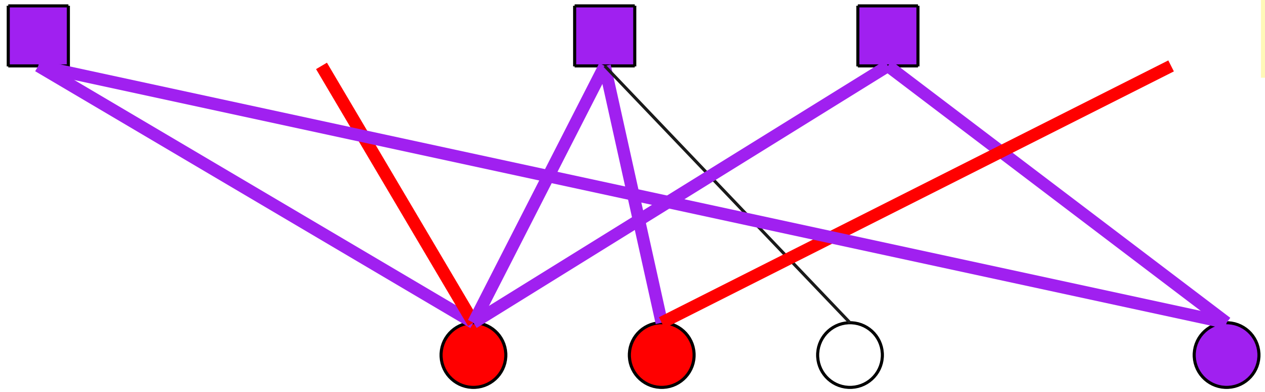

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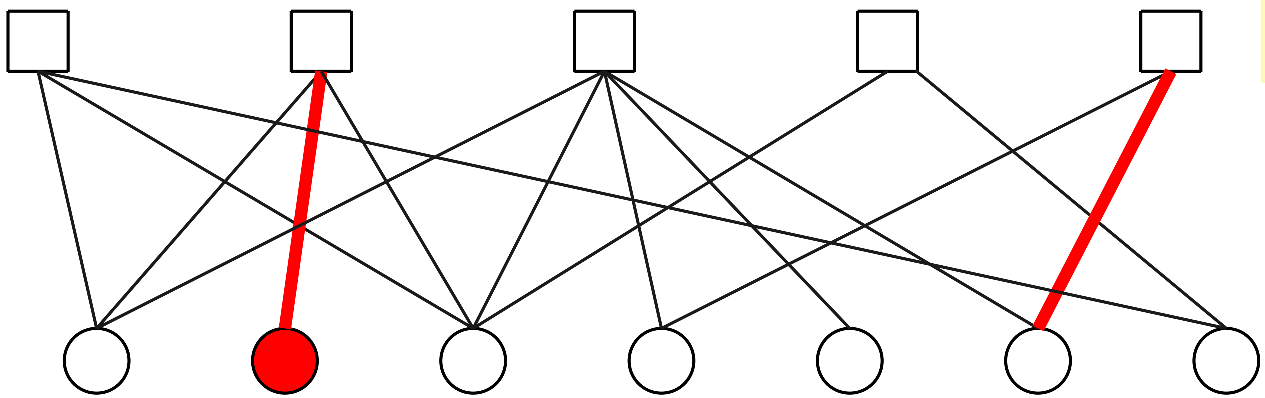

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5. Select another k edges and repeat the procedure.

Empirical Study of k-OTD(H, t)

- Complexity grows $O(n^k)$.

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- When $n \approx 500$ and rate $\frac{1}{2}$ codes, $t = 10$–12 for 1-OTS(H, t). $t = 9$–11 for 2-OTS(H, t), based on our SD(H, t).

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- **Tanner (155,64,20) code** 04: Minimal 1-out TD ≥ 12, and minimal 2-out TD $= 8$ w. multiplicity 465. All from the following by automorphisms [Tanner *et al.* 04].

$$7, 17, 19, 33, 66, 76, 128, 140$$
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$$1, 19, 63, 66, 105, 118, 121, 140$$
$$44, 61, 65, 73, 87, 98, 137, 146$$
$$31, 32, 37, 94, 100, 142, 147, 148.$$

Empirical Study of k-OTD(H,t)

- **Ramanujan-Margulis (2184,1092) Code** w. q = 13, p = 5 [Rosenthal *et al.* 00];
- Inexhaustive results — upper bounds: analytical search [Mackay *et al.* 03], error-impulse search [Hu *et al.* 04]

$$\text{Minimum Hamming distance} \leq 14$$

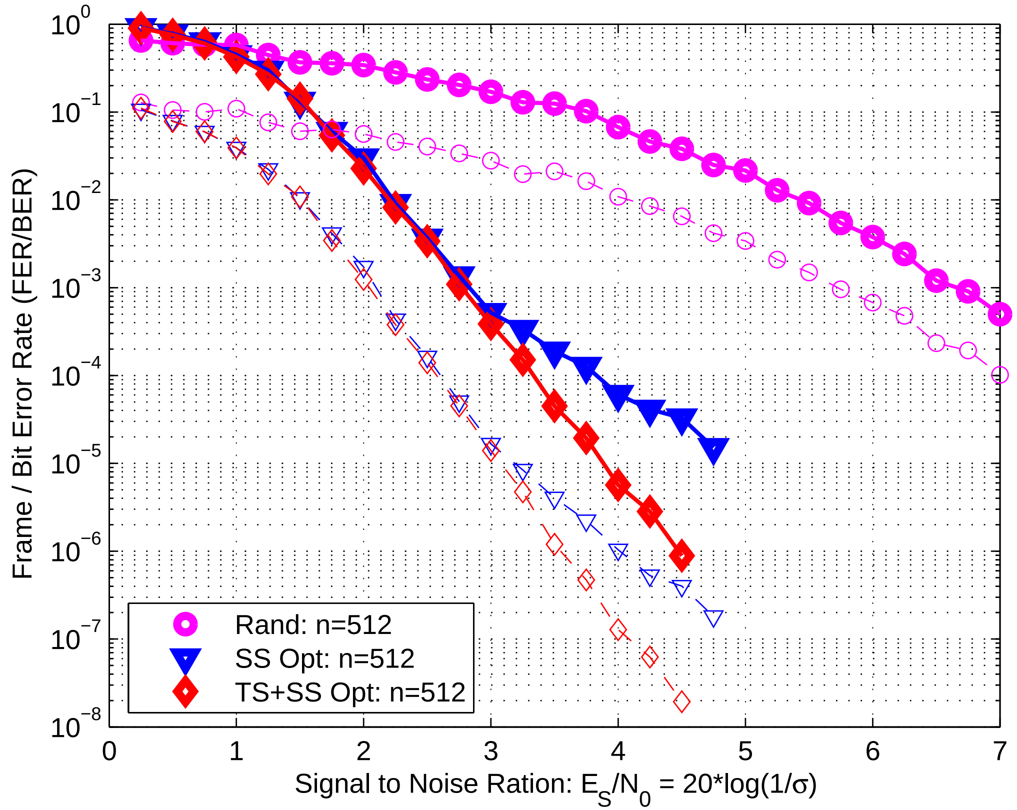
- Exhaustive results by SD(H,t) — lower bounds:

$$\text{Minimum Hamming distance} \geq \text{minimum SD} \geq 14$$
$$\text{multiplicity } 1092$$

Min. 1-out TD $\geq$ 13 and min. 2-out TD $\geq$ 10.

Impact on Error Floors

$$\lambda(x) = 0.31961x + 0.27603x^2 + 0.01453x^5 + 0.38983x^6, \;\; \rho(x) = 0.50847x^5 + 0.49153x^6$$



AWGN, $(\lambda(x), \rho(x))$, $n = 512$, 0-out/1-out trapping sets.

"Rand" $(2,1)$, $(2,8)$; "SS Opt" $(13,40)$, $(5,4)$; "SS+TS Opt" $(11,12)$, $(10,24)$.

Sum-product decoder, 80 iterations, 100 frame errors.

Insufficiency of TSs

- The relationship to error floors.
  - $n = 504$ Girth-optimized Irregular PEG code [Hu *et al.* 05], 1-out TSs of size 7 :

    52, 53, 122, 136, 178, 229, 348

    5, 42, 100, 131, 187, 199, 374

  - $n = 504$ TS-optimized irregular code w. the same deg. distr., 0/1-out TSs: $(10, 7)$/$(8, 40)$.

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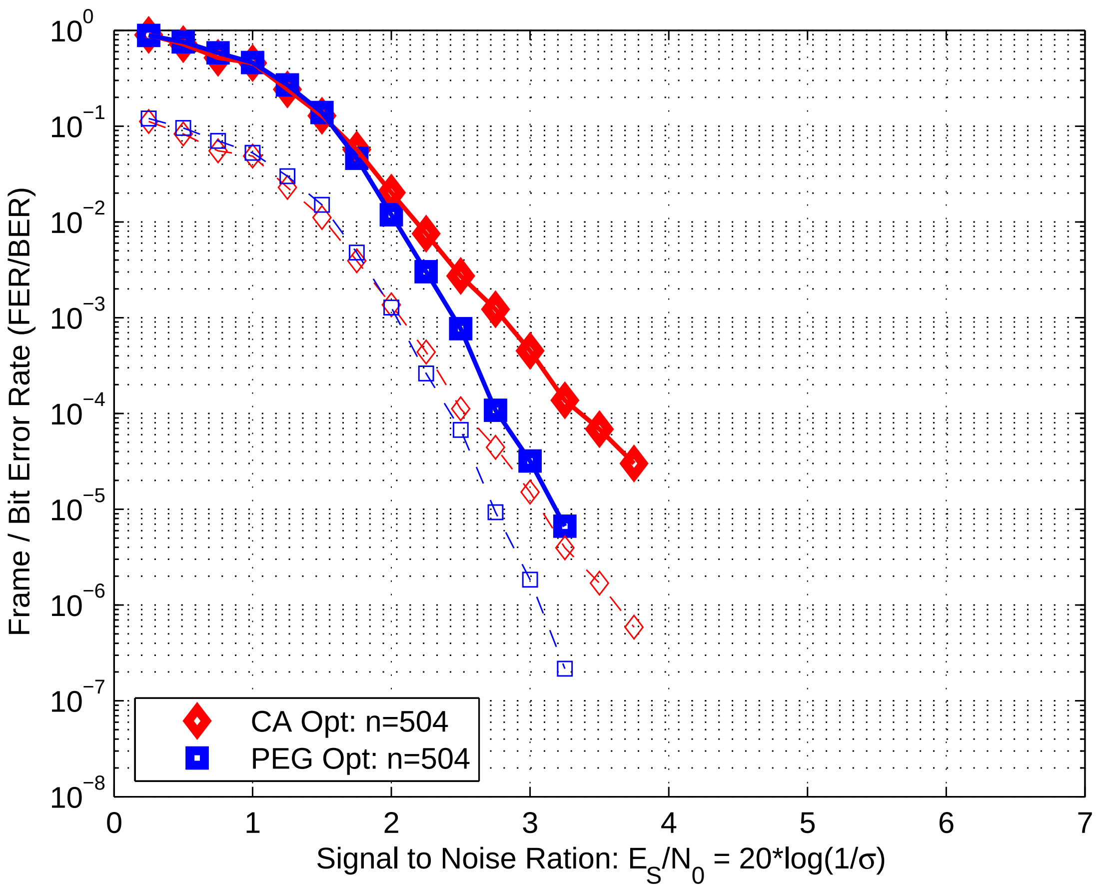
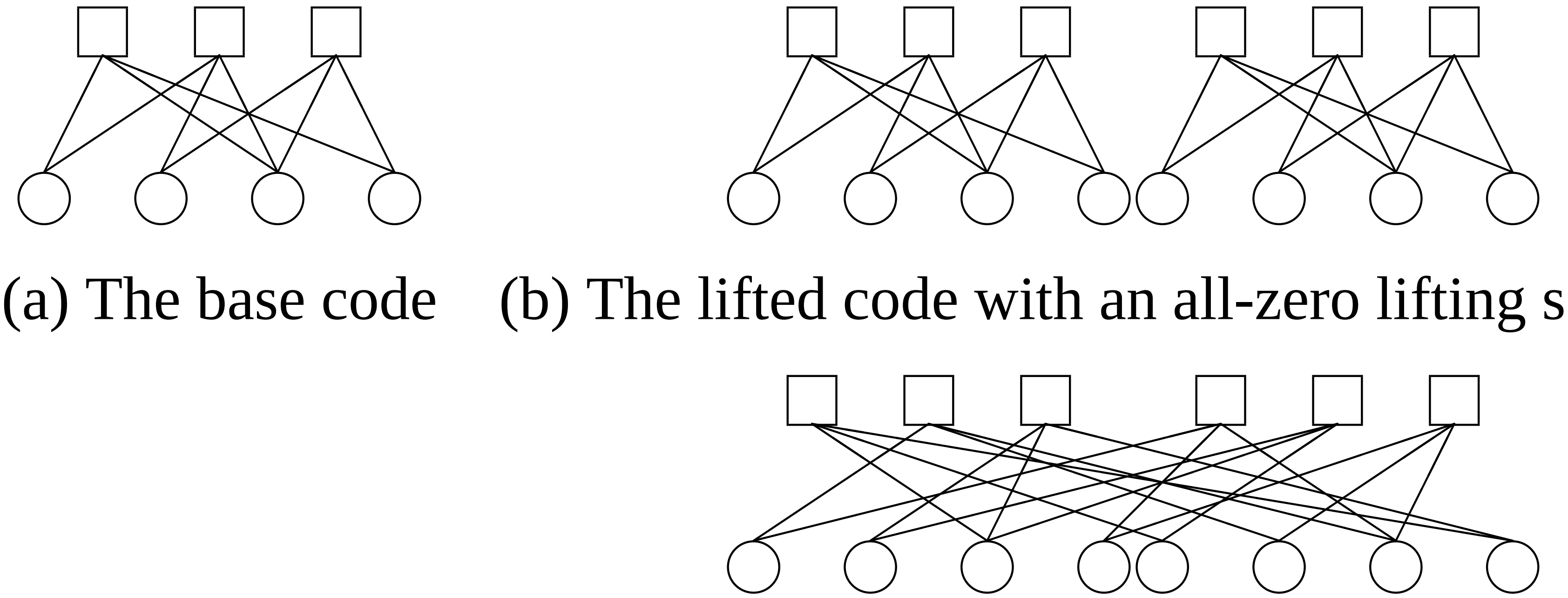


The Cyclically Lifted Ensemble

[Gross 74], [Richardson & Urbanke] and many more.



(a) The base code (b) The lifted code with an all-zero lifting sequence

(c) The lifted code with a cyclic lifting sequence.

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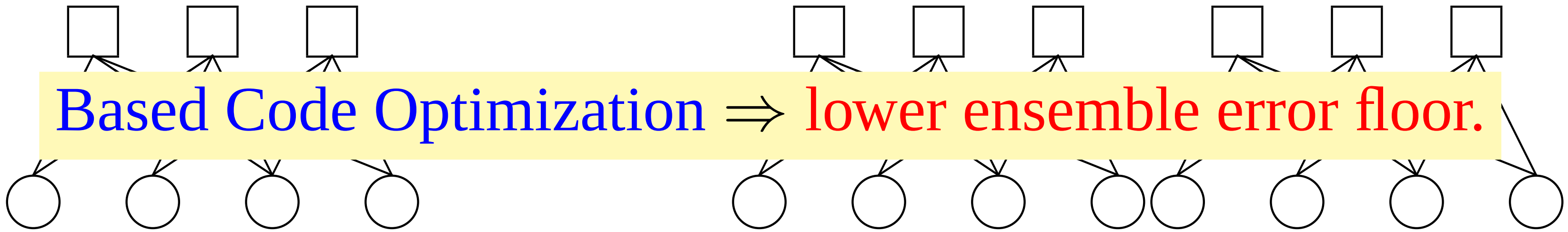

Based Code Optimization $\Rightarrow$ lower ensemble error floor.

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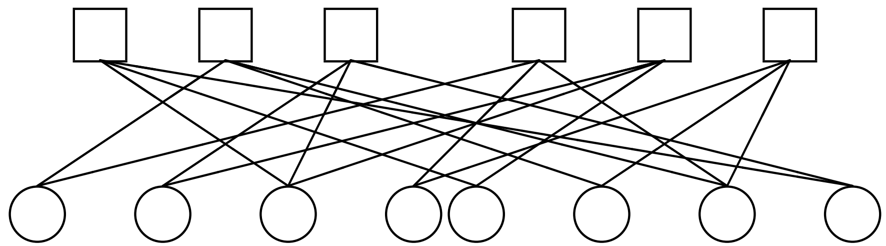

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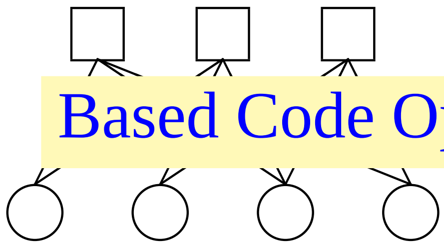

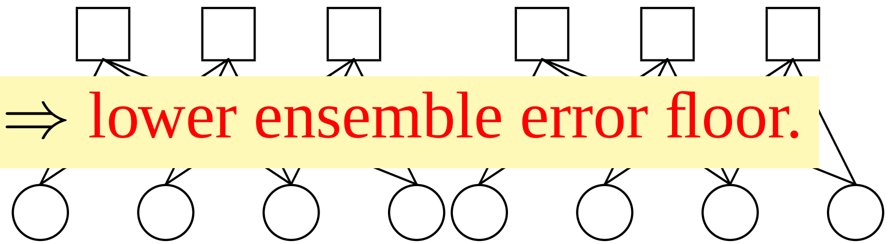

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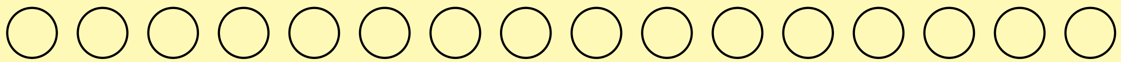
Base Code — of size n ($n = 16$)



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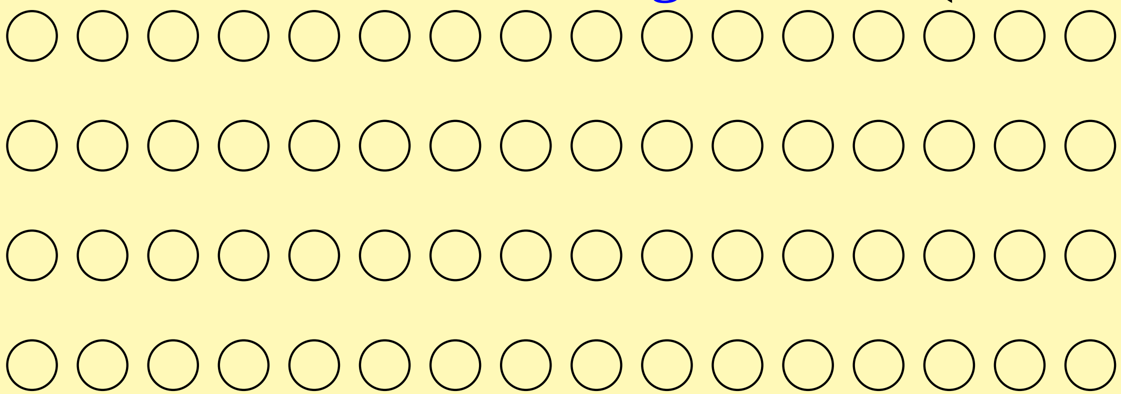

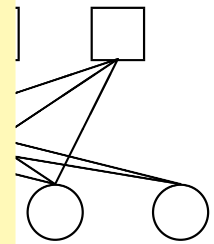

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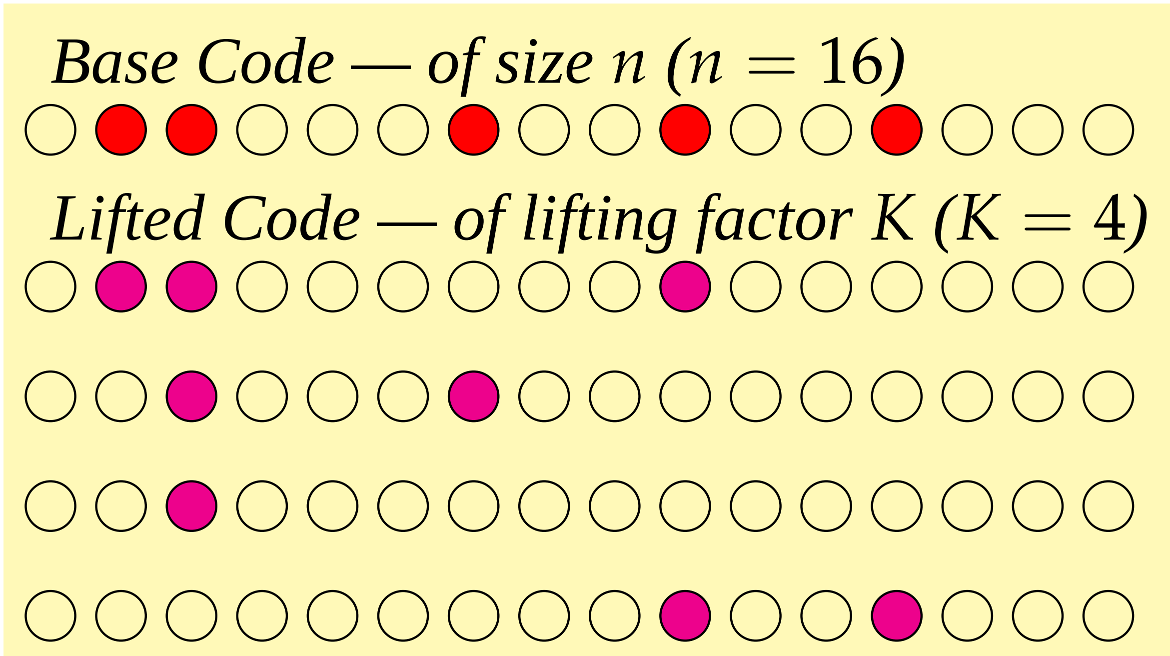
Survival of Trapping Sets

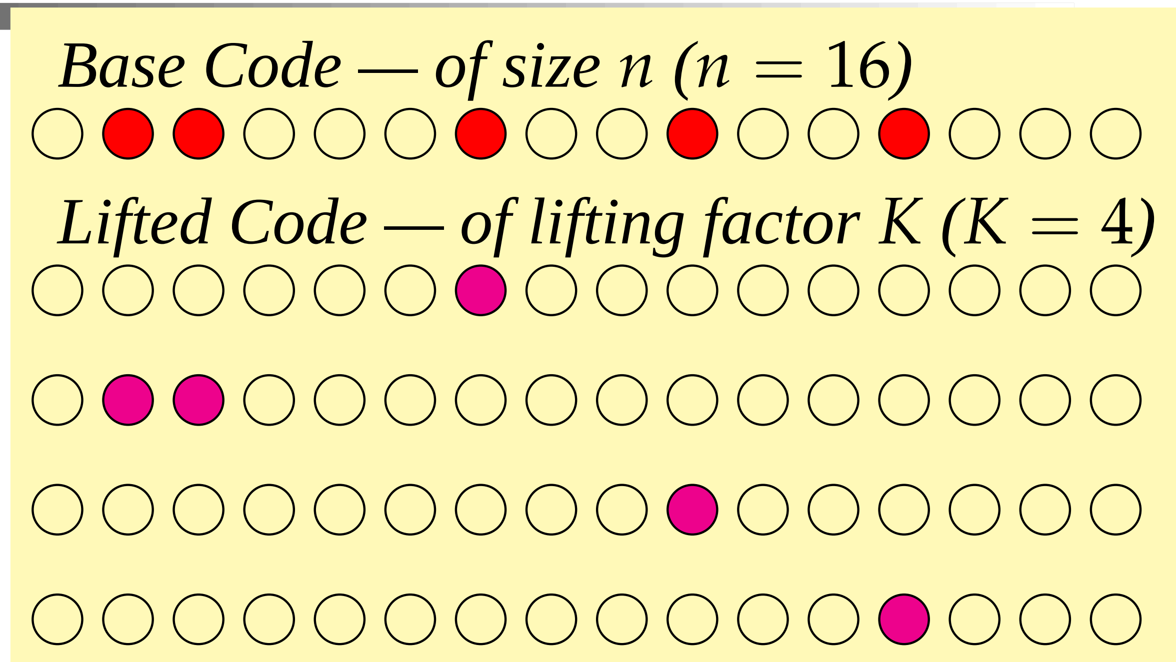
Theorem 3 *If ● forms a k_L-out trapping set for one lifted code, then ● forms a k_B-out trapping set for the base code where $k_L \geq k_B$.*



Different Orders of Survivals

Definition 2

First order survivals



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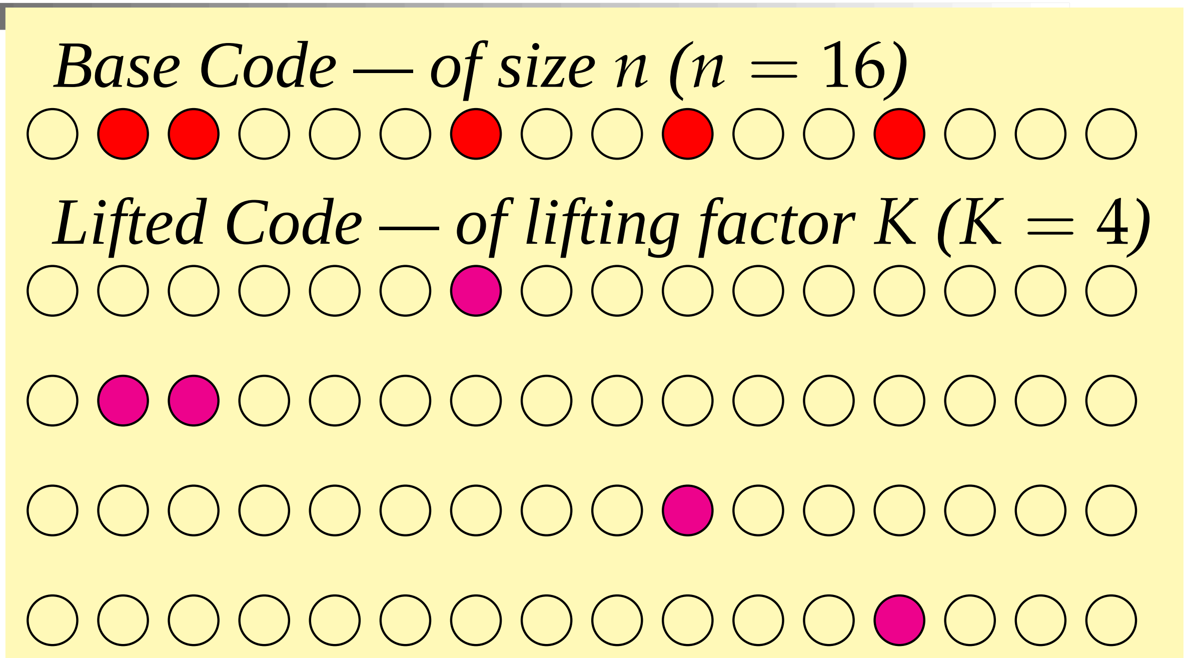

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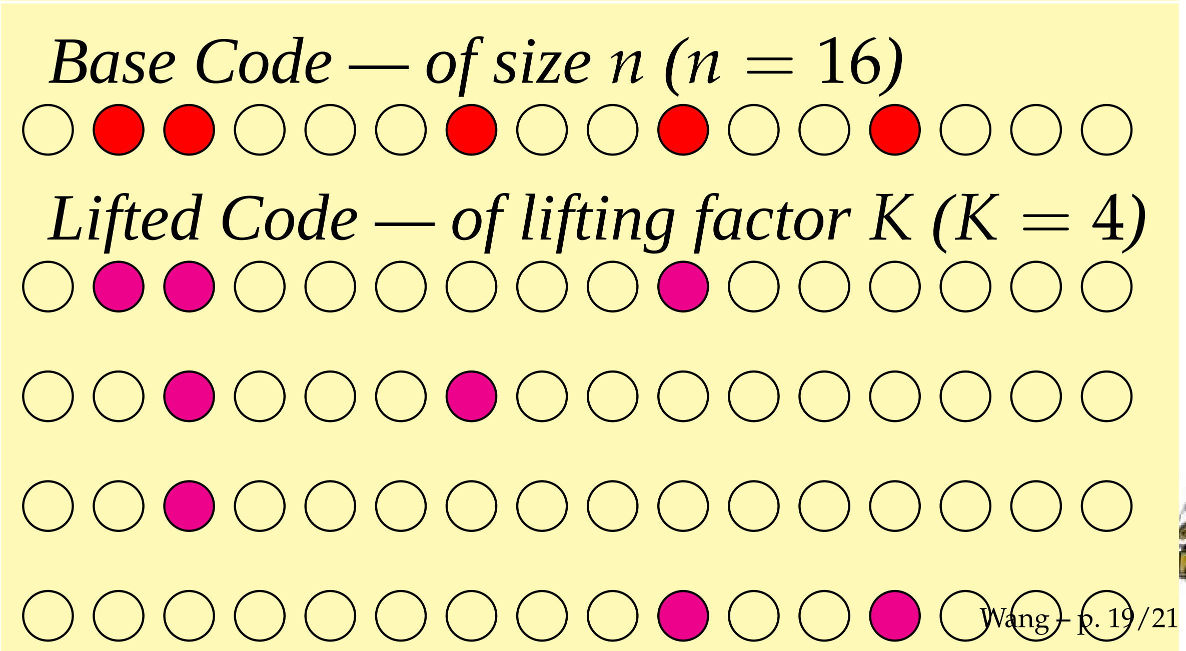

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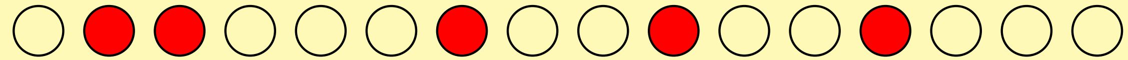

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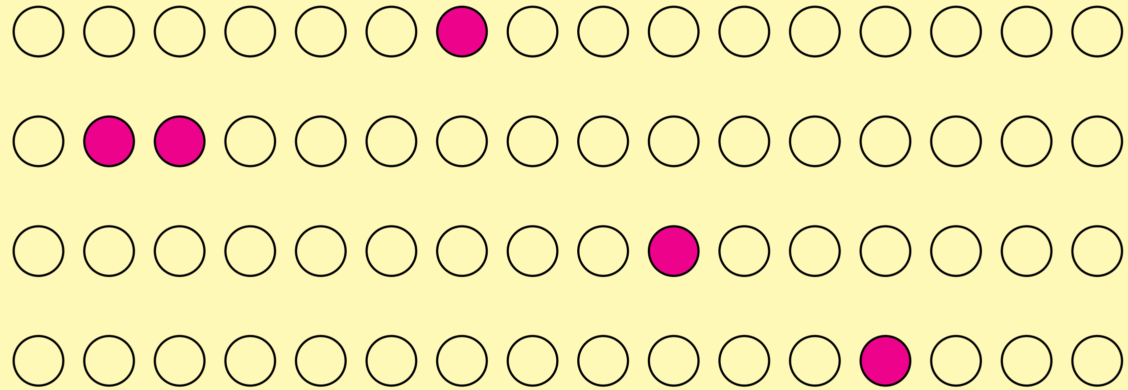

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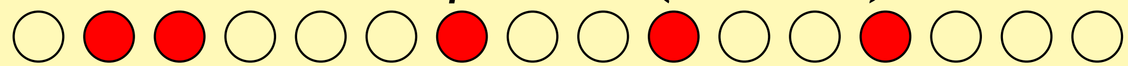

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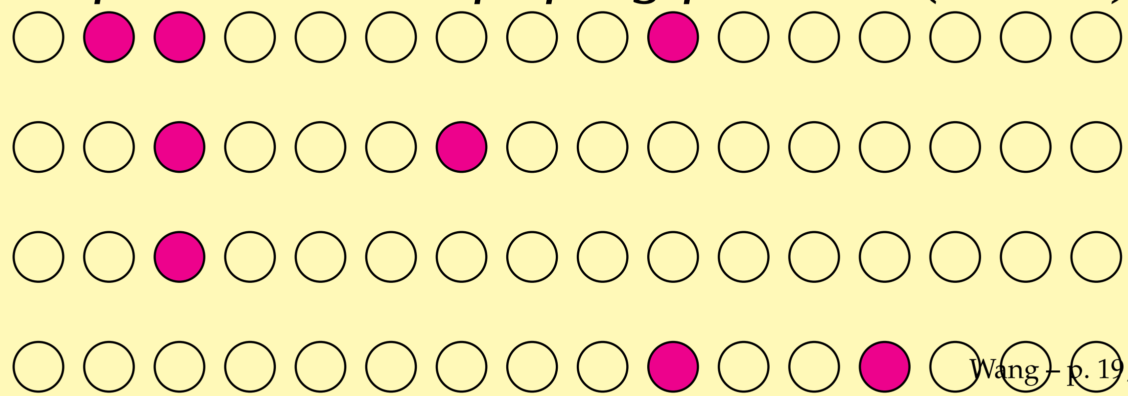

Empirically, almost all small trapping sets are of first order.

[Wang 06, Ländner 05]

First order survival

Theorem 4 ($k_L = k_B = 0$, a preliminary result) *For a fixed base code with a min. stopping set* $\mathbf{s}_B$,

$$\mathsf{E}\{|\textit{first order survivals}|\} \propto K^{-(0.5\#E - \#V + 0.5\#C_{\mathrm{odd},\geq 3})}$$

$$FER_{BEC,ensemble} = \mathsf{const} \cdot K^{-(0.5\#E - \#V + 0.5\#C_{\mathrm{odd},\geq 3})}.$$

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First order survival

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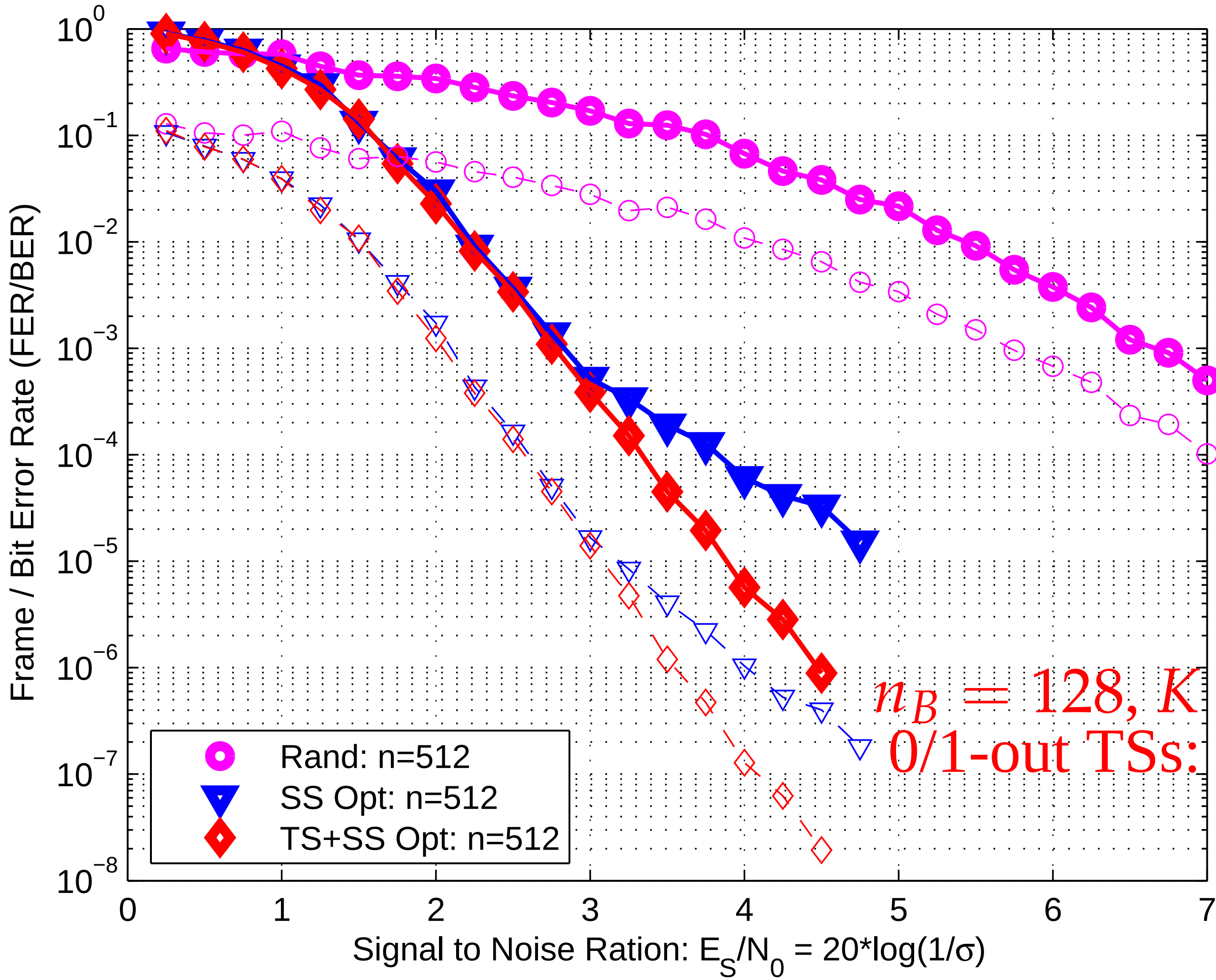

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