Capacity Regions for Multiple Unicasts Flows using Inter-session Network Coding

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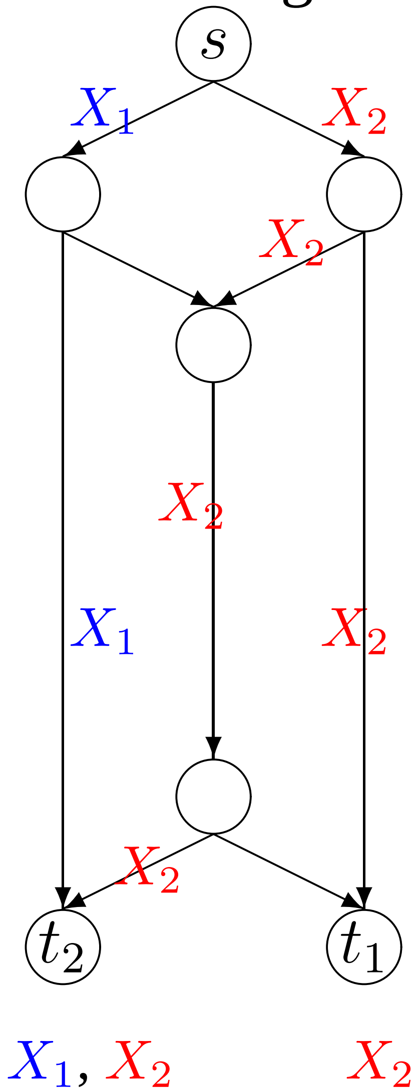
Single Session

The Routing Solution



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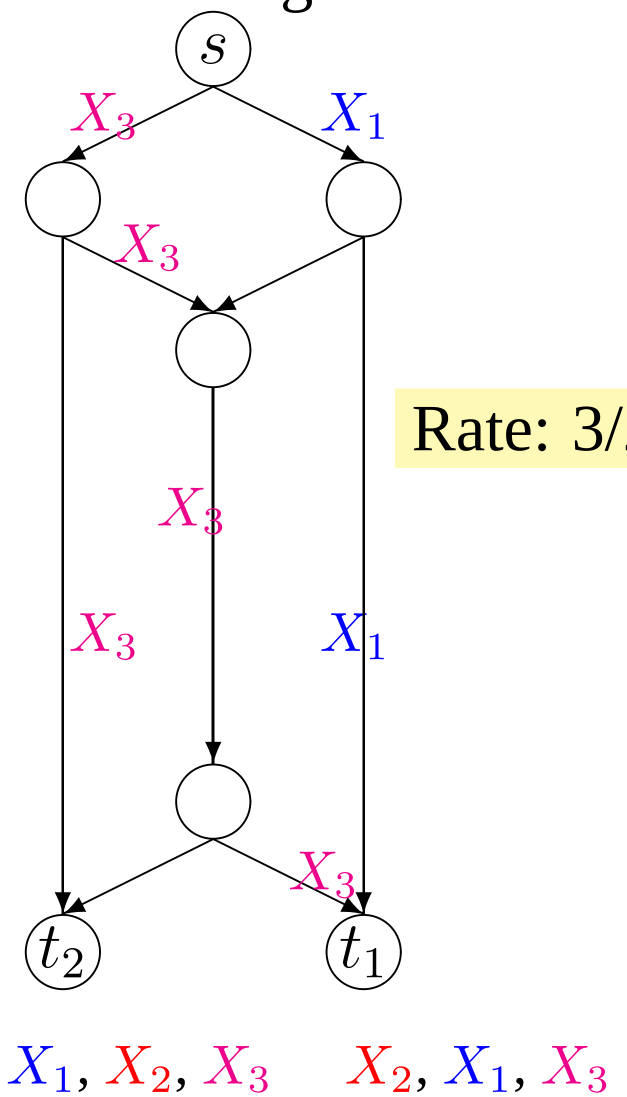

Rate: 3/2

Single Session

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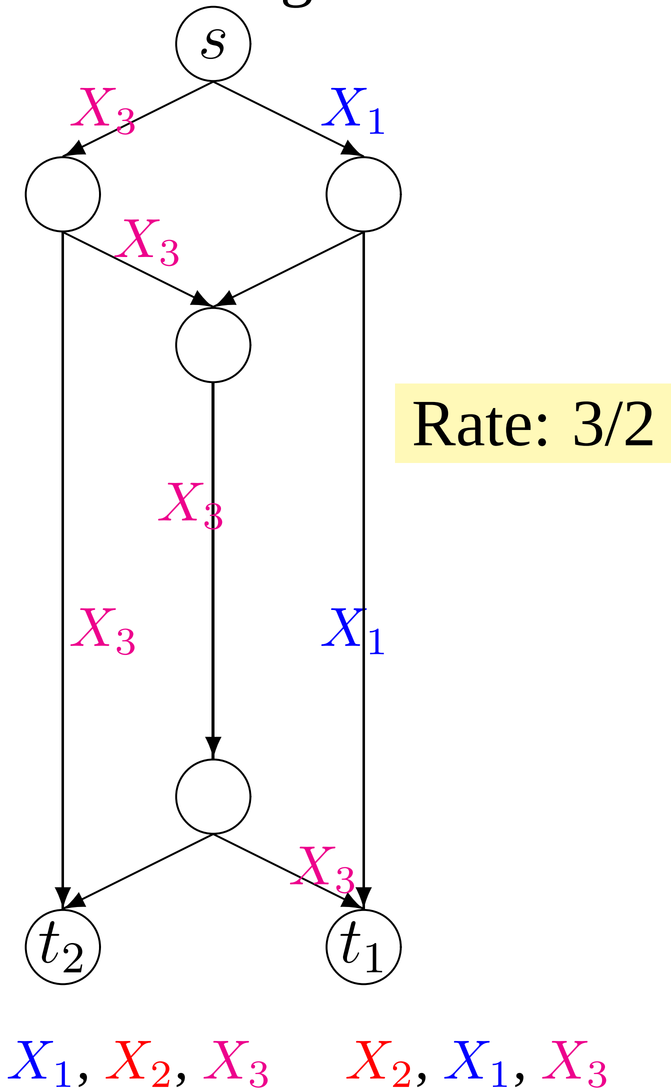
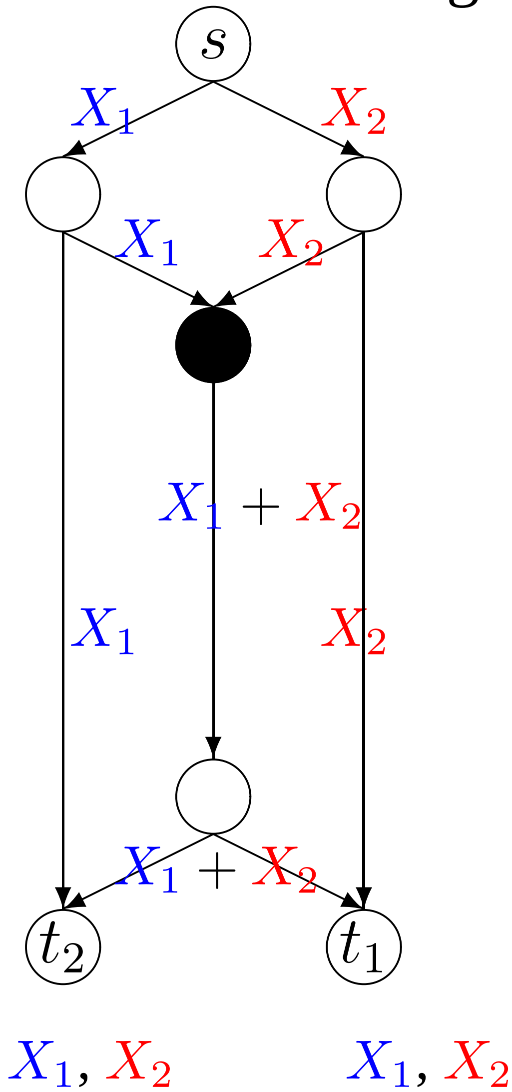


The Network Coding Solution



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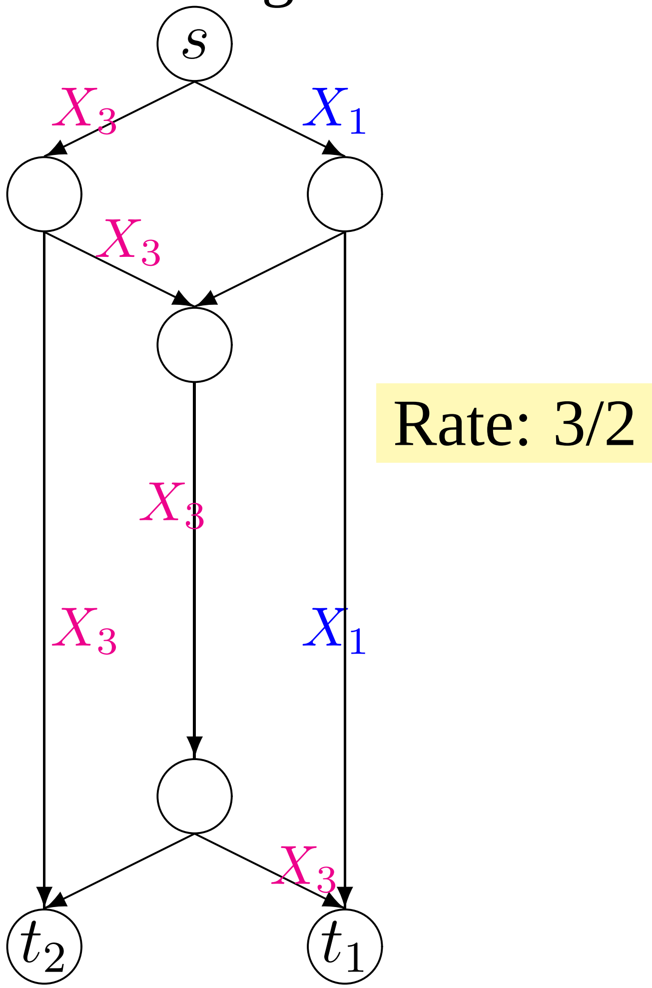

X_1, X_2, X_3 X_2, X_1, X_3

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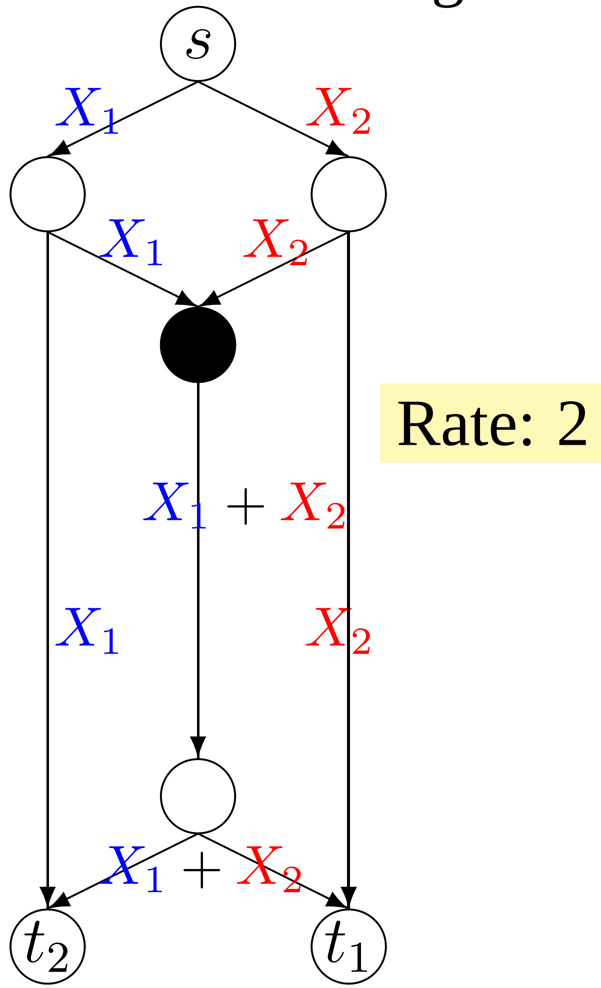

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Multicast

Theorem 1 *[Ahlswede et al. 00] For a single multicast session, rate R is achievable if for all dest. t_i, the min-cut/max-flow $\rho_G(s, t_i)$ between s and t_i satisfies*

$$R \leq \rho_G(s, t_i), \ \forall i.$$

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Intra-session Mutlicast [Chen *et al.* 07]

$$\max_{R_i} \quad \sum_i U_i(R_i)$$

subject to $\sum_i f_{i,e} \leq c_e, \ \forall e \in E$

$\forall i$, $\{f_{i,e}\}_{e \in E}$ and R_i satisfy the min-cut max-flow conditions.

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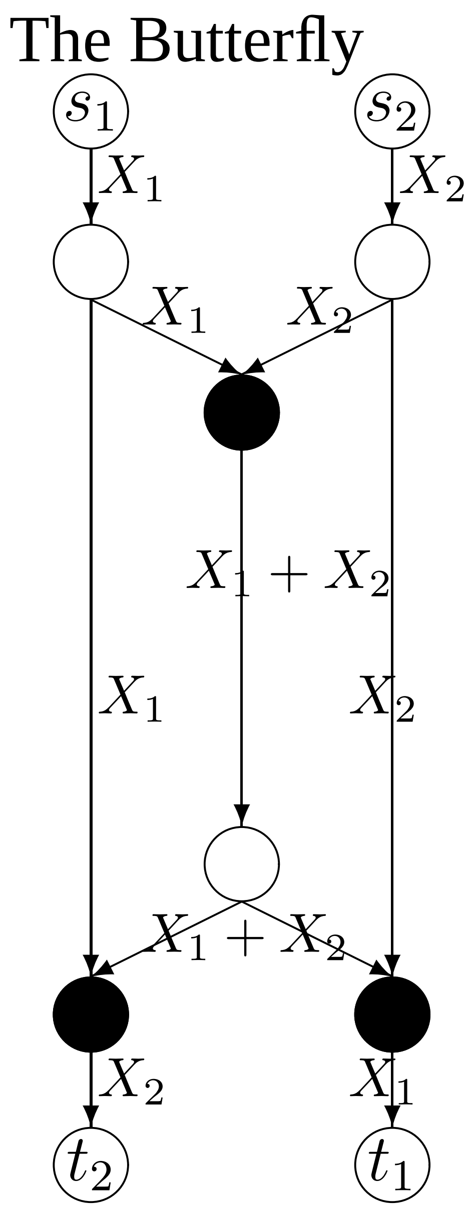
- One possible formulation

$$\sum_{n \in \Gamma_O(g)} x_n(i) - \sum_{n \in \Gamma_I(g)} x_n(i) = \begin{cases} R_i & g = s_i \\ -R_i & g = t_i \\ 0 & \text{else} \end{cases} \qquad (1)$$

$$\sum_{i=1}^{I} x_n(i) \leq C_n \qquad \forall n \in E \qquad (2)$$

Two simple unicasts



The $\mathcal{TRLKM}$ region

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- Resolves butterfly bottlenecks in the network by introducing virtual flows p, q, and r.

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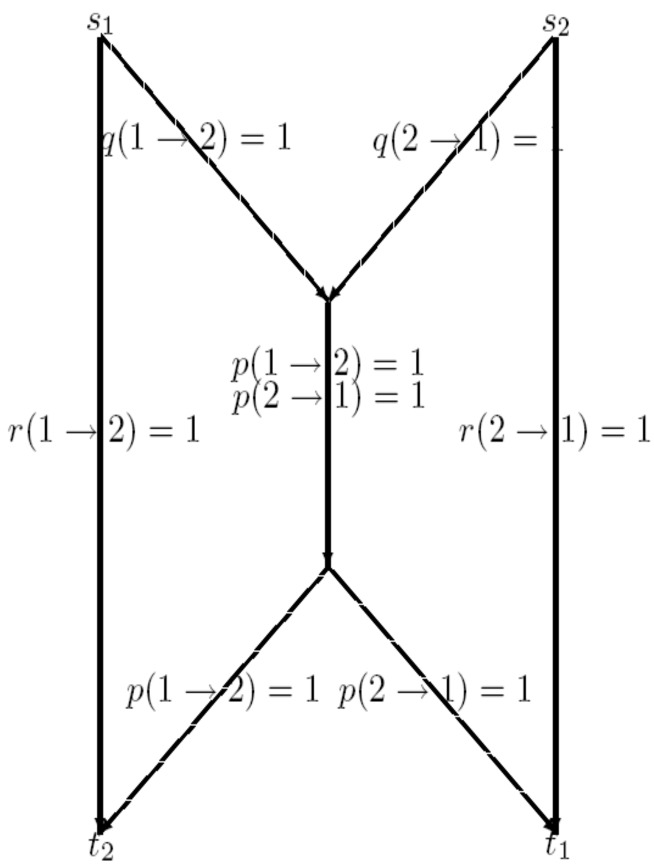

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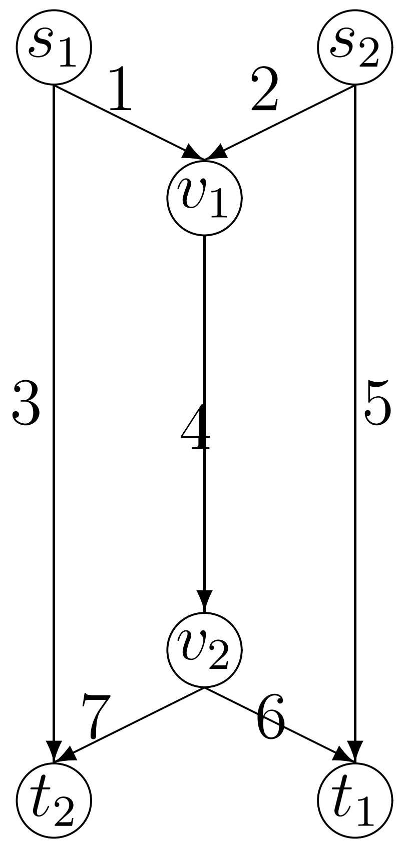
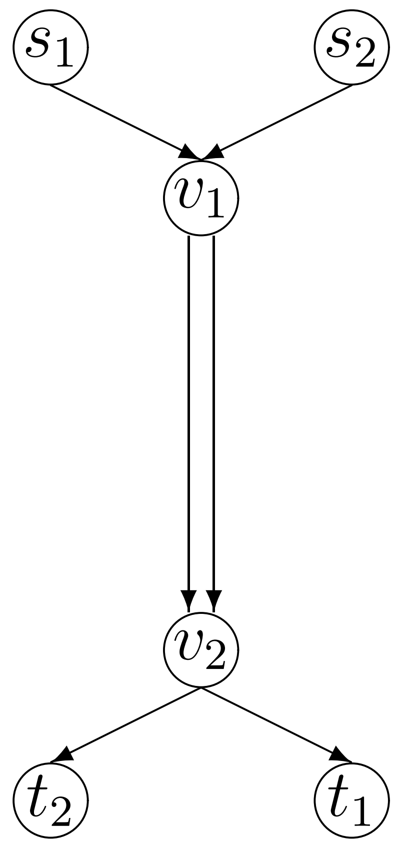
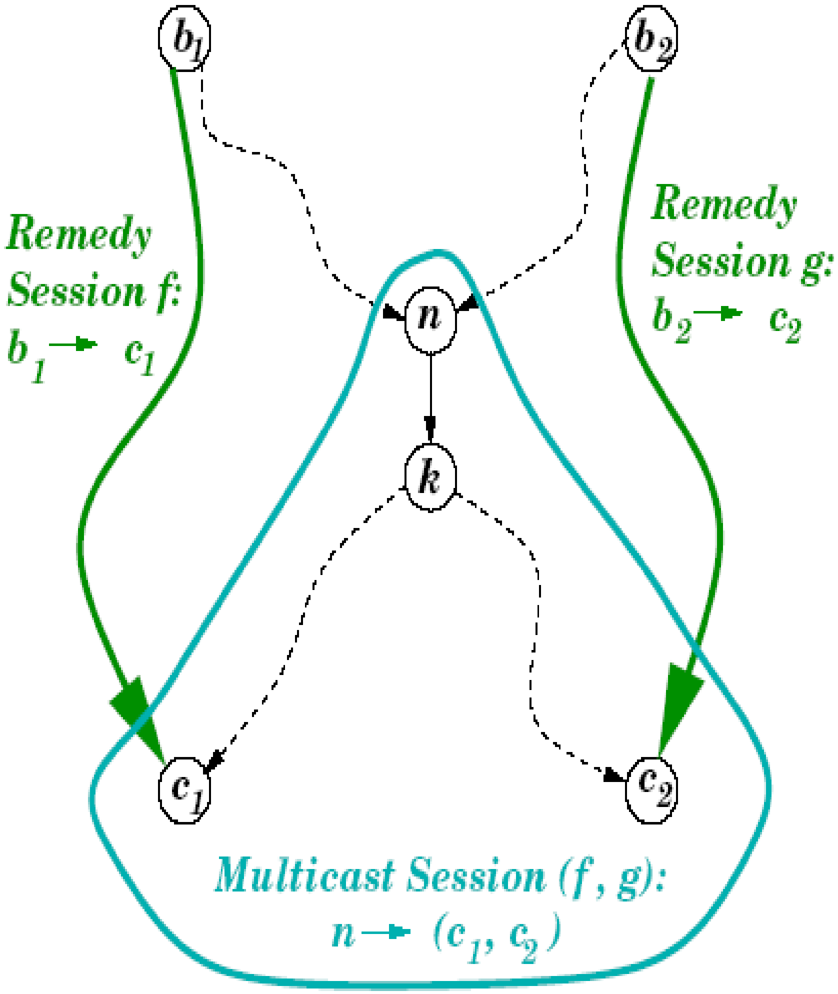




The $\mathcal{RSC}$ Algorithm



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- At each link (n, k) compute two weights by queue lengths exchange:
  - $\rho^*_{(n,k)[t]}$ corresponds to routing.

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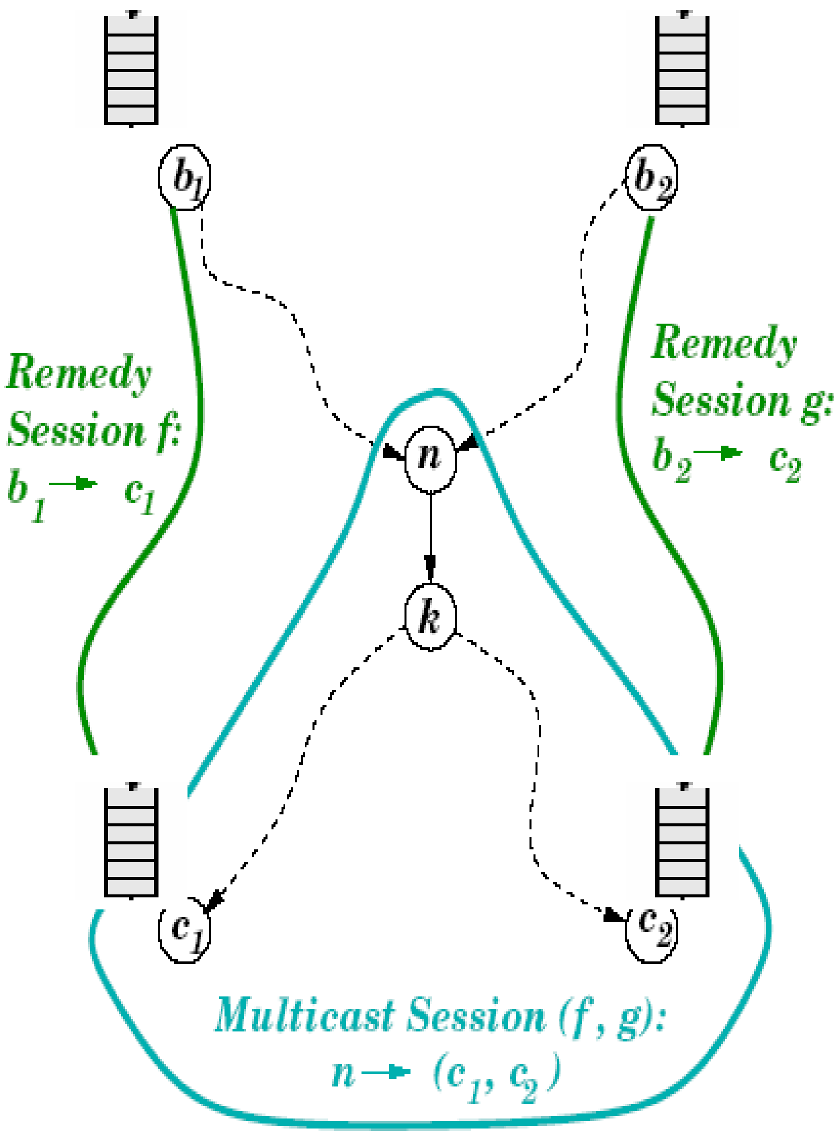

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  - $\sigma^*_{(n,k)}[t]$ corresponds to inter-session coding.

The $\mathcal{RSC}$ Algorithm cont.

- if $\rho^*_{(n,k)[t]} > \sigma^*_{(n,k)}[t]$ perform routing, otherwise do intersession coding.
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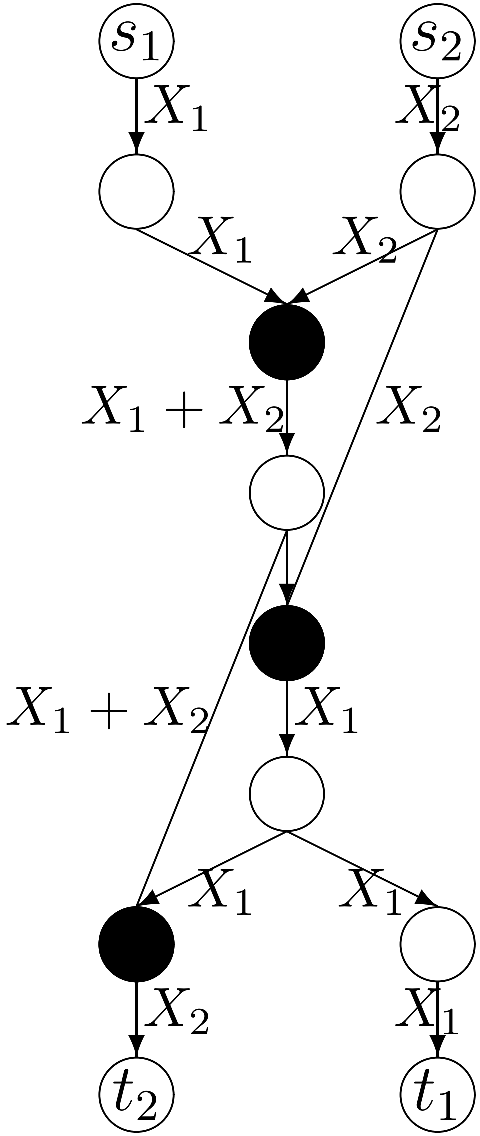
- Drawbacks:
  - High Complexity policy.
  - Coding is dependent on queuing
  - No rate control mechanism
  - Considers only butterfly coding opportunities.

Two simple unicasts

The Grail



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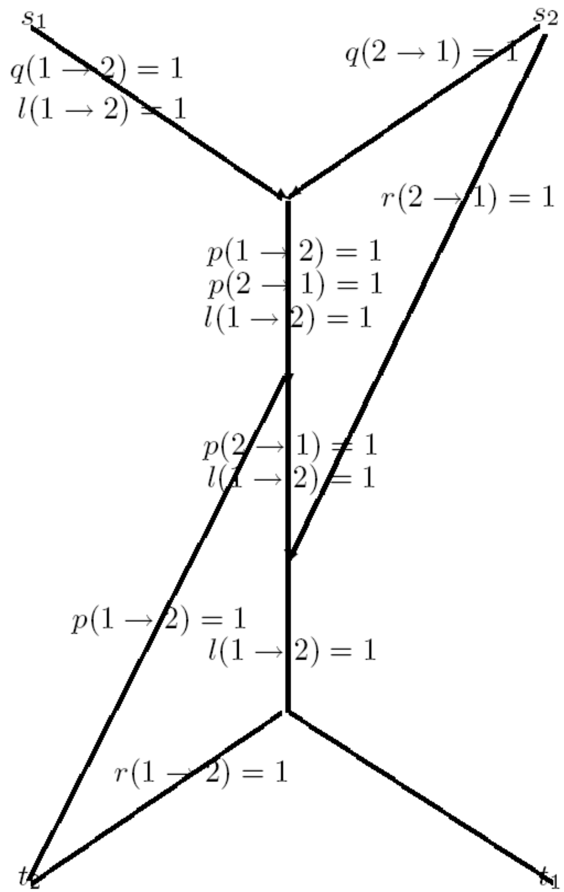

The $\mathcal{I} - \mathcal{TRLKM}$ region

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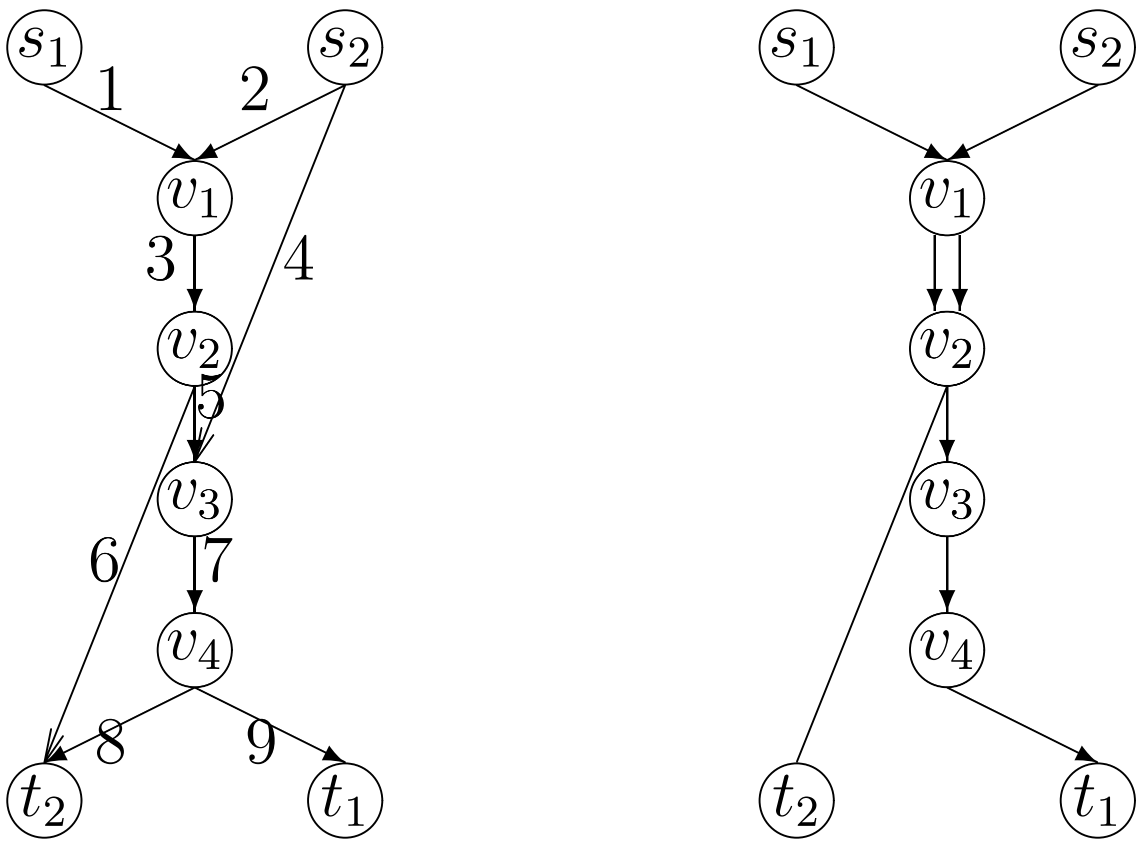

The $\mathcal{I} - \mathcal{RSC}$ Algorithm

- Compute three weights based on queue length exchange:
  - $\rho^*_{(n,k)}$ corresponds to routing.
  - $\sigma^*_{(n,k)}[t]$ corresponds to butterfly inter-session coding.
  - $\sigma^*_{1(n,k)}[t]$ corresponds to grail inter-session coding.
- If $\rho^*_{(n,k)}[t] = \max\{\rho^*_{(n,k)}[t], \sigma^*_{(n,k)}[t], \sigma^*_{1(n,k)}[t]\}$ perform routing.
- If $\sigma^*_{(n,k)} = \max\{\rho^*_{(n,k)}, \sigma^*_{(n,k)}[t], \sigma^*_{1(n,k)}[t]\}$ perform butterfly net coding.
- If $\sigma^*_{1(n,k)} = \max\{\rho^*_{(n,k)}, \sigma^*_{(n,k)}[t], \sigma^*_{1(n,k)}[t]\}$ perform grail net coding.

So Far

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- No rate control or utility maximization in the backlog algorithms.
- Coding is dependent on Queueing.
- Try path-based regions using inter-session network coding.

Preliminaries — 2 Unicasts

- Setting: General finite directed acyclic graphs, unit edge capacity, (s_1, t_1) & (s_2, t_2), two integer symbols X_1 and X_2.
- Number of Coinciding Paths of edge e: $\mathcal{P} = \{P_1, \cdots, P_k\}$, and $\mathsf{ncp}_{\mathcal{P}}(e) = |\{P \in \mathcal{P} : e \in P\}|$.

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Theorem 2 *Network coding $\Longleftrightarrow$ one of the following two holds.*

1. $\exists \mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}\}$, *such that*

$$\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 1.$$

2. $\exists \mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}, P_{s_2,t_1}\}$ *and* $\mathcal{Q} = \{Q_{s_1,t_1}, Q_{s_2,t_2}, Q_{s_1,t_2}\}$ *s.t.*

$$\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 2 \text{ and } \max_{e \in E} \mathsf{ncp}_{\mathcal{Q}}(e) \leq 2.$$

The $\mathcal{WS}$ Region

Represent the network G as a superposition of one G_r and finitely many G_p such that:

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- Routing is supported at G_r.
- Pairwise network coding is supported between two sessions on every G_p

Formulation

I:	the no. coexisting unicast sessions (s_i, t_i)
$\mathcal{P}(i)$:	the set of all (s_i, t_i) paths
$\mathbb{P}(i,j)$:	the set of all $(P_{s_i,t_i}, P_{s_j,t_i}, P_{s_j,t_j})$ tuples
$E_{e,i}^k$:	$= 1$, if link e uses the k-th path in $\mathcal{P}(i)$
	$= 0$, otherwise
$H_{e,ij}^l$:	$= 2$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 3$
	$= 1$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 1, 2$
	$= 0$, if for the l-th tuple in $\mathbb{P}(i,j)$, $\mathsf{ncp}(e) = 0$
x_i^k:	the routing rate through the k-th path of session i.
g_{ij}^{lm}:	joint coding rate between session i and j.

Formulation cont.

$$\max_{\vec{x},\vec{g}} \sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$$

$$\text{s.t.} \sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^k x_i^k + \sum_{i=1}^{I} \sum_{i<j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \max(H_{e,ij}^l, H_{e,ji}^m) g_{ij}^{lm} \leq C_e, \forall e$$

$$x_i^k \geq 0, \qquad g_{ij}^{lm} = g_{ji}^{ml} \geq 0, \qquad \forall i \neq j, l, m$$

Incorporating the Proximal Meth.

- $\sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$ may not be strictly concave.

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- The proximal method with auxiliary var. $\overrightarrow{y}, \overrightarrow{h}$:

$$\max_{\{\overrightarrow{x}, \overrightarrow{g}\}} \sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$$
$$- \sum_{i=1}^{I} \sum_{k}^{|\mathcal{P}(i)|} \frac{c_i}{2} (x_i^k - y_i^k)^2 - \sum_{i \neq j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \frac{d_i}{2} (g_{ij}^{lm} - h_{ij}^{lm})^2$$

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- The Slater condition holds.
- Solve the dual of the intermediate problem.

The Proximal Method (Cont'd)

- The Lagrangian $L_{\overrightarrow{y},\overrightarrow{h}}(\overrightarrow{x},\overrightarrow{g},\overrightarrow{\lambda},\overrightarrow{\mu})$ is

$$\sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$$

$$- \sum_{i=1}^{I} \sum_{k}^{|\mathcal{P}(i)|} \frac{c_i}{2} (x_i^k - y_i^k)^2 - \sum_{i=1}^{I} \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \frac{d_i}{2} (g_{ij}^{lm} - h_{ij}^{lm})^2$$

$$- \sum_{e} \lambda_e \left(\sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^k x_i^k + \sum_{i=1}^{I} \sum_{i<j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} \max(H_{e,ij}^l, H_{e,ji}^m) g_{ij}^{lm} - C_e \right)$$

$$- \sum_{i=1}^{I} \sum_{i<j} \sum_{l} \sum_{m} \mu_{ij}^{lm} \left(g_{ij}^{lm} - g_{ji}^{ml} \right)$$

The Proximal Method (Cont'd)

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$$-\sum_{i=1}^{I}\sum_{k}^{|\mathcal{P}(i)|}\frac{c_i}{2}(x_i^k-y_i^k)^2-\sum_{i=1}^{I}\sum_{j\neq i}\sum_{l=1}^{|\mathbb{P}(i,j)|}\sum_{m=1}^{|\mathbb{P}(j,i)|}\frac{d_i}{2}(g_{ij}^{lm}-h_{ij}^{lm})^2$$

$$-\sum_{e}\lambda_e\left(\sum_{i=1}^{I}\sum_{k=1}^{|\mathcal{P}(i)|}E_{e,i}^k x_i^k+\sum_{i=1}^{I}\sum_{i<j}\sum_{l=1}^{|\mathbb{P}(i,j)|}\sum_{m=1}^{|\mathbb{P}(j,i)|}\max(H_{e,ij}^l,H_{e,ji}^m)g_{ij}^{lm}-C_e\right)$$

$$-\sum_{i=1}^{I}\sum_{i<j}\sum_{l}\sum_{m}\mu_{ij}^{lm}\left(g_{ij}^{lm}-g_{ji}^{ml}\right)$$

- Separable!

The Distributed Solver

- Repeat the following K times:
  - Solve $D_{\overrightarrow{y}, \overrightarrow{h}}(\overrightarrow{\lambda}, \overrightarrow{\mu}) = \max_{\overrightarrow{x}, \overrightarrow{g}} L_{\overrightarrow{y}, \overrightarrow{h}}(\overrightarrow{x}, \overrightarrow{g}, \overrightarrow{\lambda}, \overrightarrow{\mu})$ via separability.
  - Solve the dual problem $\min D_{\overrightarrow{y}, \overrightarrow{h}}(\overrightarrow{\lambda}, \overrightarrow{\mu})$ by the gradient method with step size α.
- Update $\overrightarrow{y} \leftarrow \overrightarrow{x}^*$, $\overrightarrow{h} \leftarrow \overrightarrow{g}^*$, and go back to the beginning.

Algo $\mathcal{A}$ Summary

- Source Algorithm:

$$\{\overrightarrow{x}(t,r), \overrightarrow{g}(t,r)\} = \arg \max_{\{\overrightarrow{x}, \overrightarrow{g}\} \geq 0}$$

$$U_i\left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{i \neq j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm}\right) - \sum_{k}^{|\mathcal{P}(i)|} \frac{c_i}{2}(x_i^k - y_i^k)^2$$

$$-\sum_{i \neq j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,j)|} \frac{d_i}{2}(g_{ij}^{lm} - h_{ij}^{lm})^2 - \sum_{k} \left(\sum_{e} E_{e,i}^{k} \lambda_e\right) x_i^k$$

$$-\sum_{i \neq j} \sum_{l} \sum_{m} \left(\sum_{e} \max(H_{e,ij}^{l}, H_{e,ji}^{m}) \lambda_e\right) g_{ij}^{lm} - \sum_{i<j} \sum_{l} \sum_{m} \mu_{ij}^{lm} g_{ij}^{lm}$$

$$+\sum_{i>j} \sum_{l} \sum_{m} \mu_{ji}^{ml} g_{ij}^{lm}$$

Algo $\mathcal{A}$ Summary

- Link Algorithm:

$$\lambda_e(t,r+1) = [\lambda_e(t,r) + \alpha_e(\sum_{i=1}^{I}\sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^k x_i^k(t,r) + \sum_{i=1}^{I}\sum_{i\neq j}\sum_{l=1}^{|\mathbb{P}(i,j)|}\sum_{m=1}^{|\mathbb{P}(j,i)|} \max(H_{e,ij}^l, H_{e,ji}^m) g_{ij}^{lm}(t,r) - C_e)]^+.$$

- Sink Algorithm:

$$\mu_{ij}^{lm}(t,r+1) = \mu_{ij}^{lm}(t,r) + \beta_{ij}^{lm}(g_{ij}^{lm}(t,r) - g_{ji}^{ml}(t,r)) \qquad \forall i < j.$$

The Convergence Result

Theorem 3 *If the step size α of the gradient method (for the dual) and the proximal method coefficients c_i and d_i satisfy the following:*

$$\alpha \left(2 + \sum_{i=1}^{I} \sum_{k=1}^{|\mathcal{P}(i)|} E_{e,i}^{k} + \frac{1}{4} \sum_{i=1}^{I} \sum_{i \neq j} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(ji)|} (\max(H_{e,ij}^{l}, H_{e,ji}^{m}))^2 \right) < 2 \min_i \min(c_i, d_i),$$

then as $K \to \infty$, the proximal method converges to the optimal $\overrightarrow{x}_{\text{opt}}$ and $\overrightarrow{g}_{\text{opt}}$ for the original problem.

- For bounded K, the convergence is verified by simulations.
- It can also be proved similar to that in [Lin and Shroff 06].

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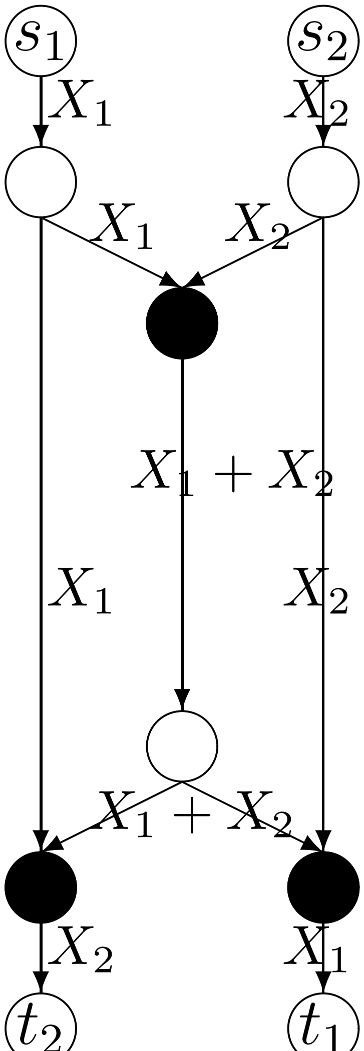

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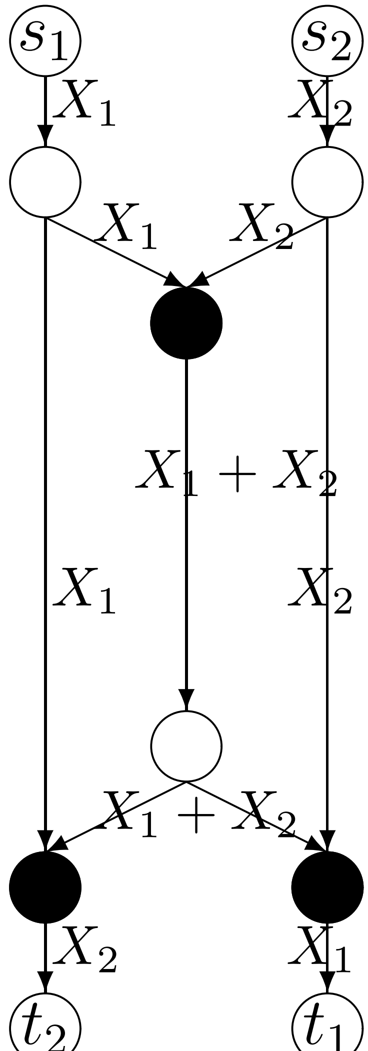

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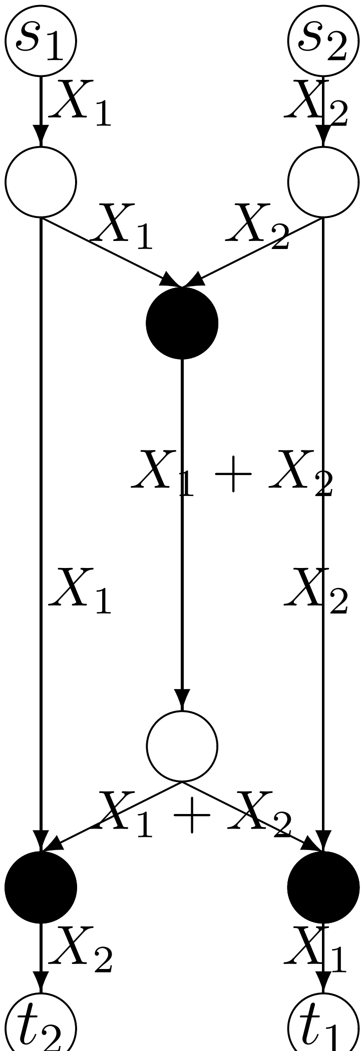

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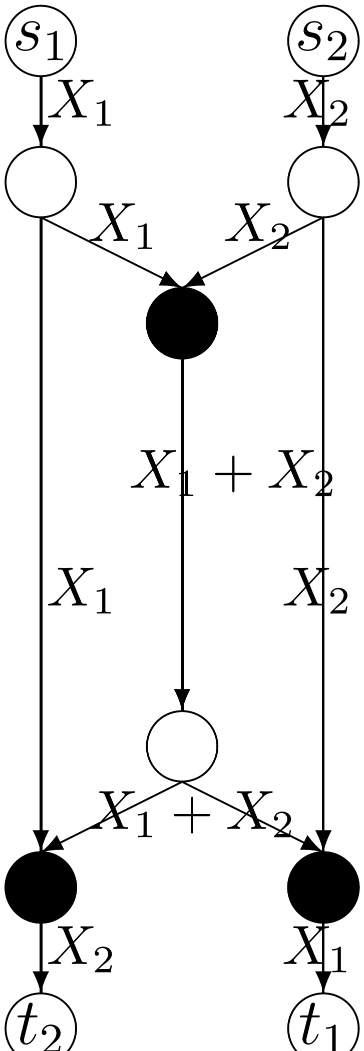

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Coding is independent of Queuing & rate allocation!

The Implementation Issues

- The control messages to collect the info. nec. for maximizing the Lagrangian.

$$\sum_{i=1}^{I} U_i \left(\sum_{k=1}^{|\mathcal{P}(i)|} x_i^k + \sum_{j \neq i} \sum_{l=1}^{|\mathbb{P}(i,j)|} \sum_{m=1}^{|\mathbb{P}(j,i)|} g_{ij}^{lm} \right)$$

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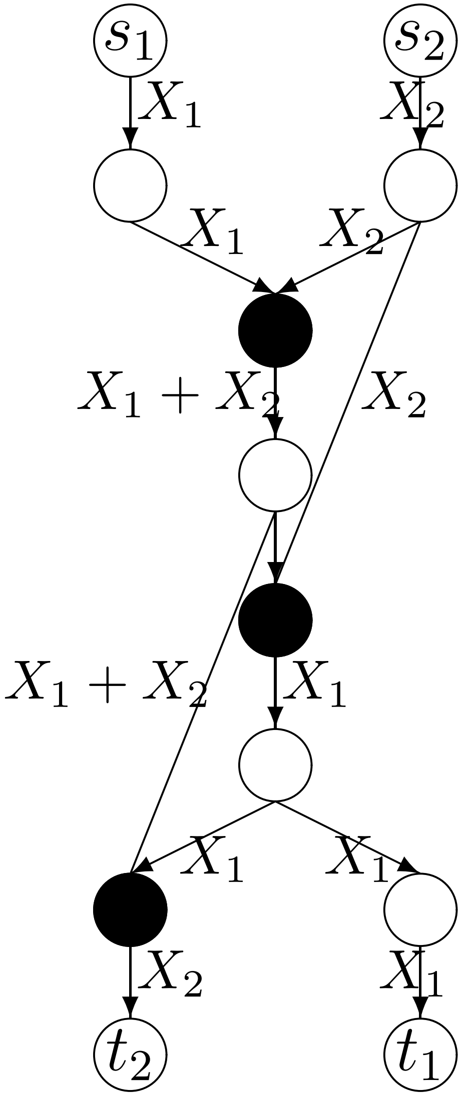
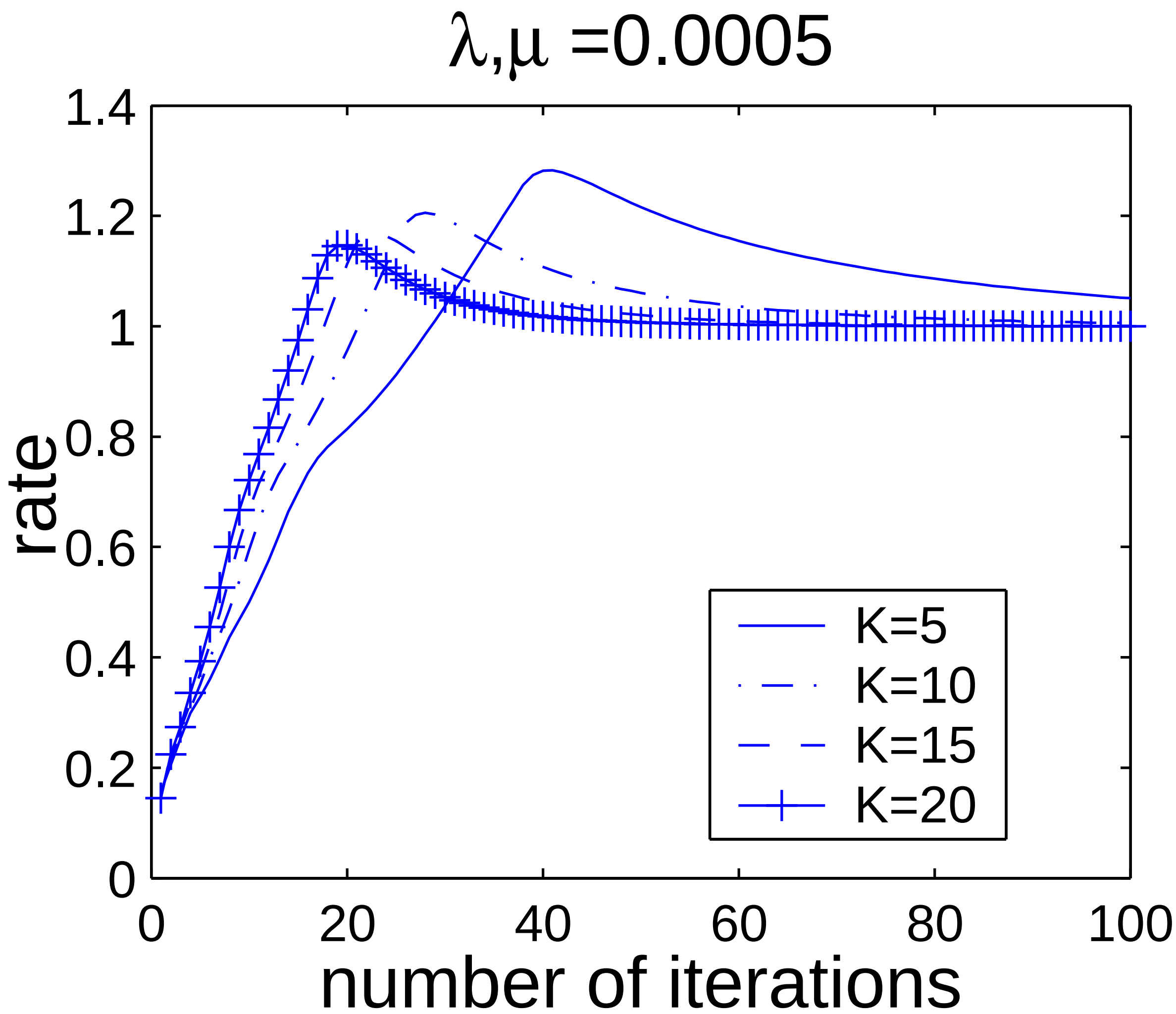
- Adaptively select $\mathcal{P}(i)$ and $\mathbb{P}(i,j)$.

Numerical Experiments





Capacity

Theorem 5 *For any network with I unicast sessions, any rate vector $(R_1, \ldots, R_I)$ that is achievable with the $\mathcal{TRLKM}$ or the $\mathcal{WS}$ region is also achievable with the $\mathcal{I}-\mathcal{TRLKM}$ region.*

Capacity & Fairness

- Utility gain $\mathcal{UG}$: $\frac{\sum U_i(\text{intersession NC.}) - \sum U_i(\text{routing})}{\sum U_i(\text{routing})}$.
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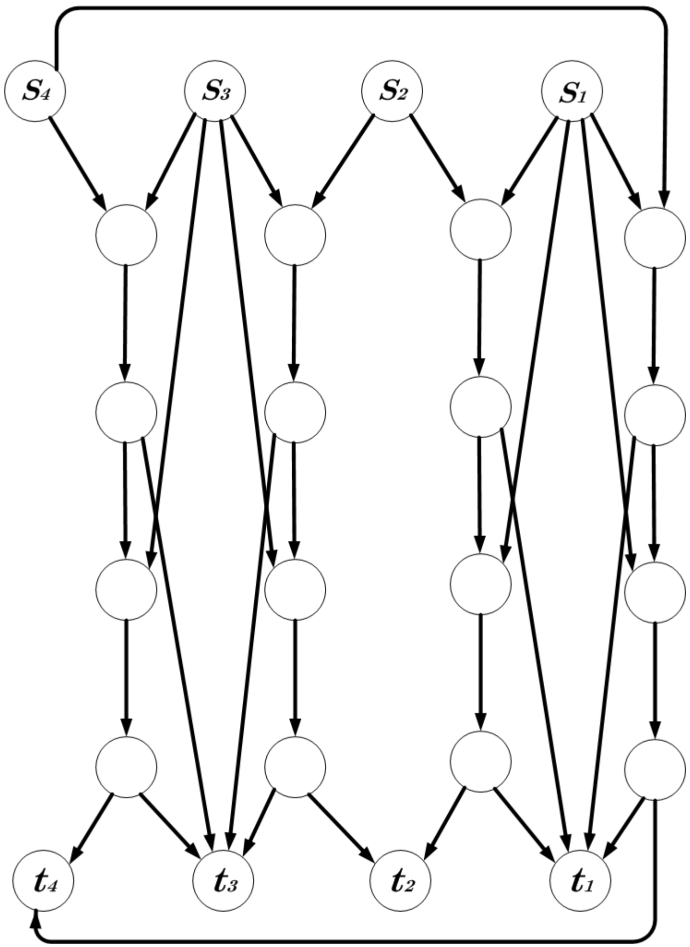

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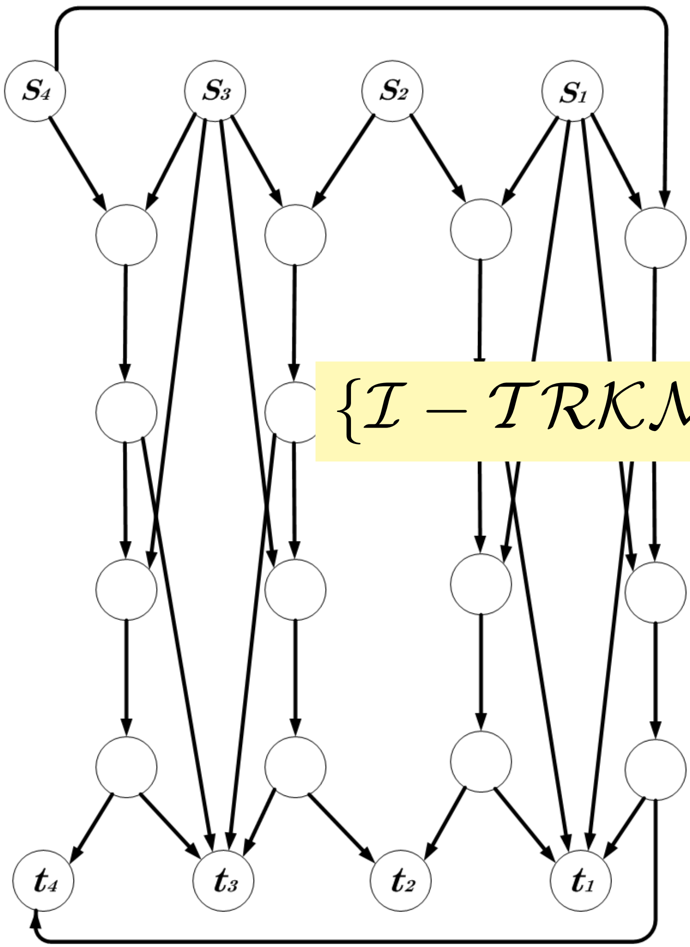
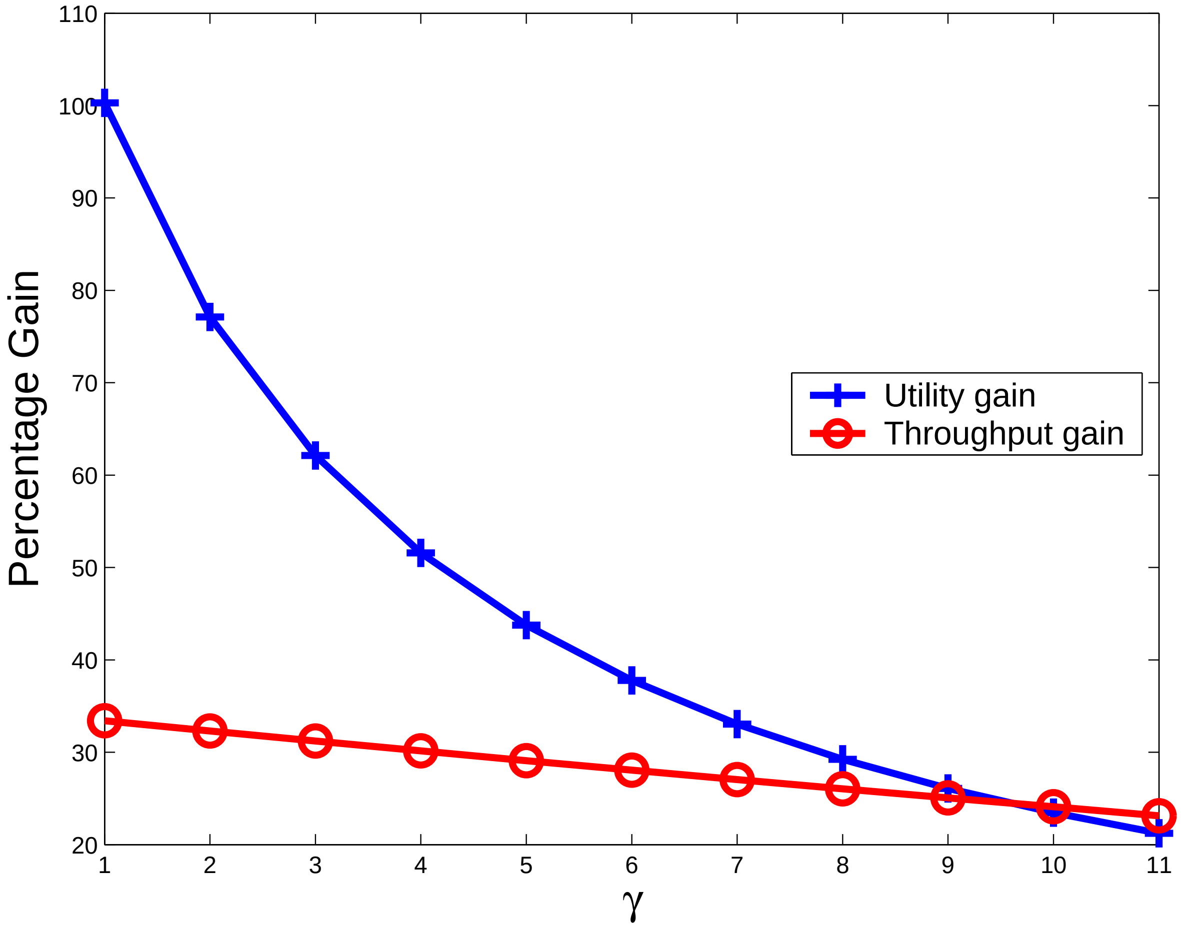


$\{\mathcal{I}-\mathcal{TRKM}, \mathcal{WS}\} > \{\mathcal{TRLKM}\}$

Sim. Results



$$\log(\gamma + r_i)$$

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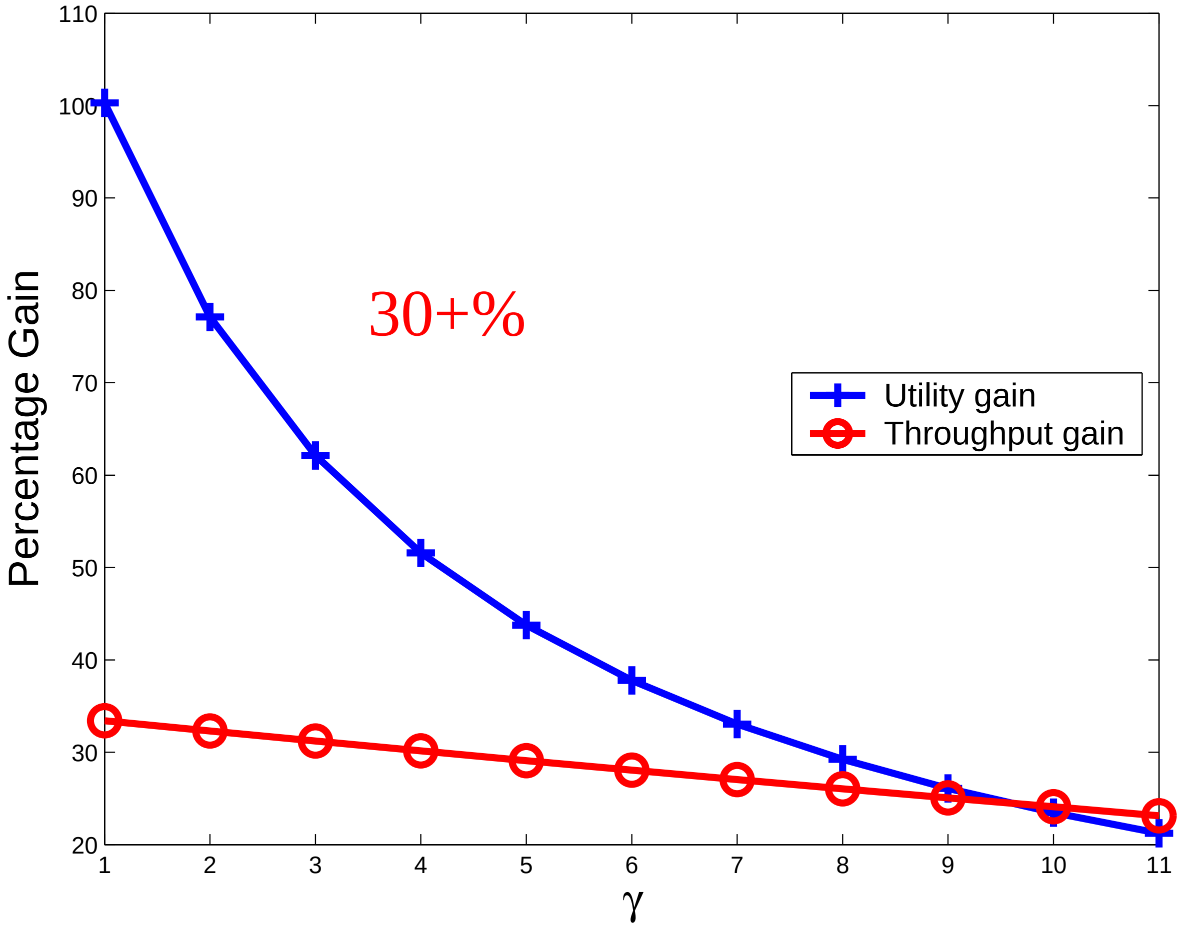

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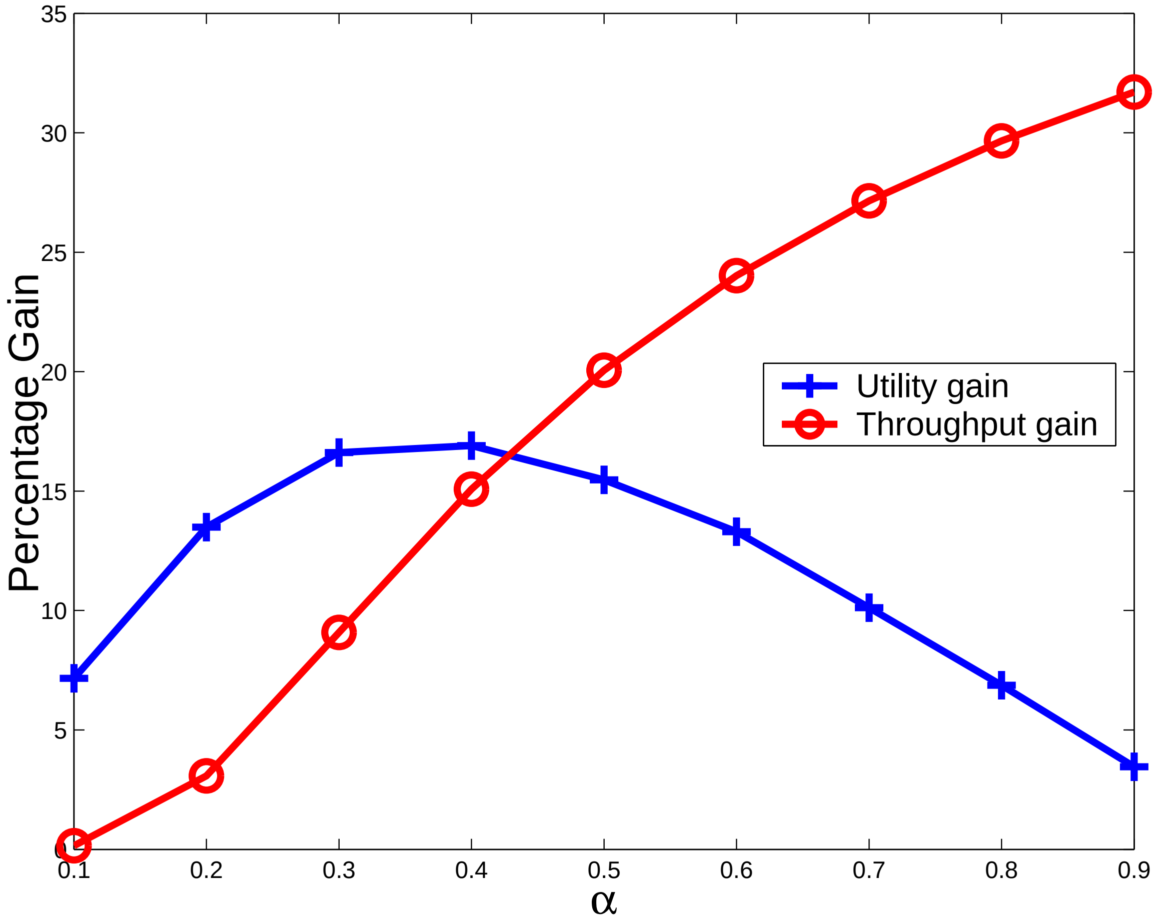

$$\frac{r_i^{1-\alpha}}{1-\alpha}$$

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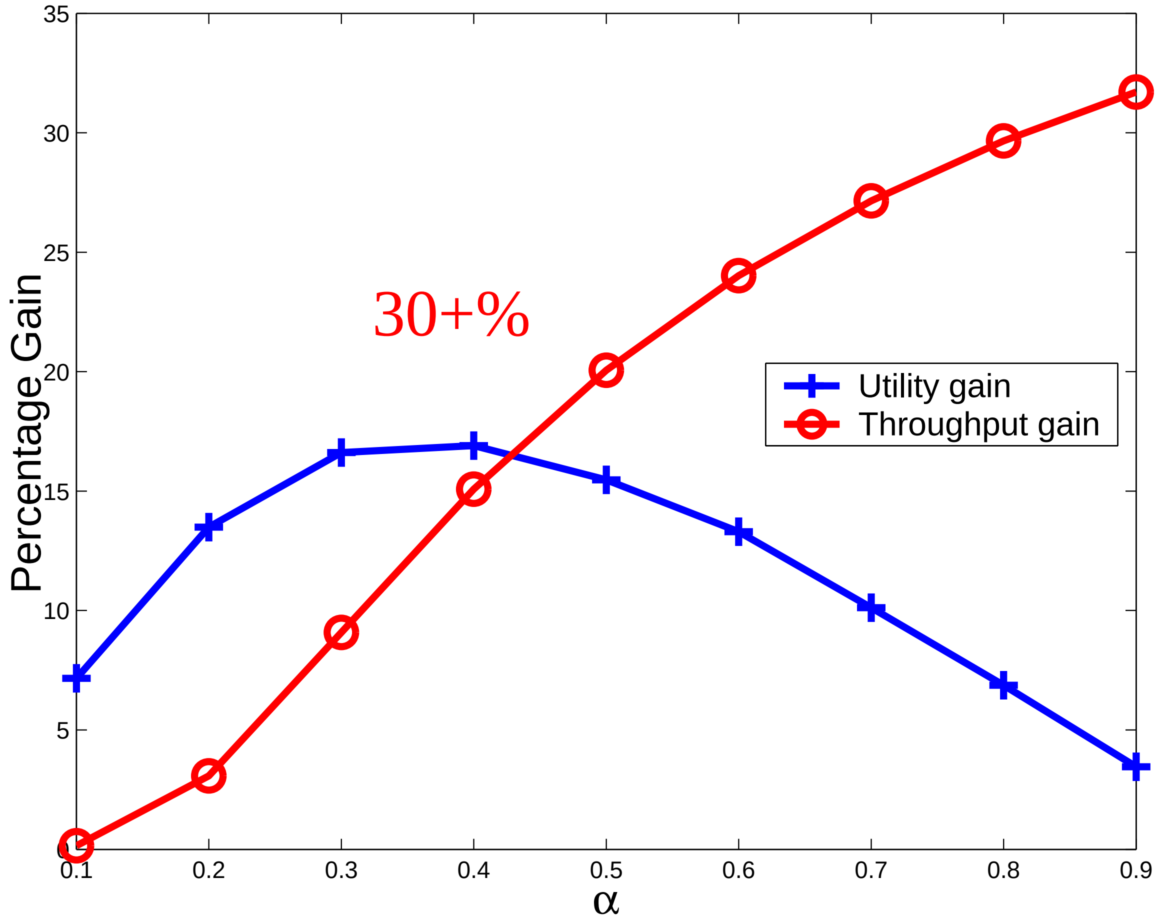

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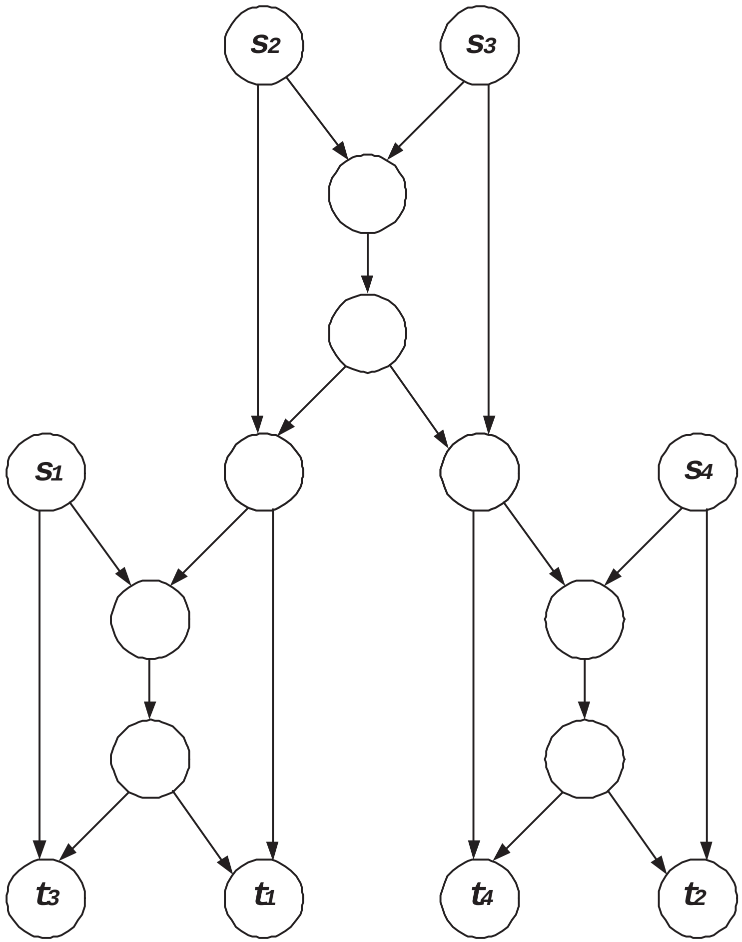

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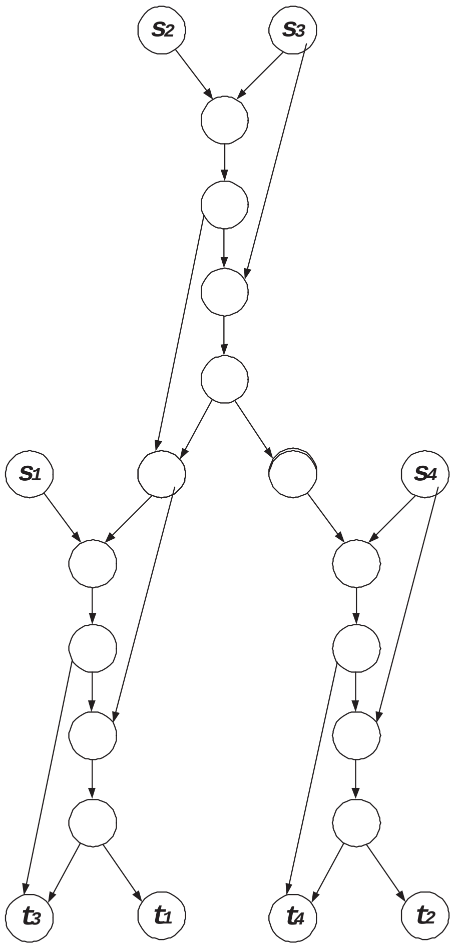

Complexity & Dist. Implementation

	Constraints	# Variables	# Constraints	Coding scheme
$\mathcal{TRLKM}$	Linear	$> 3\|E\|\|V\|(I^2 - I)$	$> (3\|V\| + \|E\|)\|V\|(I^2 - I)$	Limited to XOR
$\mathcal{WS}$	Linear	$\sum_{i=1}^{I} \sum_{i<j}^{I} J(i,j)J(j,i) + \sum_i J(i)$	$\|E\|$	Random
$\mathcal{I} - \mathcal{TRLKM}$	Non Linear	$> 3\|E\|(\|V\|)^3(I^2 - I)$	$> (3\|V\| + \|E\|)\|V\|(I^2 - I)$	Limited to XOR

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	Approach	Rate control	Rate alloc and coding	Queues exchange	Adaptive complexity reduction	Coding scheme
$\mathcal{RSC}$	Structure based	NO	Dependent	YES	hard	Limited to XOR
Algorithm$\mathcal{A}$	Path based	YES	Independent	NO	easy	Random
$\mathcal{I} - \mathcal{RSC}$	Structure based	NO	Dependent	YES	hard	Limited to XOR

Conclusions

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