On Wireless Network Scheduling with Intersession Network Coding

How far can we borrow the wisdom from the routing paradigm?

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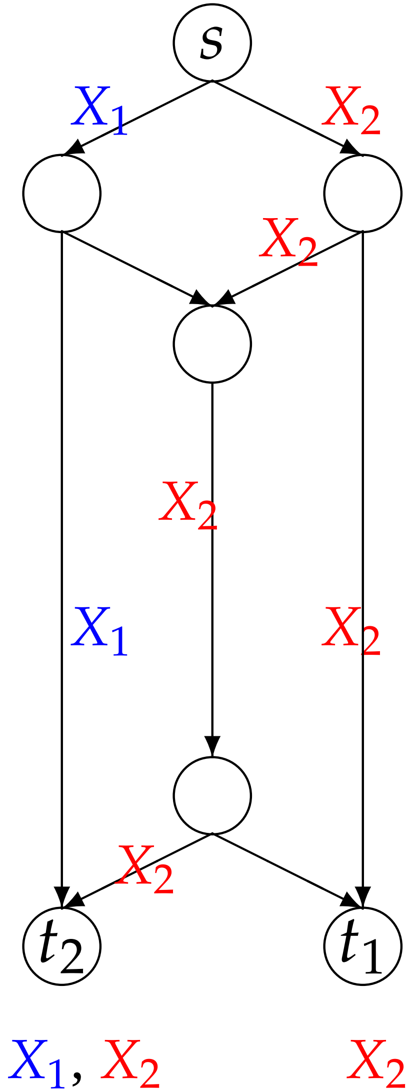
- Two different philosophies. Significant 30–80% throughput improvements. Combination of both?
- How far can we borrow the wisdom from the routing (non-coded) paradigm for network coding?

Content

- Review of existing results.
  - Intrasession network coding.
  - Cross-layer Optimization with intrasession network coding.
- New understanding of pairwise intersession network coding (PINC)
- A cross layer optimization framework for wireless networks with PINC.
- Impact of imperfect scheduling for PINC.

Coding Within A Single Session

The Non-Coded Solution



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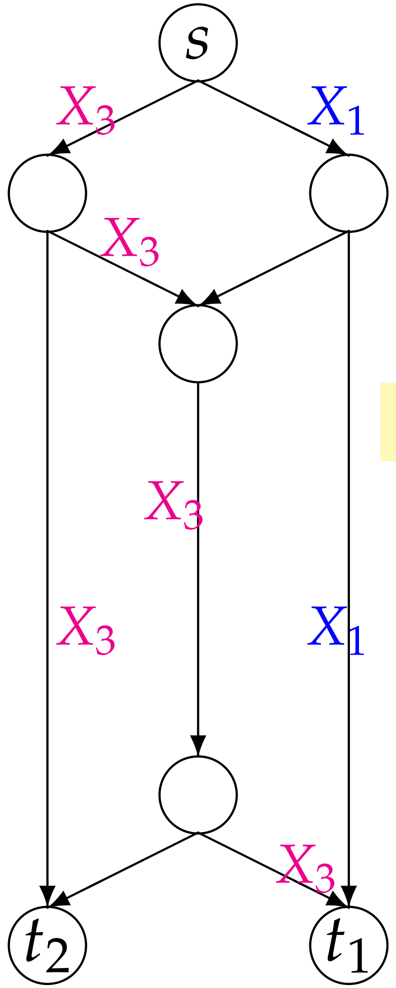

Rate: 3/2

X_1, X_2, X_3 X_2, X_1, X_3

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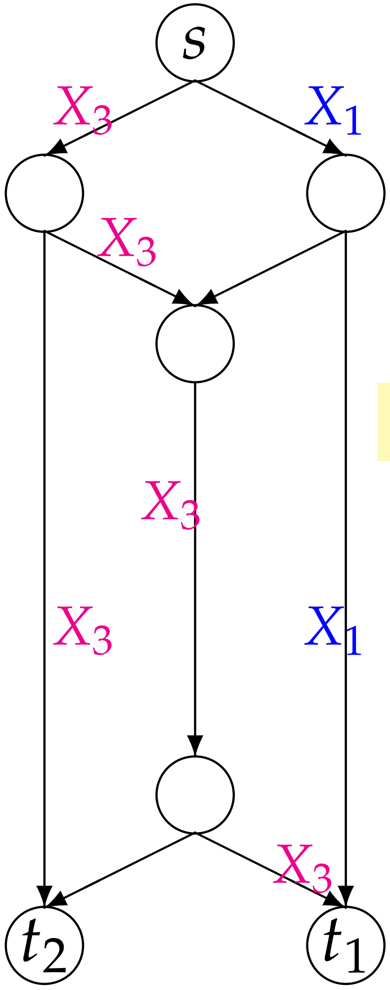

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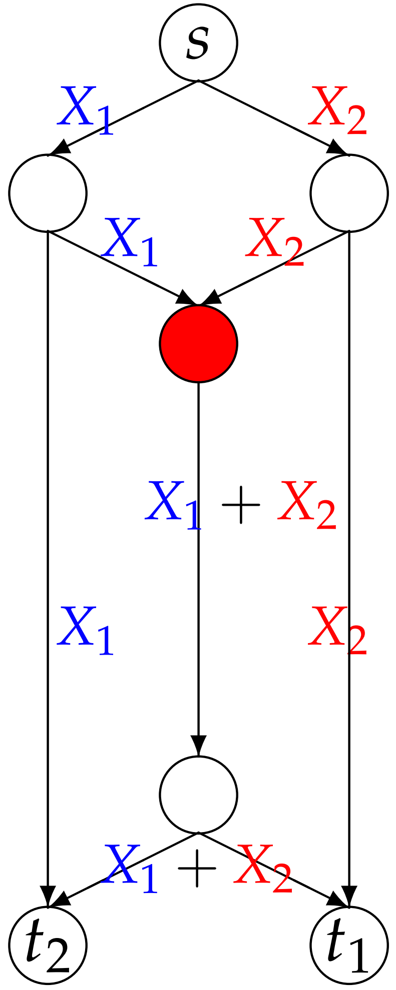

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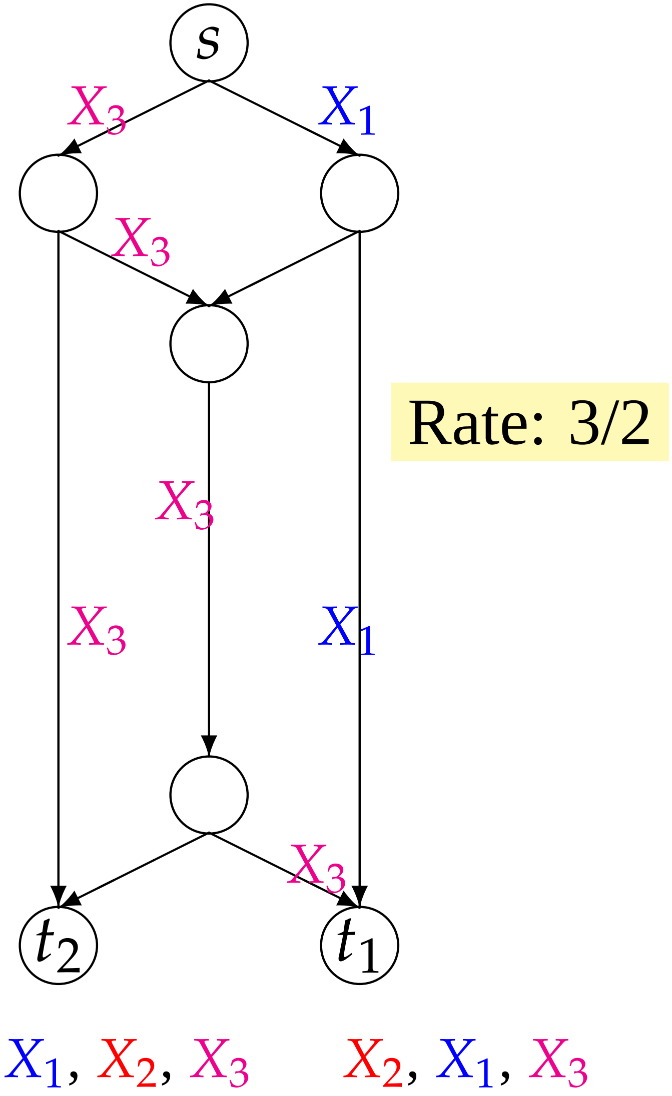

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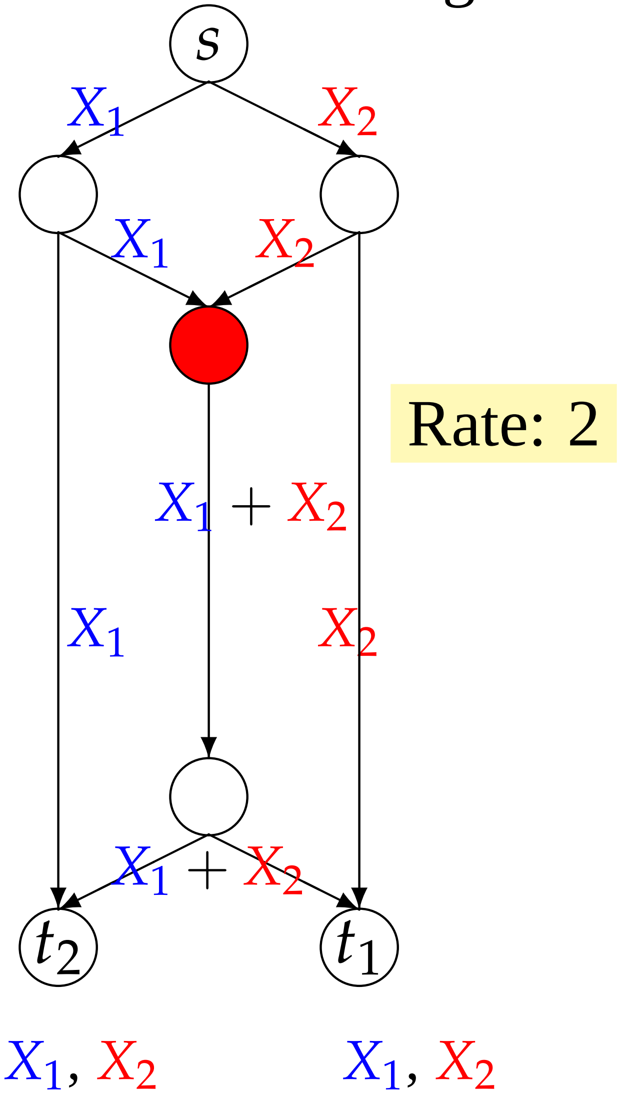

Achievable Rate Characterization

Theorem 1 *[Ahlswede et al. 00] For a single multicast session, rate r is achievable if for all dest. t_i, the min-cut/max-flow $\rho_G(s,t_i)$ between s and t_i satisfies*

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Cross-layer optimization with a single multicast session [Wu *et al.* 06]:

- Directed acyclic graph: $G = (V, E, \{c_e\}_{e \in E})$
- $$\begin{aligned} \max_{r, \{c_e\}} \quad & U(r) - \sum_{e \in E} p_e(c_e) \\ \text{subject to} \quad & r \leq \rho_G(s, t_i), \ \forall i \\ & 0 \leq c_e \leq \mathsf{ub}_e, \ \forall e \in E. \end{aligned}$$

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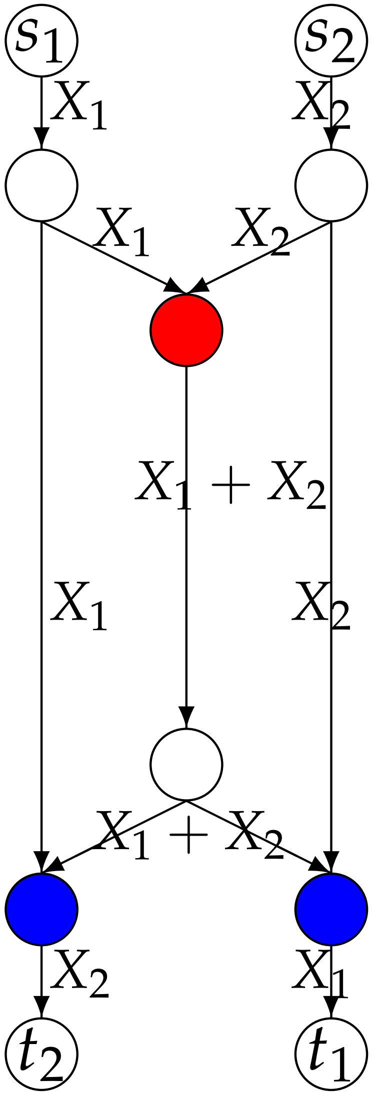
- Collapse to non-coded cross-layer optimization when only unicast sessions are present.

Coding Across Diff. Sessions

- Inter-session network coding: The benefit is apparent.



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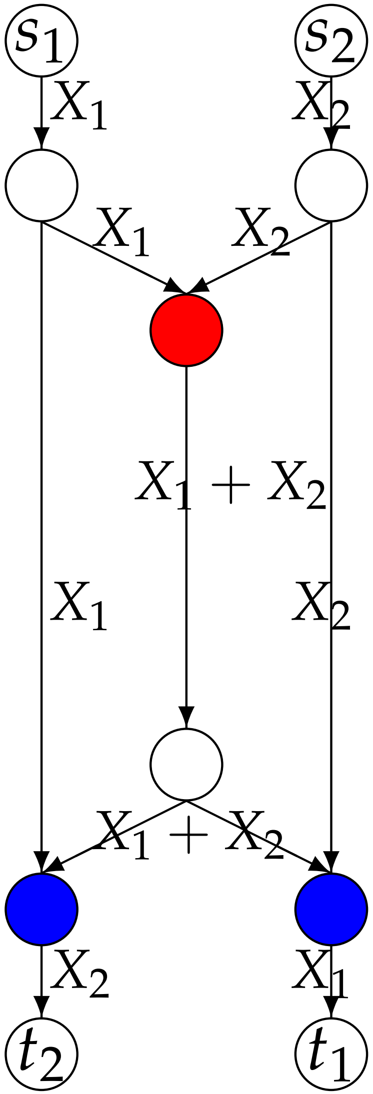

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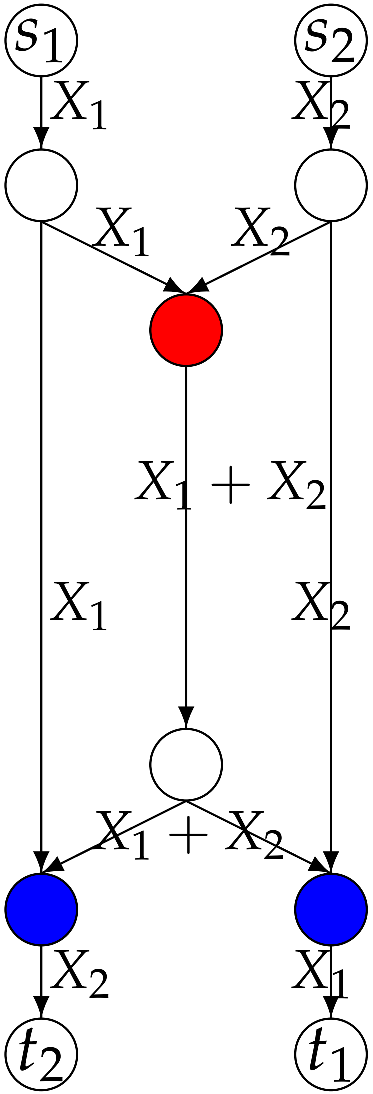
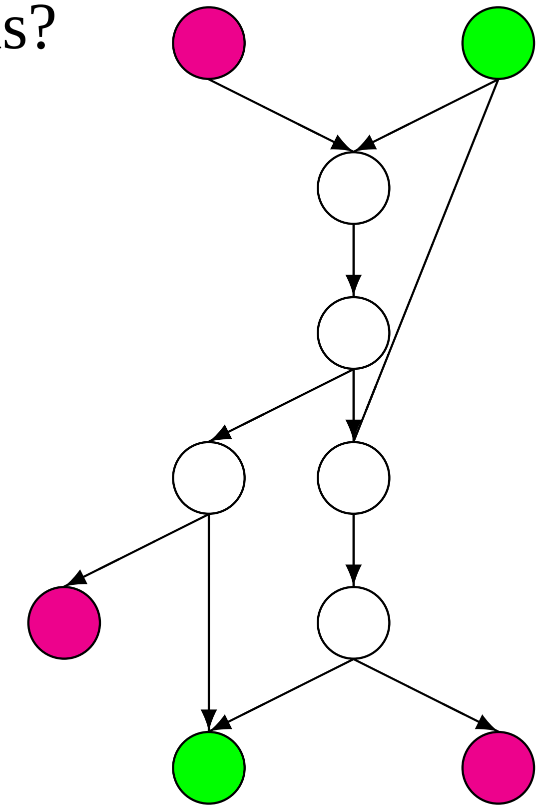


- Characterization?
- Multiple multicast sessions?



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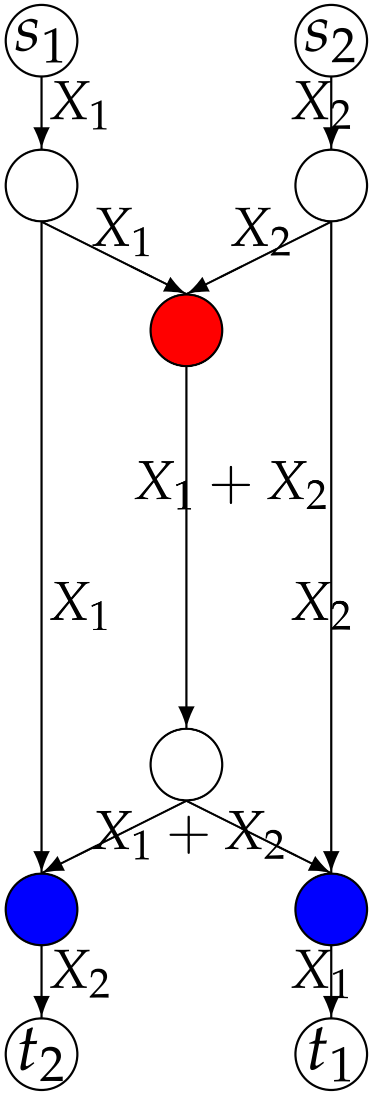

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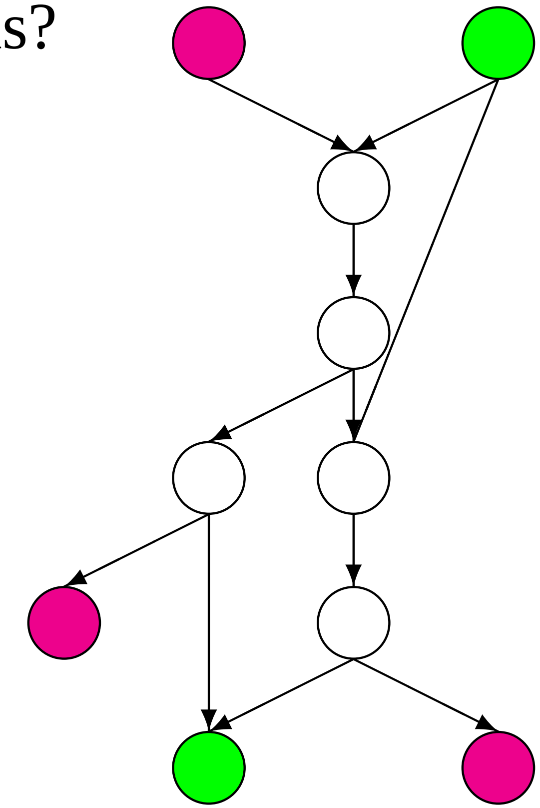

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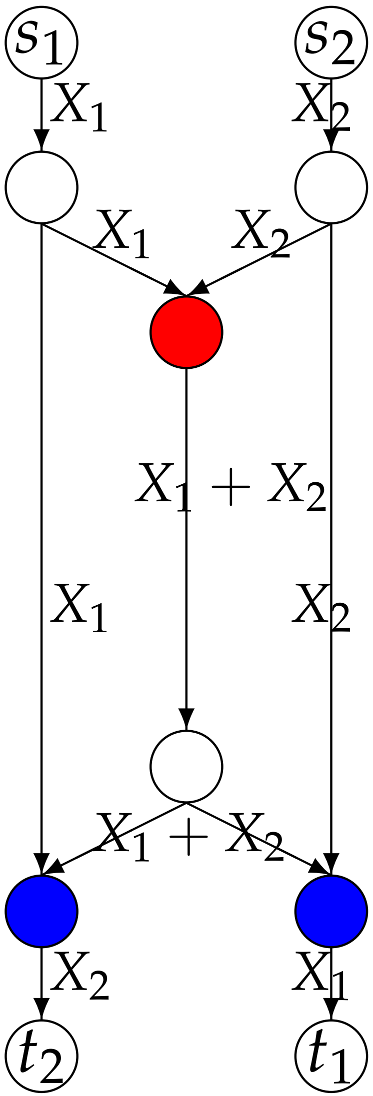

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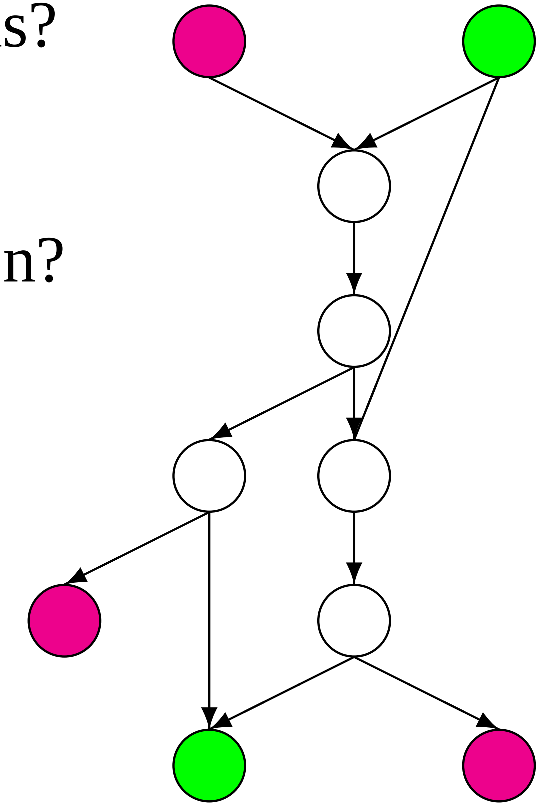
- Cross-layer optimization?
- Distributed implementation?



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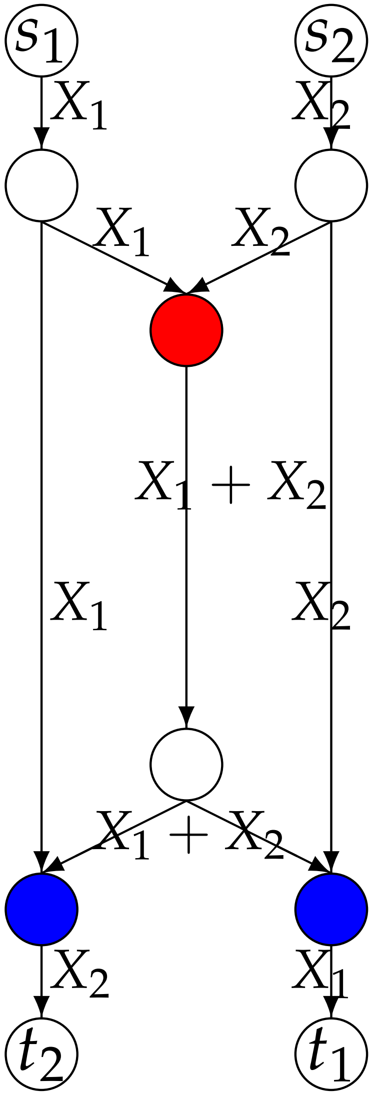

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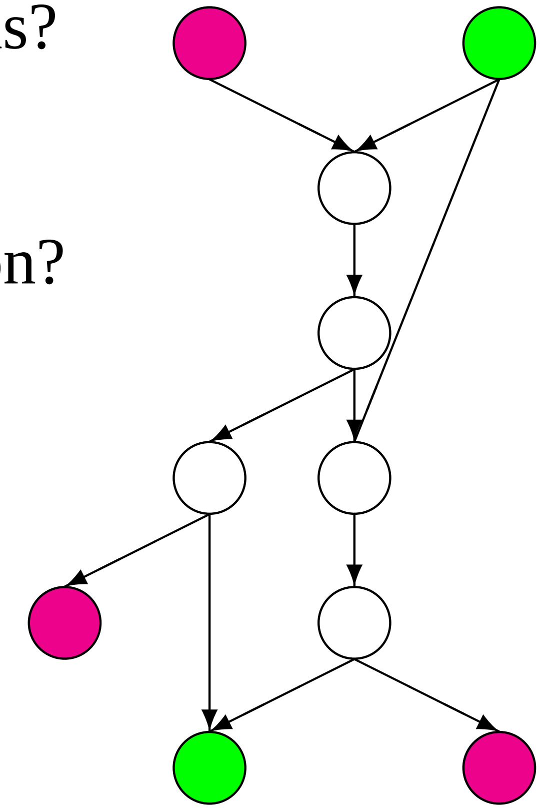
- Multiple multicast sessions?
- Cross-layer optimization?
- Distributed implementation?
- Imperfect scheduling?



Existing Multiple Session Results — Searching for Butterflies

- [Traskov *et al.* 06], [Ho *et al.* 06], [Eryilmaz *et al.* 07].

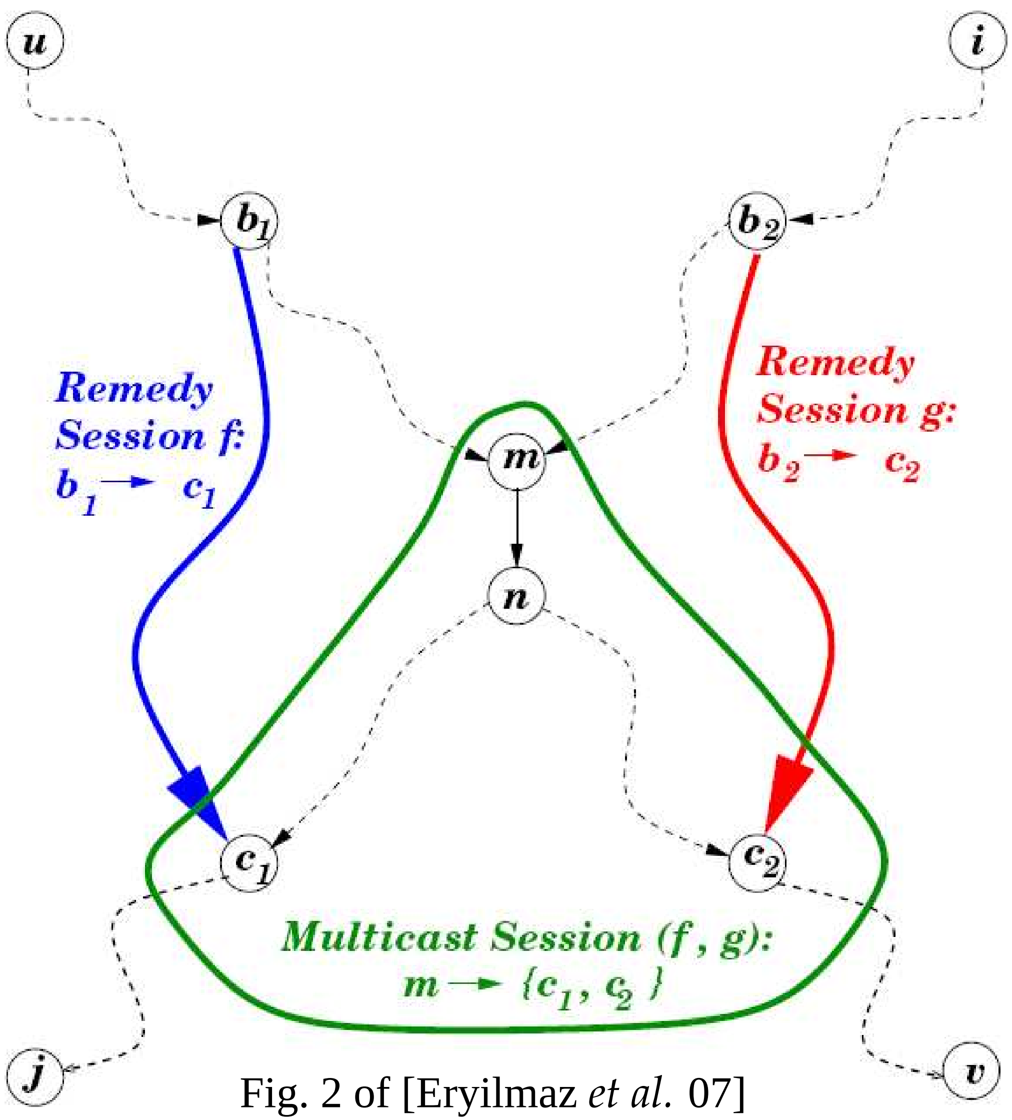


Fig. 2 of [Eryilmaz *et al.* 07]

- Create artificial flows p, q, w for each butterfly‡.
- $$\max_{r_i} \quad \sum_i U(r_i)$$
  subject to p, q, w, r_i
  satisfy the butterfly flow cond.
- With queues for each (artificial) flow, a backlog algorithm can distributively stabilize any rates in the above region.

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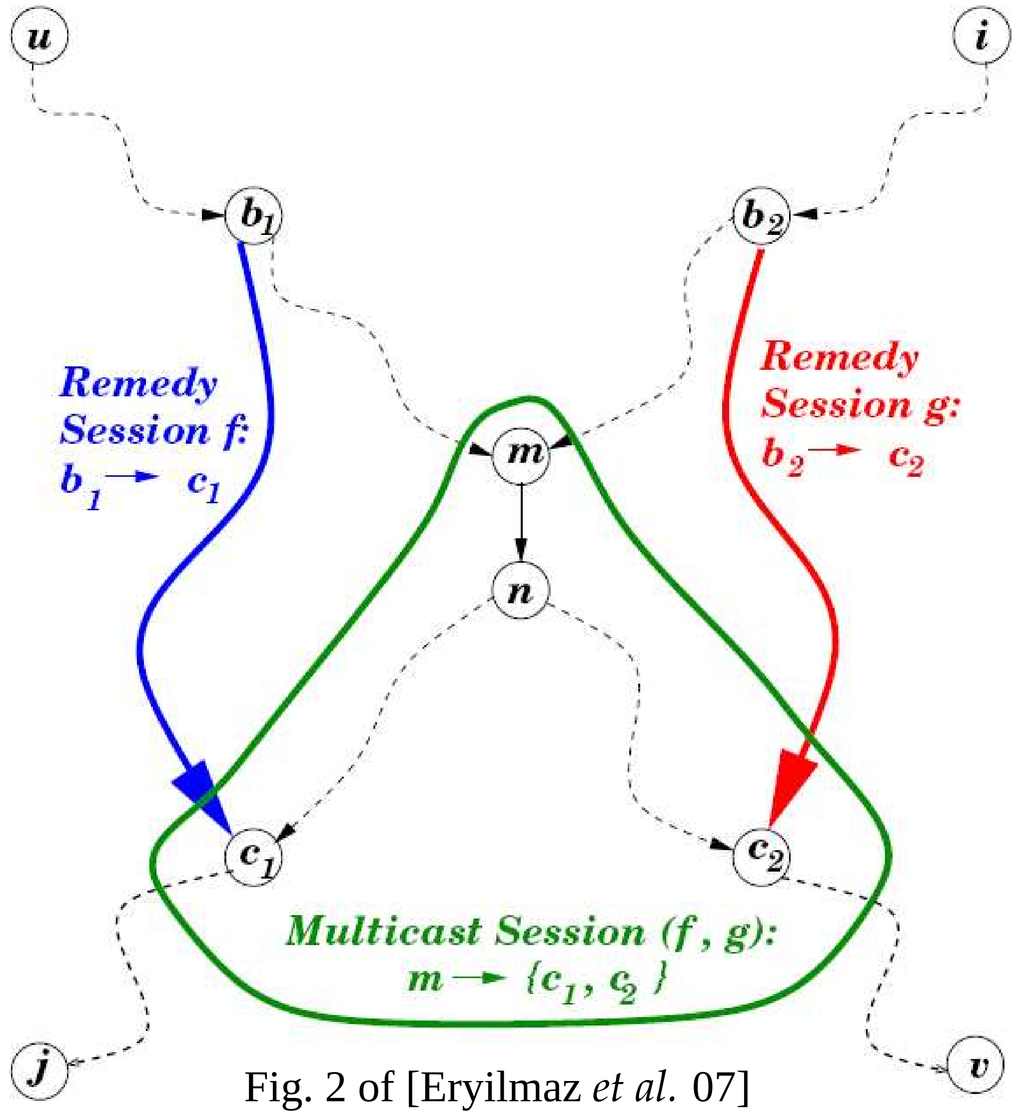
Another beneficial structure



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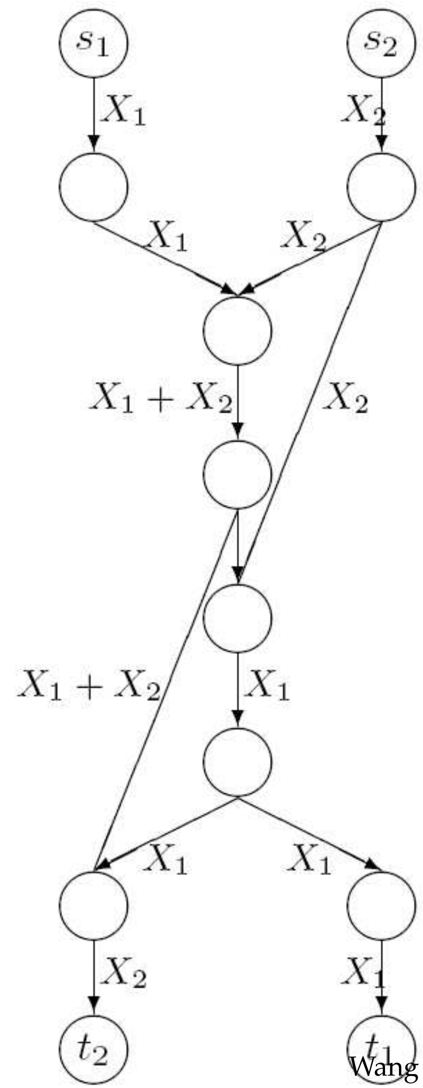

New Char. for 2 Unicasts

- Setting: General finite directed acyclic graphs, unit edge capacity, (s_1, t_1) & (s_2, t_2), two packets X_1 and X_2.
- Number of Coinciding Paths of edge e: $\mathcal{P} = \{P_1, \cdots, P_k\}$, and $\mathsf{ncp}_{\mathcal{P}}(e) = |\{P \in \mathcal{P} : e \in P\}|$.

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Theorem 2 *[Preliminary results, Wang* et al. *07] Network coding* $\Longleftrightarrow$ *one of the following two holds.*

1. $\exists \mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}\}$, *such that* $\max_{e \in E} \mathsf{ncp}_{\mathcal{P}}(e) \leq 1.$
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Routing: edge disjointness vs. Network coding: controlled overlaps.

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Flow-Based Characterization

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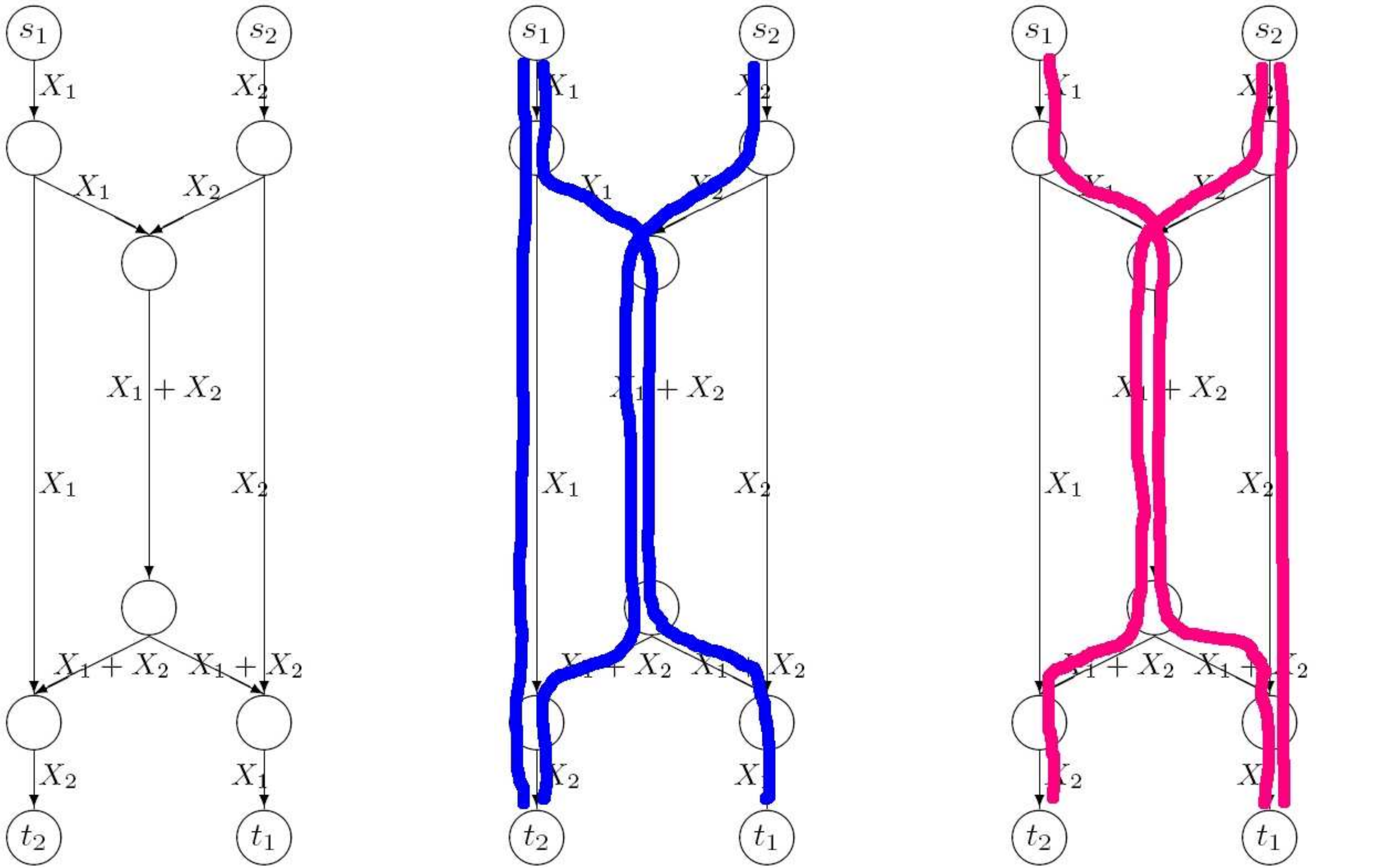
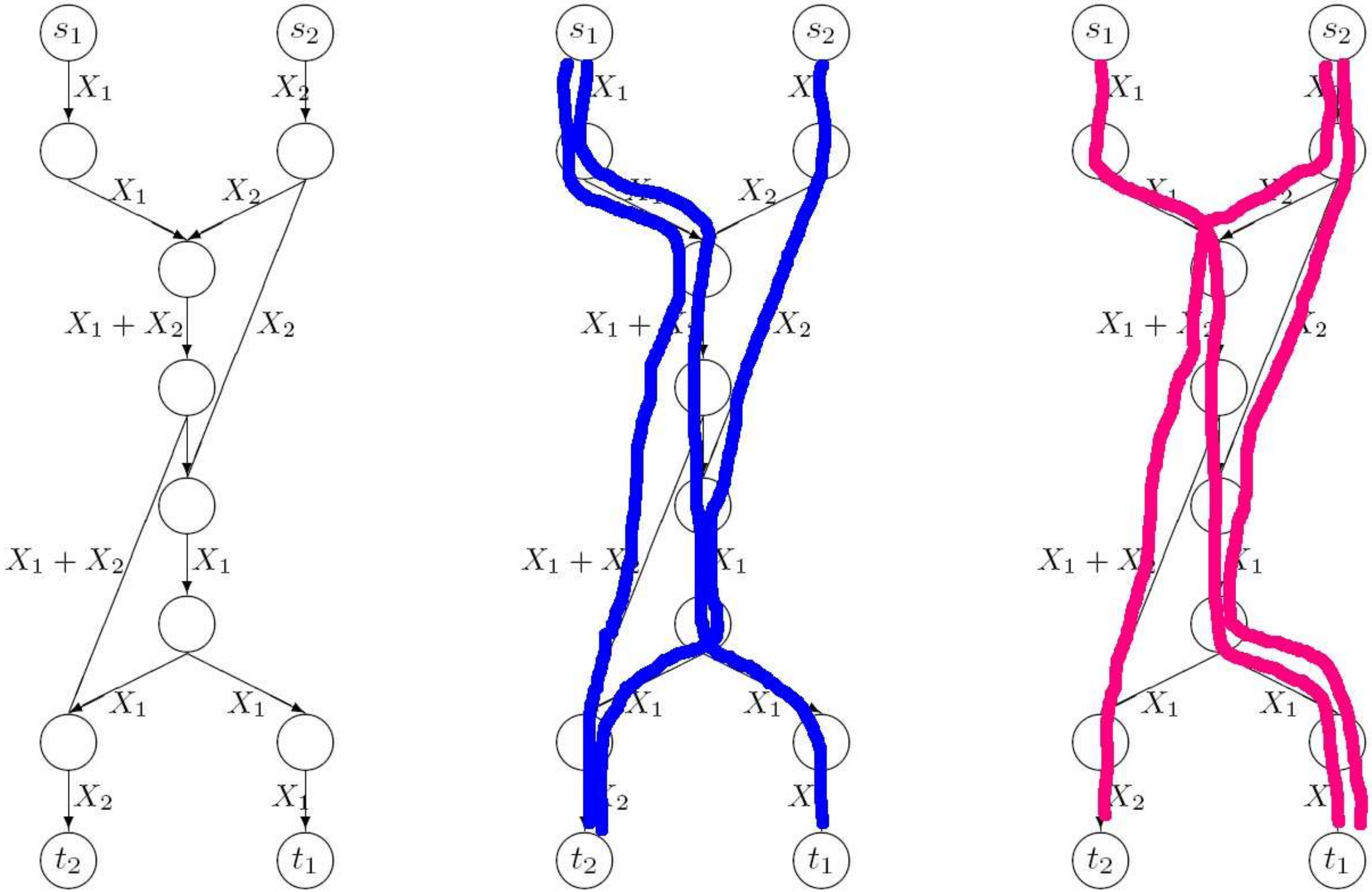
Flow-Based Characterization

Feasible Example: The Butterfly



$\mathcal{Q} = \{Q_{s_1,t_1}, Q_{s_2,t_2}, Q_{s_1,t_2}\}$ $\mathcal{P} = \{P_{s_1,t_1}, P_{s_2,t_2}, P_{s_2,t_1}\}$

Feasible Example 2: The Grail



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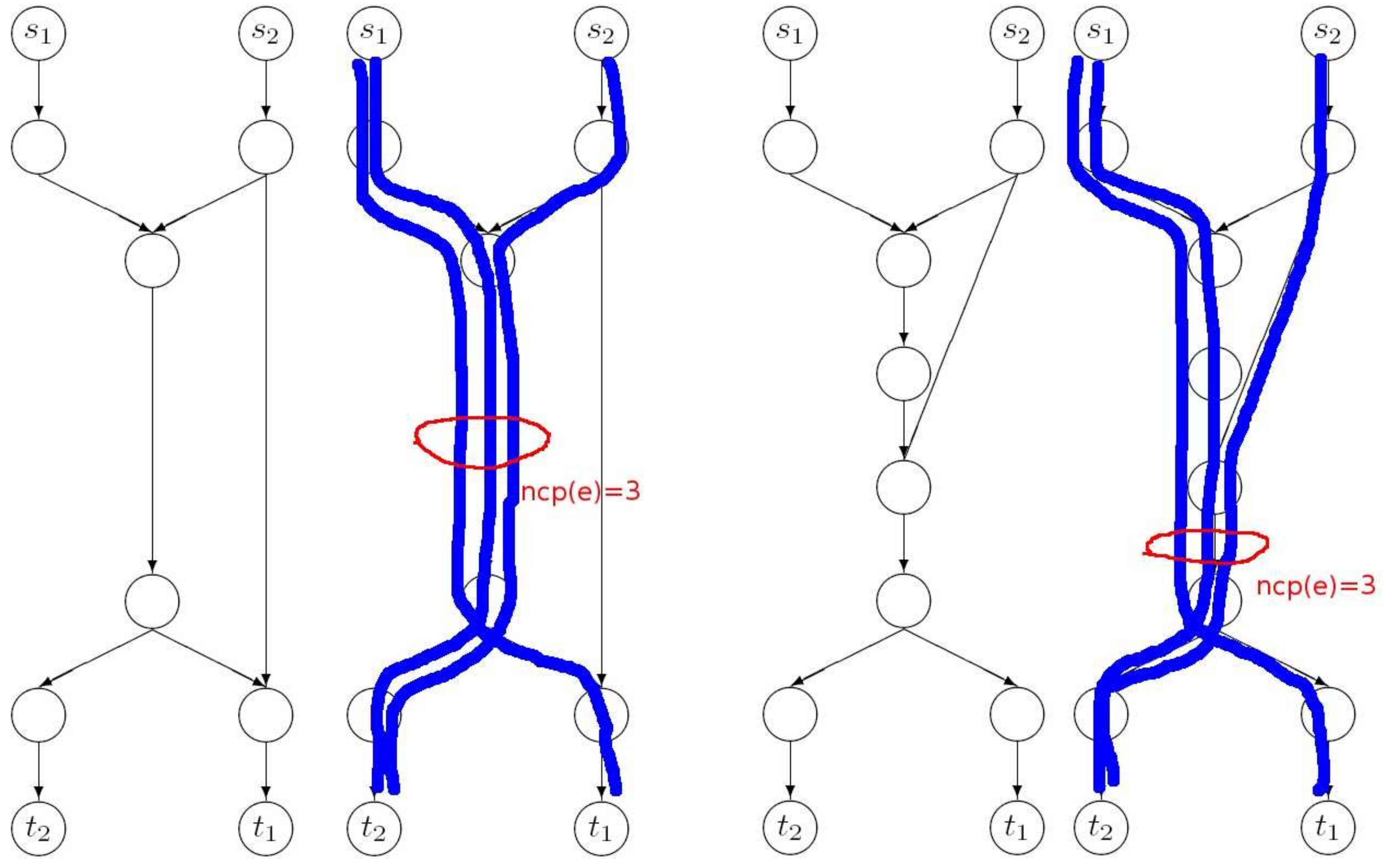
Infeasible Examples



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such that

$$\max_{e \in E} \mathsf{ncp}_{\{P_{s_1,t_{1,i}}, P_{s_2,t_{1,i}}, P_{s_2,t_{2,j}}\}}(e) \leq 2, \quad \forall i, j,$$

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such that

Choose paths for i and j separately.

$$\max_{e\in E} \mathsf{ncp}_{\{P_{s_1,t_{1,i}}, P_{s_2,t_{1,i}}, P_{s_2,t_{2,j}}\}}(e) \leq 2, \ \ \forall i, j,$$

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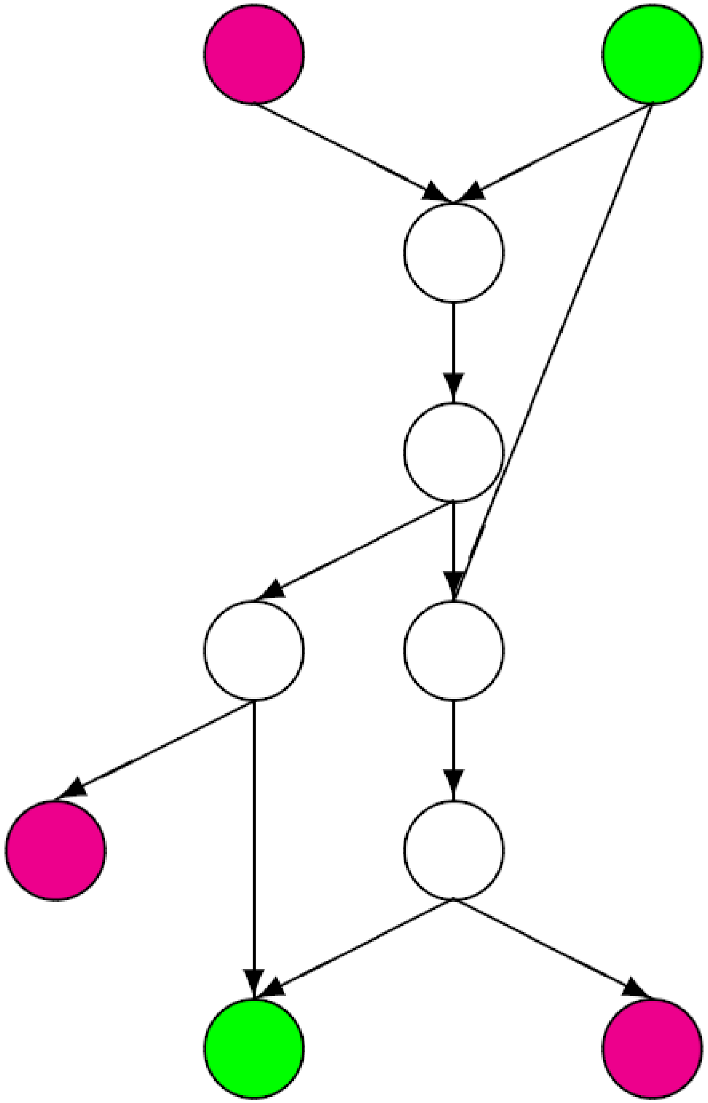
Then the conditions have to be satisfied for all (i, j) combinations.

An Infeasible Example



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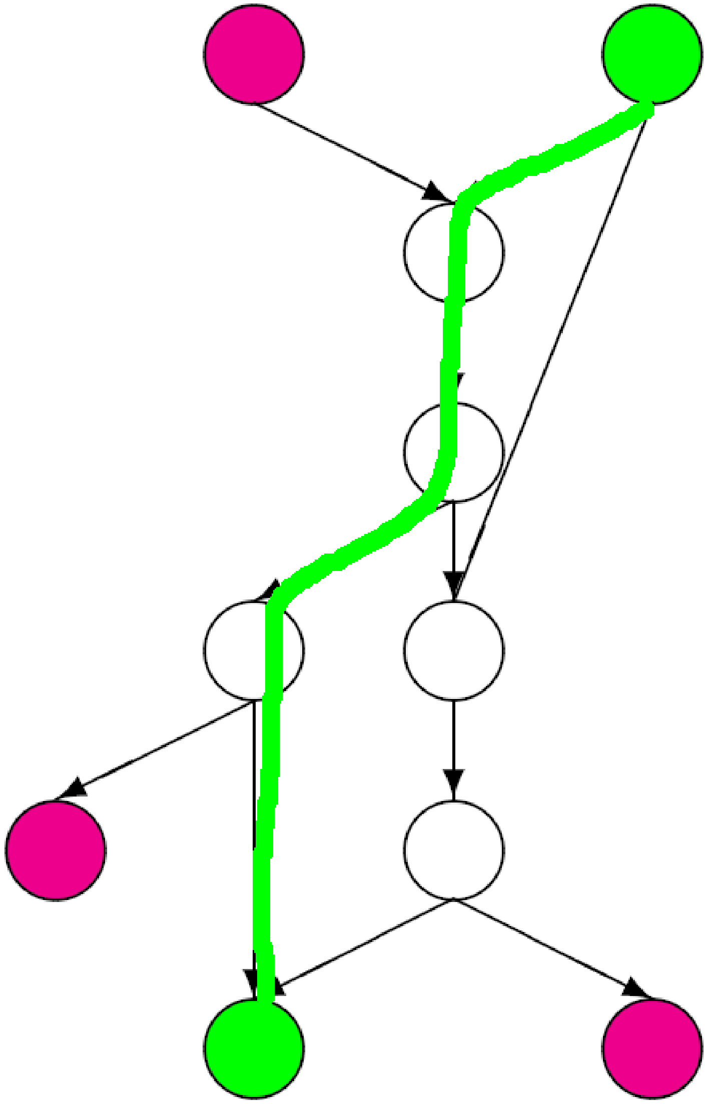

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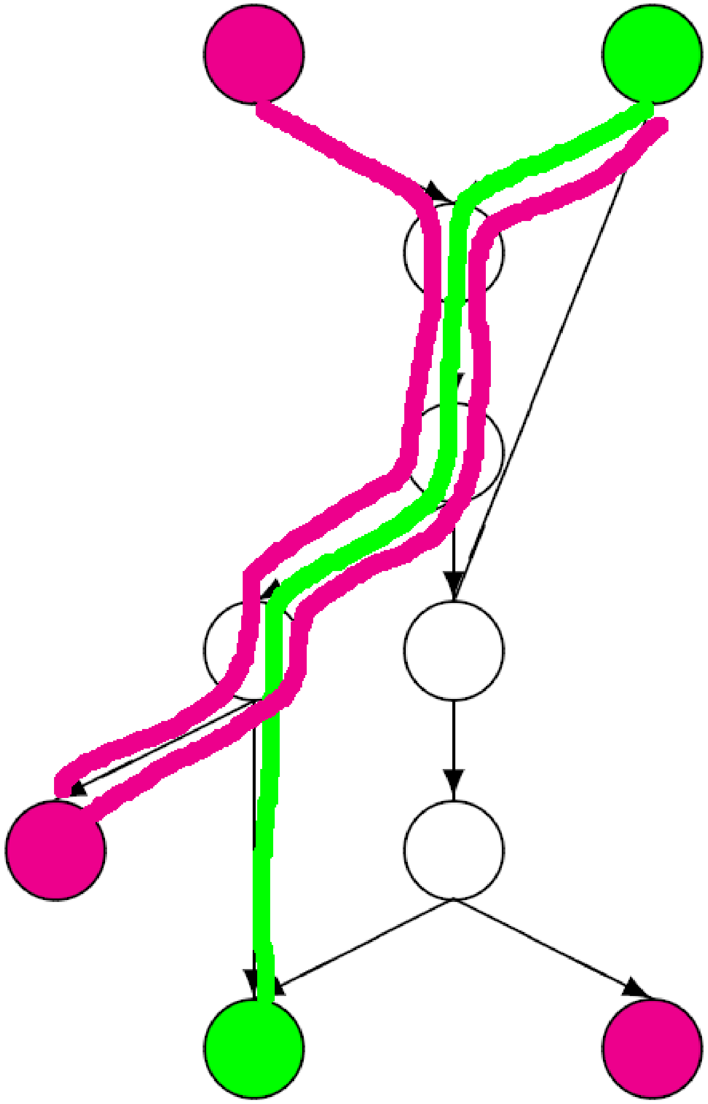

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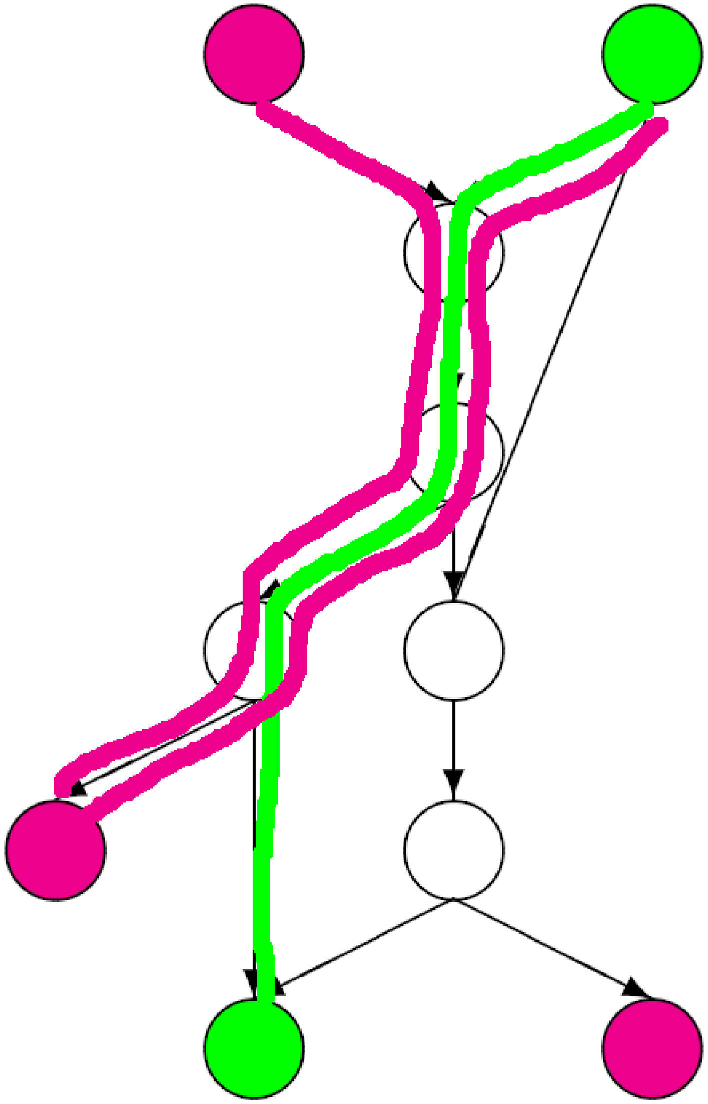

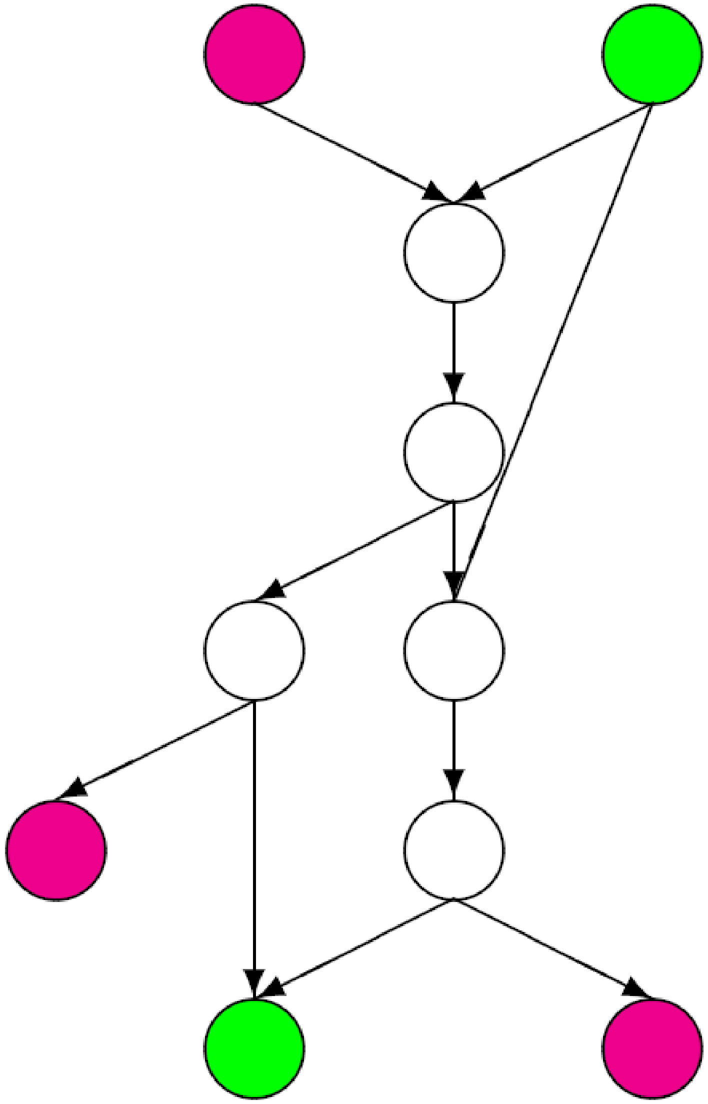

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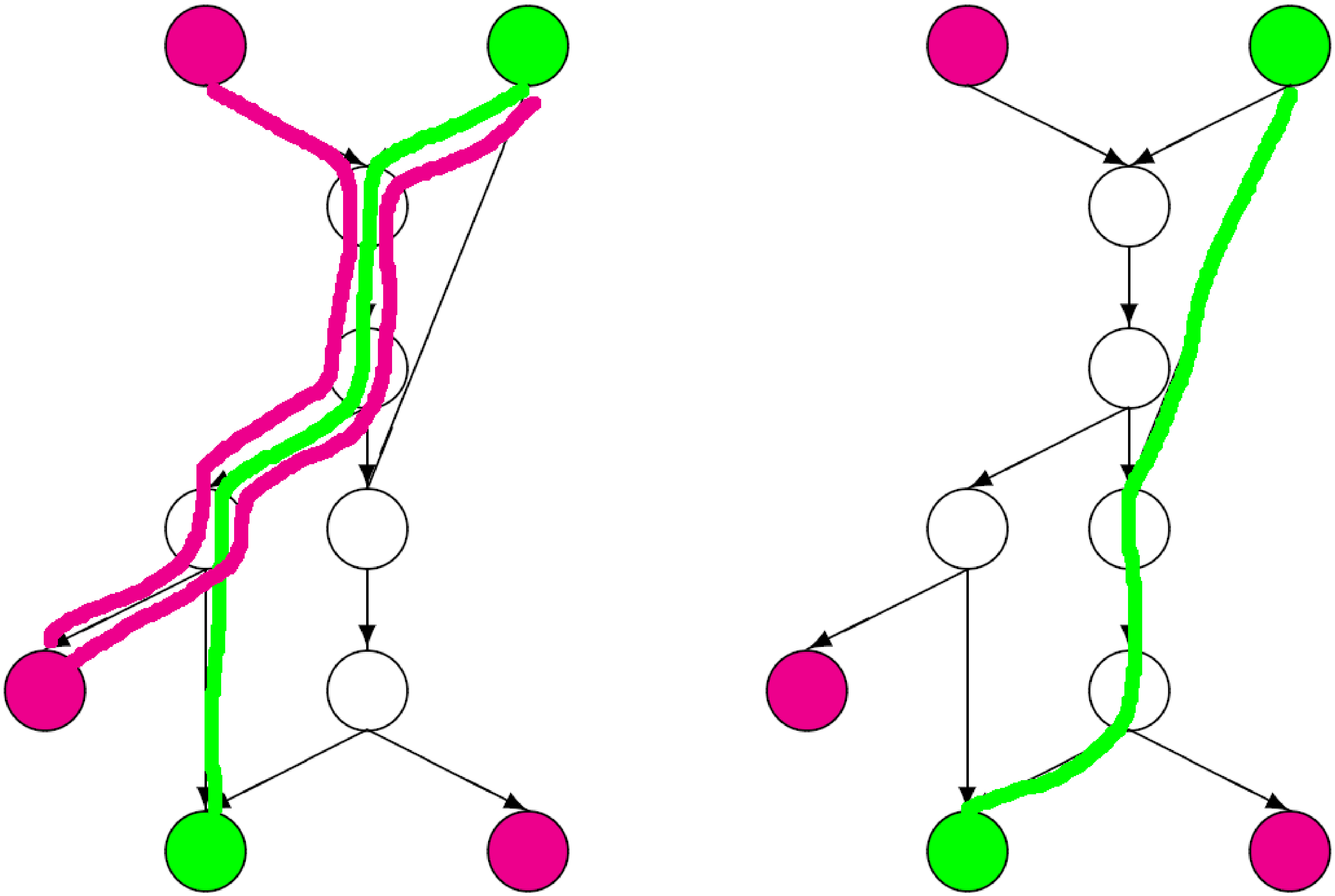

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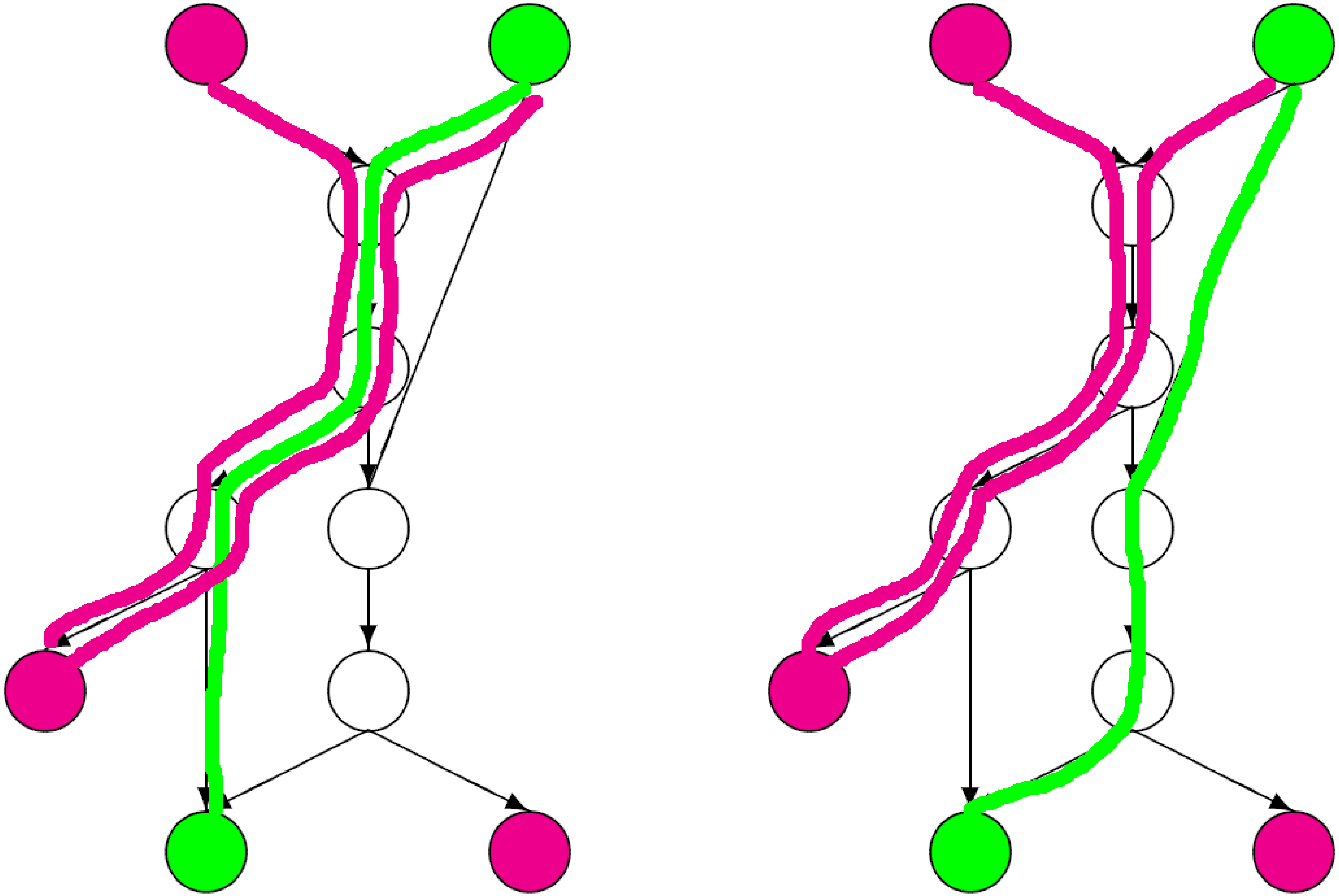

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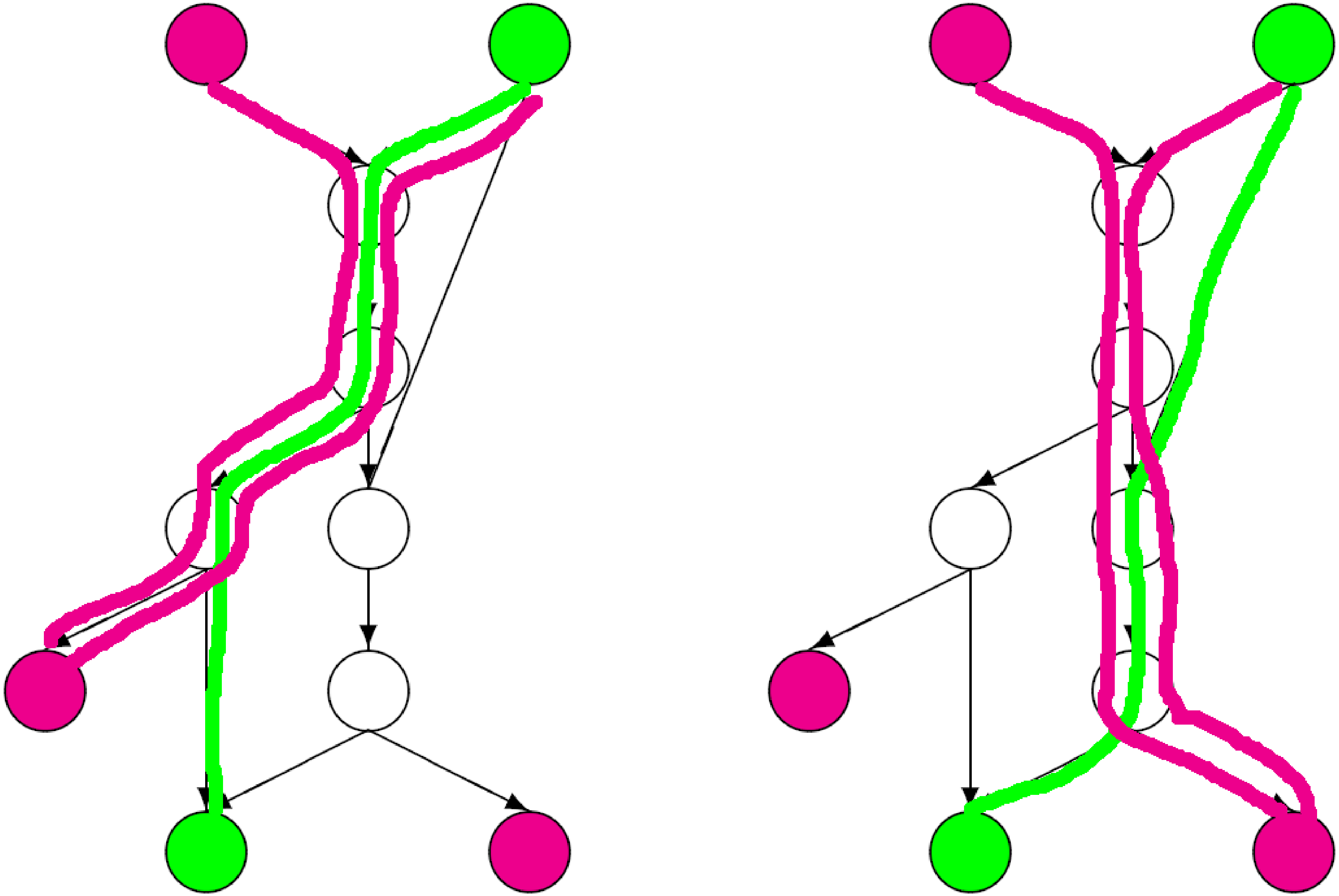
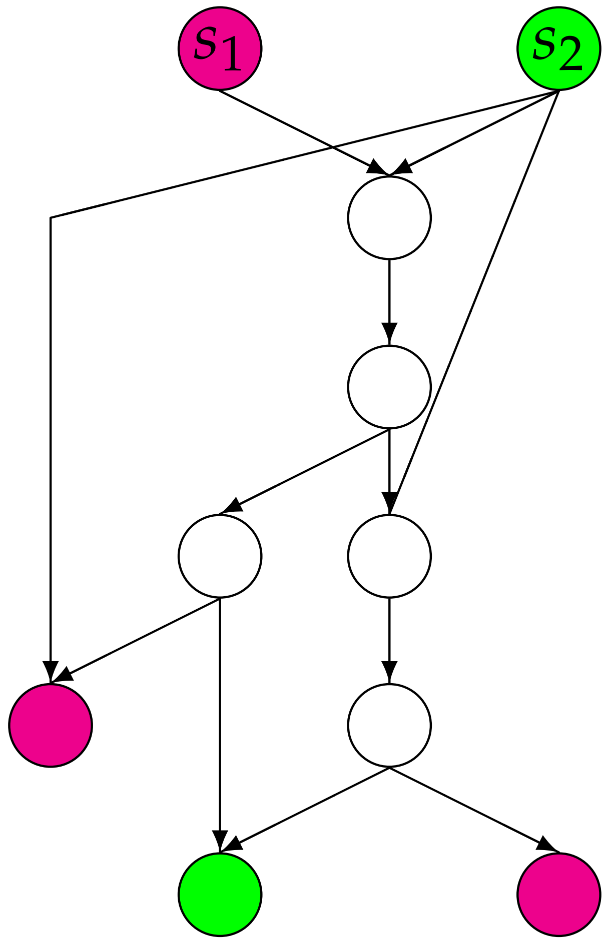


A Feasible Example



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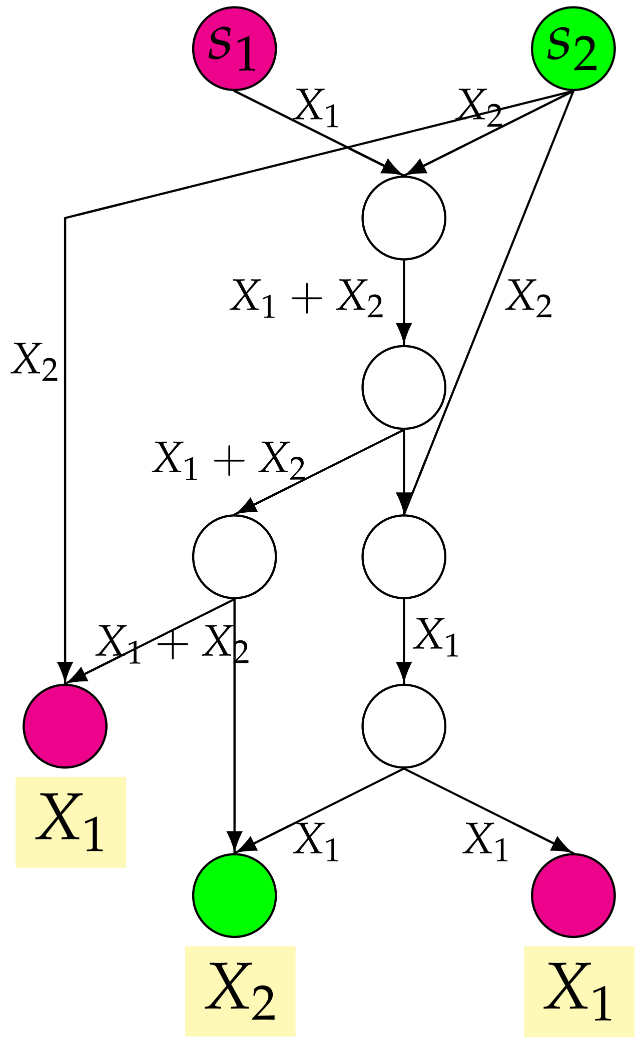
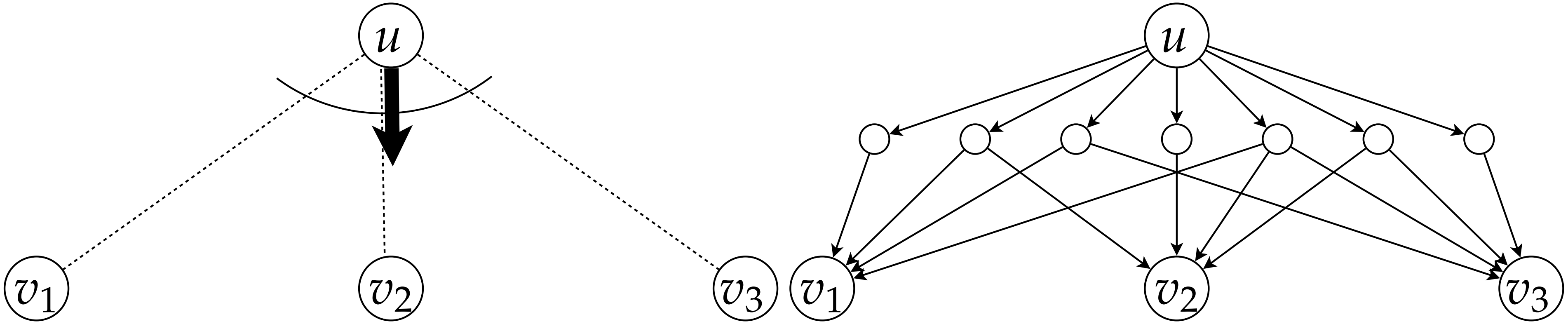


Converting Wireless to Wireline Networks



(a) Wireless Broadcast (b) Wireline Counterpart

Figure 1: Modelling the wireless mulitcast advantage

A commonly used framework, [Wu *et al.* 05] and many others.

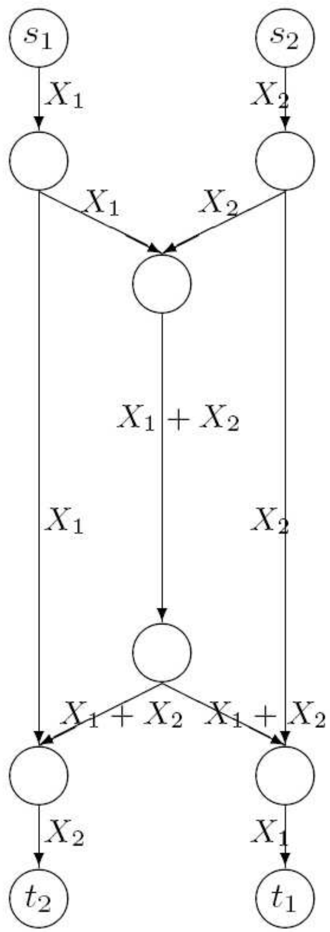
- Each auxiliary node is associated with different power profile.
- Wireless networks become wireline networks with additional scheduling constraints.

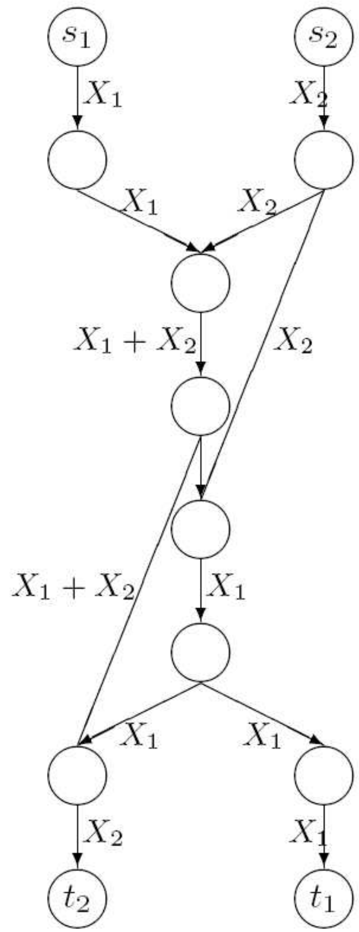
Cross-Layer Optimization

PICC$_{ij}$: Pairwise Intersession network Coding Configuration

The subgraph induced by the six paths $\mathcal{P}$ and $\mathcal{Q}$ between sessions i and j

$\mathcal{PICC}_{ij}$: The set of all PICC$_{ij}$.





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$\mathcal{P}_i$:	the set of all (s_i, t_i) paths
$H_i^k(e)$:	The indicator of the k-th path in $\mathcal{P}_i$
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$$\max_{\mathbf{x}\geq\mathbf{0},\mathbf{r}\in\mathrm{Co}(\mathcal{R})} \sum_{i=1}^{N} U_i\left(\sum_{k=1}^{|\mathcal{P}_i|} x_i^k + \sum_{j:j\neq i}\sum_{l=1}^{|\mathcal{PICC}_{ij}|} x_{ij}^l\right)$$

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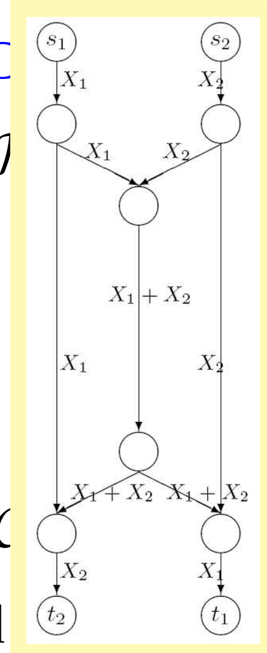

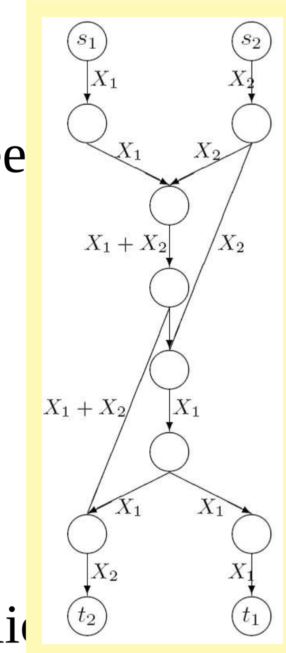

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A Decoupled Opt. Algorithm

Rate (Source s_i) Update

$$\mathbf{x}_i[\tau] = \arg\max_{\mathbf{x}_i \geq \mathbf{0}} U_i \left(\sum_{k=1}^{|\mathcal{P}_i|} x_i^k + \sum_{j:j\neq i} \sum_{l=1}^{|\mathcal{PICC}_{ij}|} x_{ij}^l \right)$$

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Queue-length (Edge) Update

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Rate Balance (Destination) Update **New!**

$$q^l_{ij}[\tau+1] = q^l_{ij}[\tau] + \beta_i \left(x^l_{ij}[\tau] - x^l_{ji}[\tau] \right), \forall j : j > i, \forall l.$$

- The updates are coupled implicitly via the queue lengths.

A Decoupled Opt. Algorithm

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- **Theorem 4** *The decoupled optimization converges to the optimal solution.*

Stability Region

- The decoupled algorithm can also be used as a rate-stabilizing algorithm.
  - Replace the **Rate Update** by the given system rates.
  - **Theorem 5** Any rate $\mathbf{x}$ that satisfies the following can be stabilized.

$$\sum_{i=1}^{N}\sum_{k=1}^{|\mathcal{P}_i|} H_i^k(e)x_i^k + \sum_{(i,j):i\neq j}\sum_{l=1}^{|\mathcal{PICC}_{ij}|} \frac{H_{ij}^l(e)x_{ij}^l}{2} \leq r_e, \forall e \in E$$

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Denote this optimal stability region as Λ.

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- A remarkable similarity between routing (non-coded) and network coding.

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- Results by [Lin *et al.* 06] show that in terms of the stability region, one can still achieve tractable results.
- How far can we borrow the wisdom of routing (non-coded) scheme for pairwise intersession network coding?

Imperfect Scheduling w. PINC

- **Theorem 6** When used as a rate-stabilizing scheme, any rate vector **x** that is within $\gamma\Lambda$ can be stabilized by γ-imperfect scheduling.

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- Wireless networks with stochastic arrivals and departures:
  - N classes of users with Poisson arrival rates λ_i, exponential file sizes with average $\frac{1}{\mu_i}$
  - System load is $\left\{\left(\frac{\lambda_i}{\mu_i}\right) : i = 1, \cdots, N\right\}$.

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- **Theorem 7** When used as a joint rate-control and scheduling scheme, any system load within $\gamma\Lambda$ can be stabilized by γ-imperfect scheduling.

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- Maximum weighted matching: Centralized solver in $\mathcal{O}(N^3)$.
- Direct 1/2 approximations:
  - (Centralized) Greedy maximal matching: Among unmatched edges, choose the one with the largest weight.
  - Distributed locally heaviest matching based on [Preis 01]:
    Step 1: Each node v scans and record the q_e for its unmatched edges.
    Step 2: v then sends a matching request along the unmatched edge with the largest weight. For any edge e, if both its end nodes choose to send a matching request along e, we put e in the current schedule.
    Step 3: Repeat until a maximal matching is found.
    The convergence within at most $N/2$ time slots.

$\frac{1}{2}$-Approximations of MWM

- To further accelerate the convergence (with some tradeoff in performance), a 1/2 approximation scheme based on maximal matching can be constructed.
- The time-multiplexing upper bound of Λ:
  For all $\mathbf{x} \in \Lambda$,

$$\forall v, \quad \sum_{e:e \in E(v)} \frac{1}{c_e} \left(\sum_{i=1}^{N} \sum_{k=1}^{|\mathcal{P}_i|} H_i^k(e) x_i^k + \sum_{(i,j):i \neq j} \sum_{l=1}^{|\mathcal{PICC}_{ij}|} \frac{H_{ij}^l(e) x_{ij}^l}{2} \right) \leq 1.$$

  where $E(v)$ contains all edges adjacent to v counting both incoming and outgoing edges.
- The node-based, double-counting nature of the upper bound enables 1/2 indirect approximation based on maximal matching.

Summary of Imperfect Scheduling Results

- All existing wisdoms of the state-of-the-art imperfect scheduling can be carried over for PINC.
  - Stability for deterministic arrivals.
  - Stability for stochastic arrivals.
  - Maximum weighted matching for the node exclusive interference model.
  - [1/2] Greedy maximal matching.
  - [1/2] A new locally heaviest edge algorithm.
  - [1/2] Maximal matching on time-multiplexing upper bound.

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- (Imperfect) scheduling on wireless networks w. PINC