

# Cross-Layer Optimizations for Inter-session Network Coding on Practical 2-Hop Relay Networks

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joint work with Ness B. Shroff (The OSU) and  
Abdallah Khreishah (Purdue)



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- NC has been formulated for 10+ years. [Ahlswede *et al.* 98].
- **Many promised advantages:**
  - Throughput,
  - Energy and power savings, [Cui *et al.* 08], [Goseling *et al.* 09], etc.
  - Security (cryptography) [Bhattad *et al.* 06], [Ngai *et al.* 09], etc.
  - Error correction [Ahlswede *et al.* 09], [Silva *et al.* 08], etc.
  - Network tomography [Sattari *et al.* 09], [Gjoka *et al.* 08], etc.
  - Speed up computation of the min-cuts and min-cut values [Wu *et al.* 06] [Wang *et al.* 09].
  - Storage [Wu 09], P2P [M. Wang *et al.* 07], etc.
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- For comparison: the capacity-achieving turbo and LDPC codes are commercialized in the span of 5 years.





# The goal of this paper

- Several practical schemes, e.g. COPE and MORE, demonstrate improvement in certain scenarios.
- Different competing techniques: 1-hop intersession NC (COPE), spatial-diversity-based opportunistic routing (MORE), cross-layer optimizations.
- Question: How much gain can we expect from coding? How does coding fare when combined with other techniques?
  - The capacity region of COPE is still unknown (let alone with cross-layer optimizations).
  - Many difficulties when combining COPE and MORE.
- We will **try to answer these questions analytically** for a somewhat restricted, but quite **practical scenario**.



# Outline

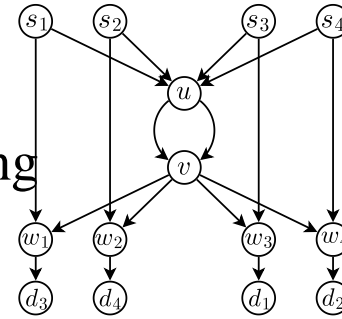
- Existing results on **multi-hop wireline networks**:
  - Cross-layer, Intersession NC, the achievability results.
  - Modeling wireless channels: From wireline to wireless?
- A more practical setting on **2-hop relay networks**:
  - **Packet erasure channels** (PECs) and the **freq. of feedback**.
- Capacity results of
  - Broadcast PECs with side information.
  - Intersession NC on 2-hop relay networks.
  - LP-based solutions with cross-layer optimization
- Simulation results that compare the gains of **cross-layer optimization**, **opportunistic routing**, **intersession NC**.



# Achievable rates with Multi-hop networks

Achievable rates for general topology:

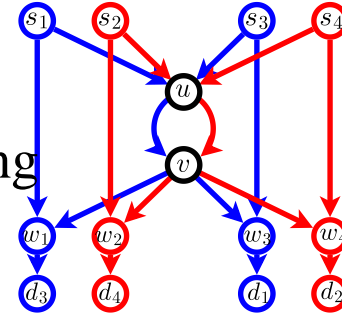
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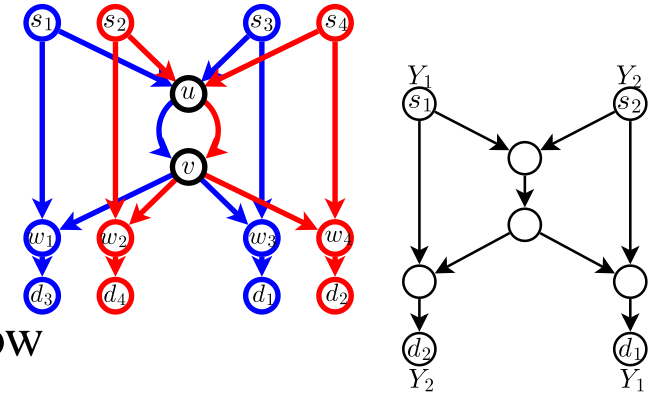
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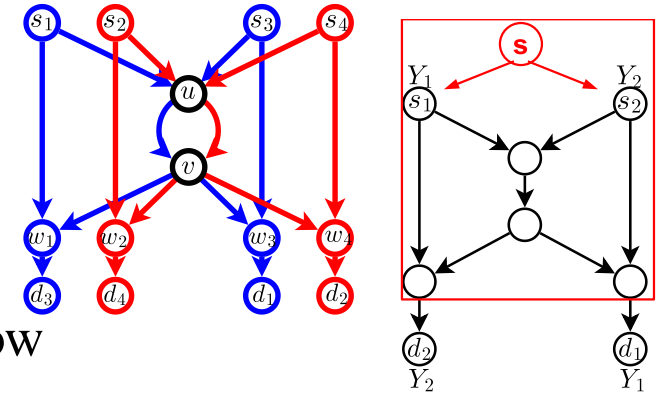
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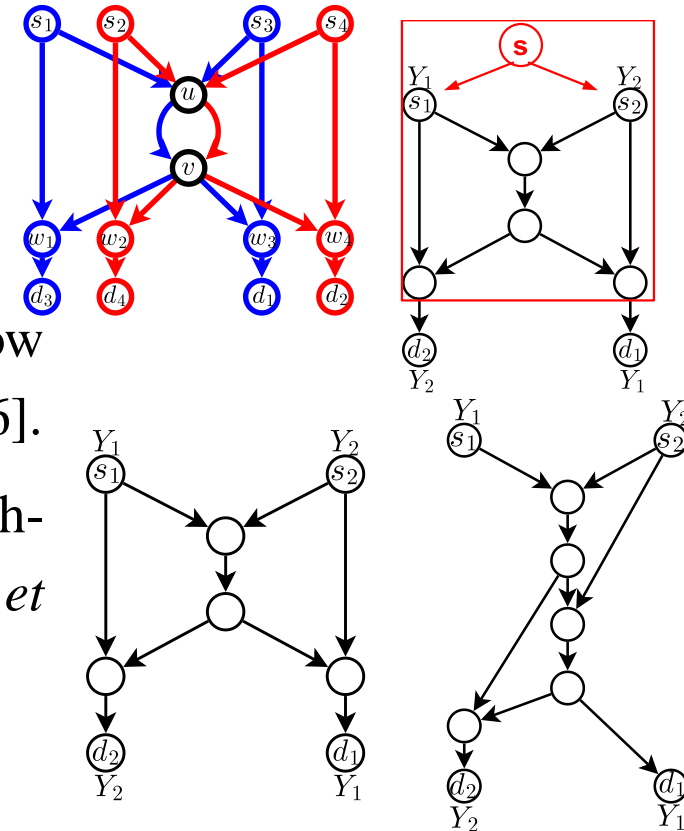
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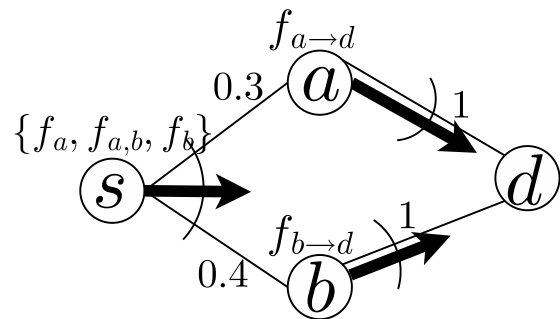
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- When limiting to two multicast symbols, the graph-theoretic capacity characterization is known [Wang *et al.* 07]. + LP superposition.

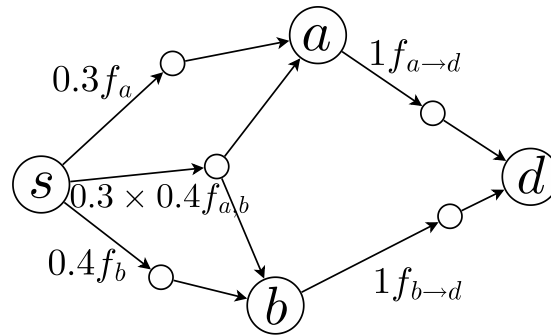
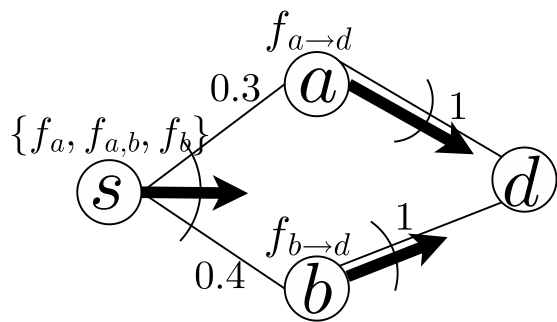


# From Wireline to Wireless





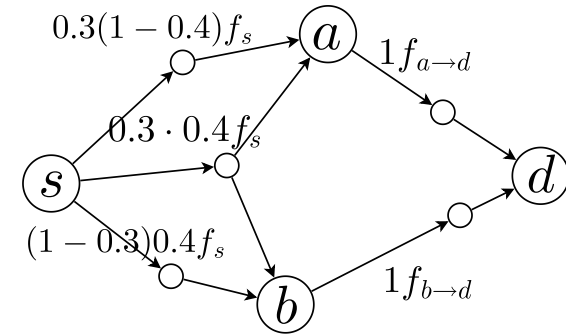
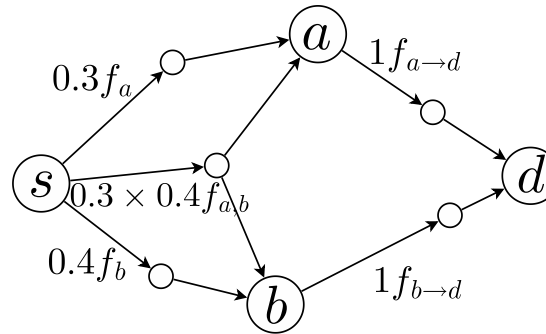
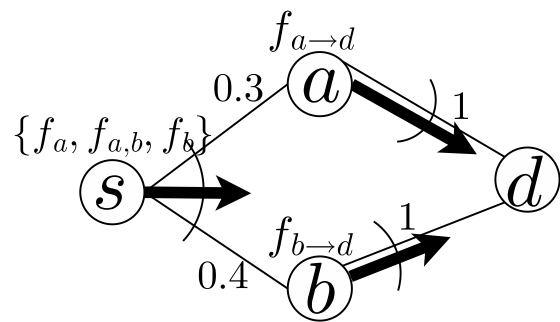
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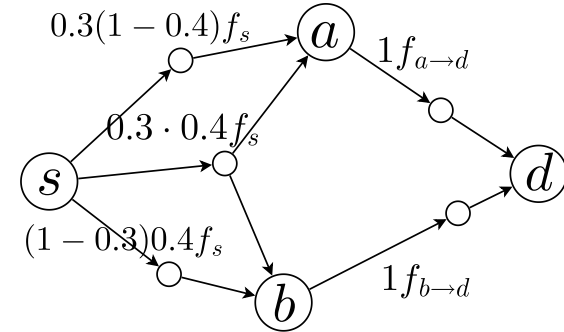
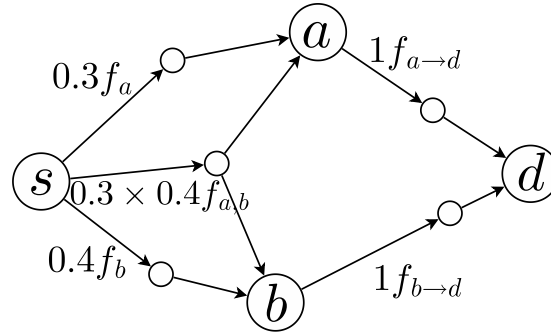
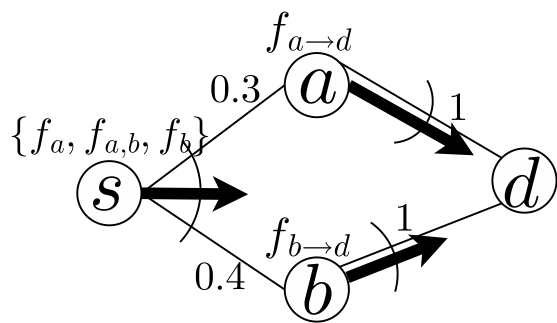
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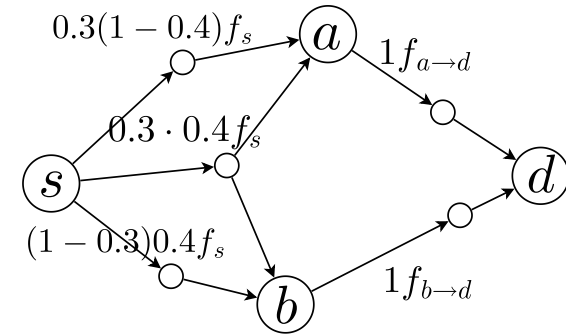
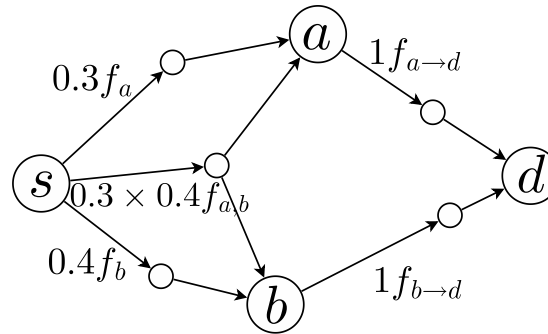
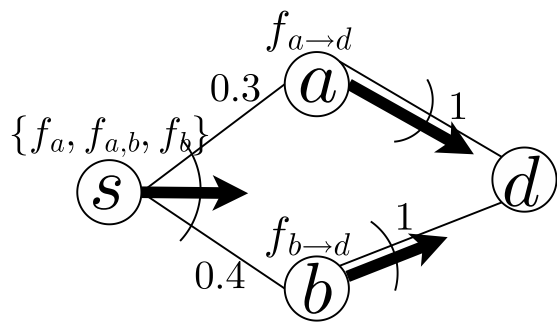
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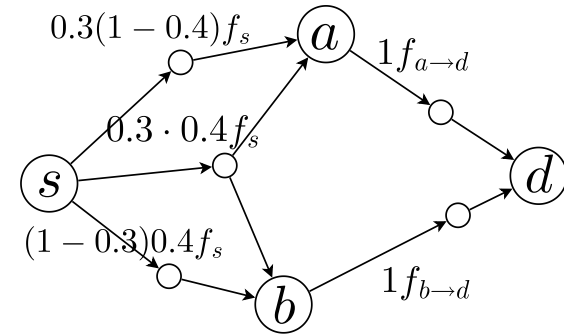
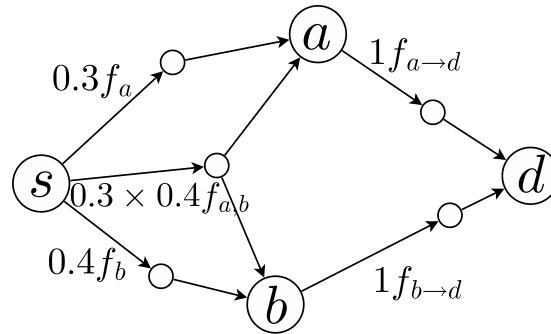
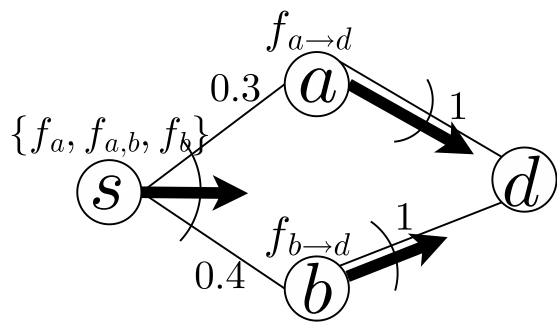
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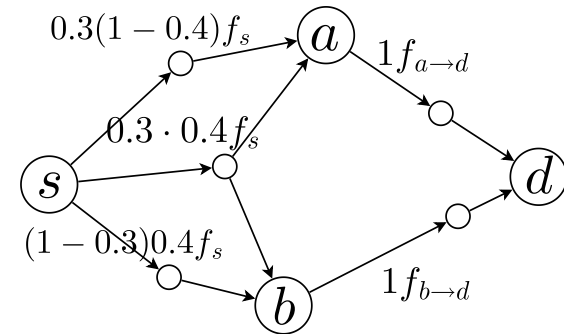
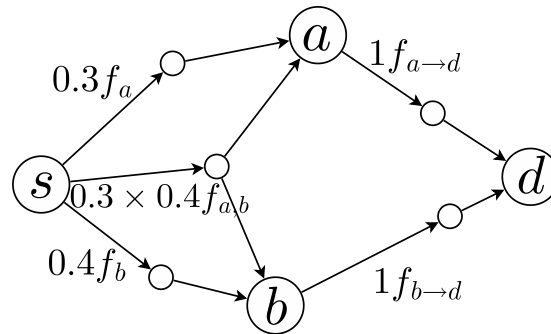
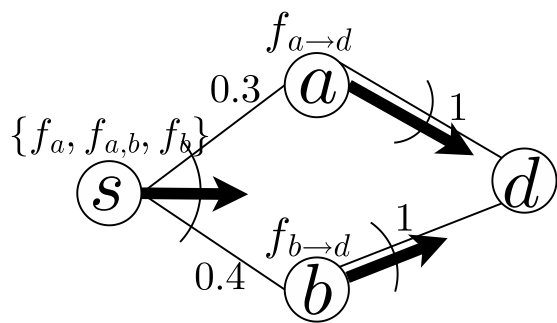
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- To closely model the scenario, we consider the packet erasure channels for 2-hop, single-relay networks.



# Our setting

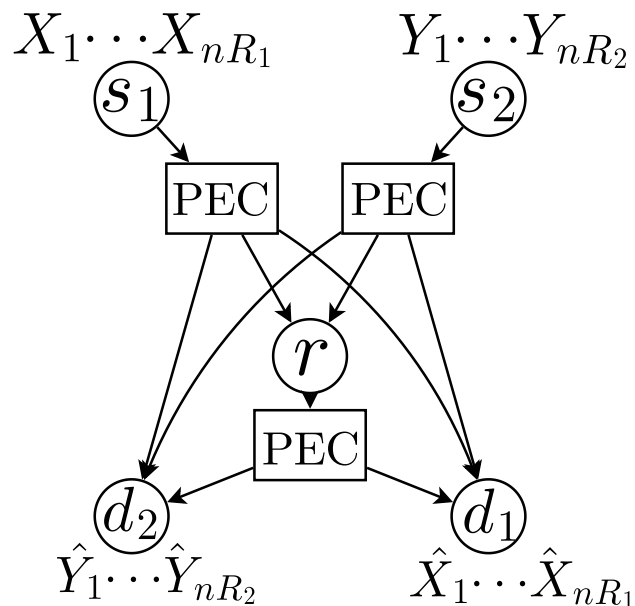
Memoryless packet erasure broadcast channels: Example: A 1-to-2  
PEC is governed by the **success probabilities**  $p_{s;12}, p_{s;12^c}, p_{s;1^c2}, p_{s;1^c2^c}$ .



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Two-hop relay networks:



- Sequentially,  $s_1$ ,  $s_2$ , and  $r$  each can send  $n$  packets.
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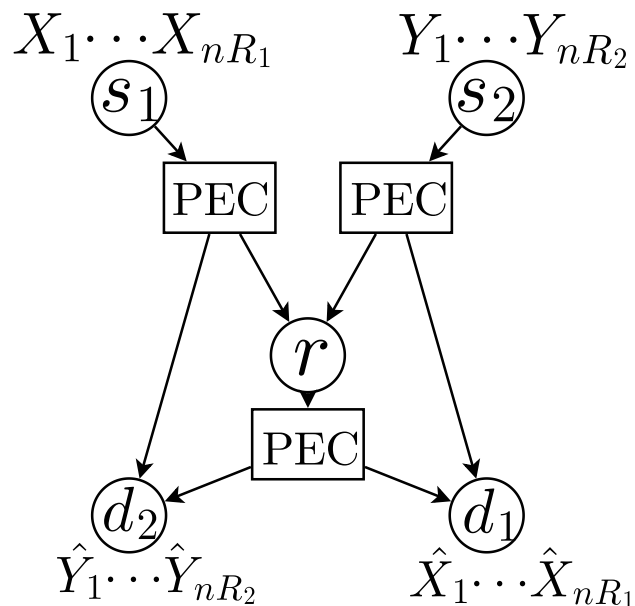




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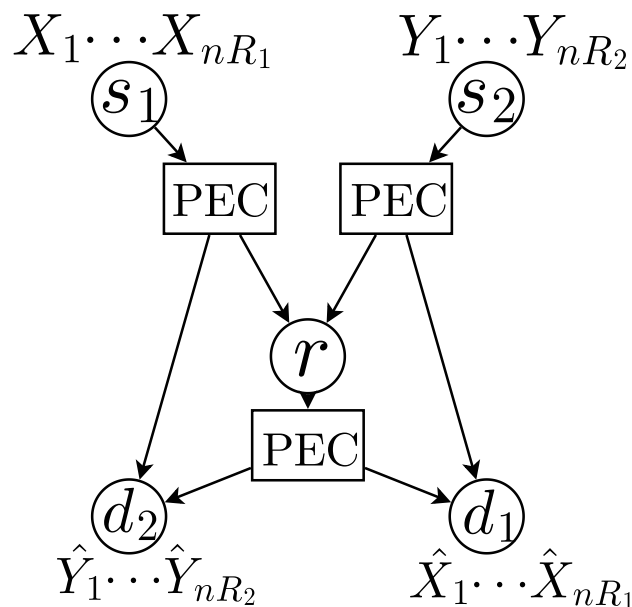
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PEC parameters:

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$$p_{r;12}, p_{r;12^c}, p_{r;1^c2}, p_{r;1^c2^c}.$$

$$p_{r;1} \triangleq p_{r;12} + p_{r;12^c}, \quad p_{r;1^c} \triangleq p_{r;1^c2} + p_{r;1^c2^c};$$

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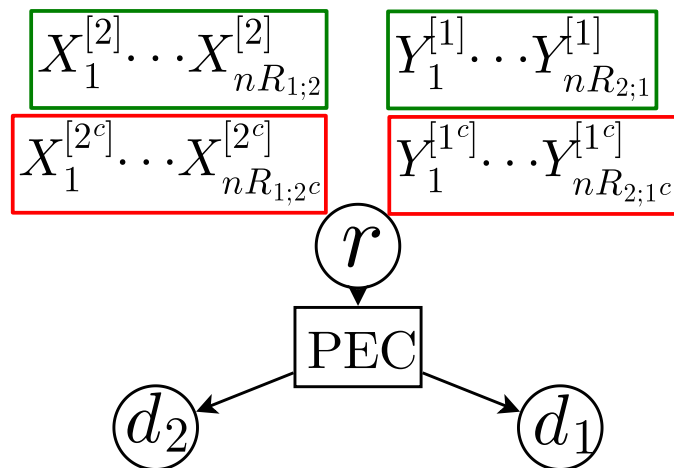
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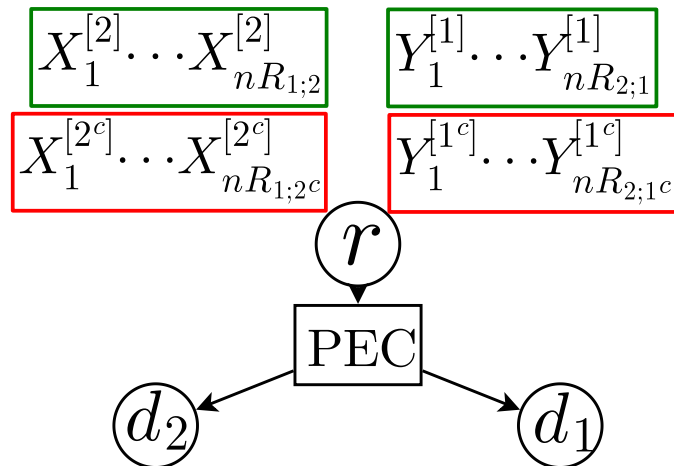
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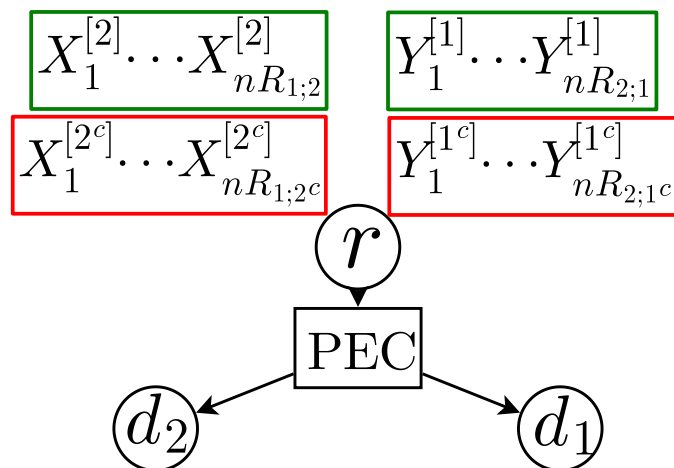
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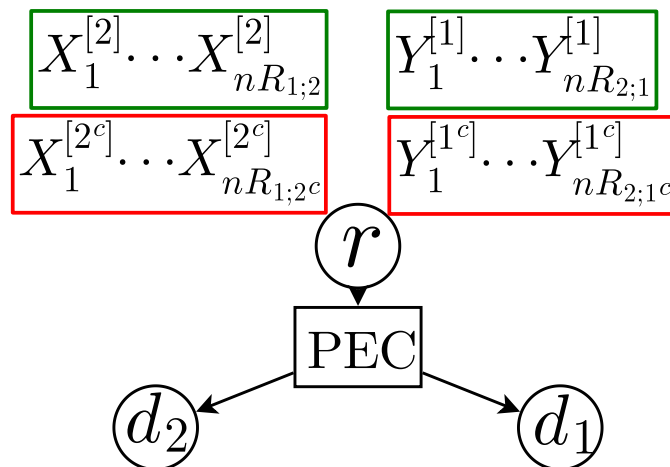
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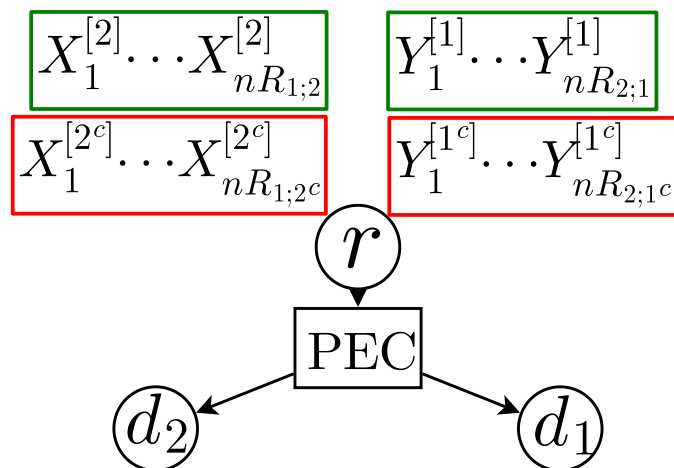
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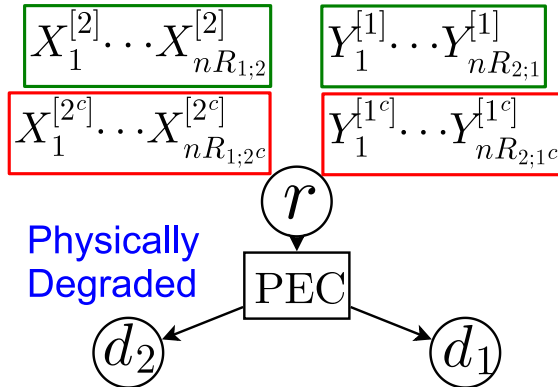
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- Cap. of 3-user PEC BC w. SI., & 3-session 2-hop relay networks.



# An Intuitive Argument



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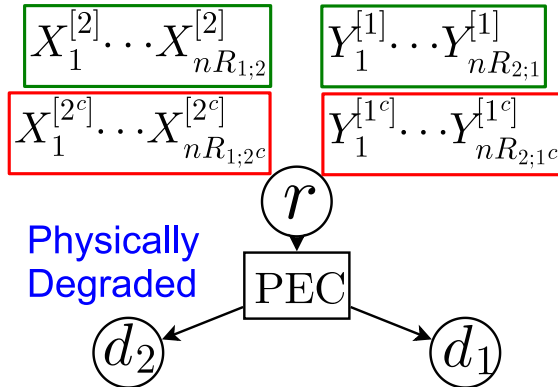
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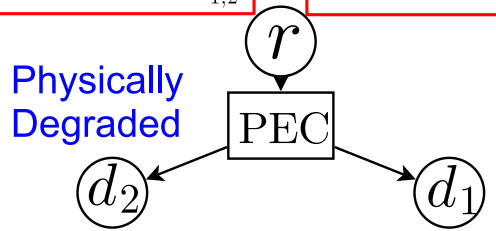
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$$\begin{array}{|c|} \hline X_1^{[2]} \dots X_{nR_{1;2}}^{[2]} \\ \hline \end{array} \quad \begin{array}{|c|} \hline Y_1^{[1]} \dots Y_{nR_{2;1}}^{[1]} \\ \hline \end{array}$$

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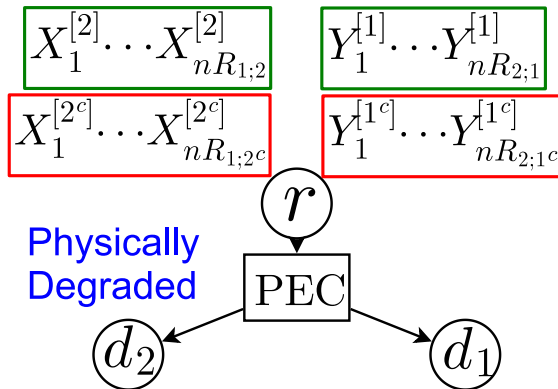
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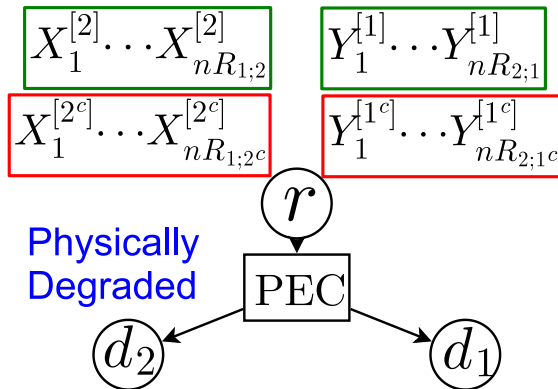
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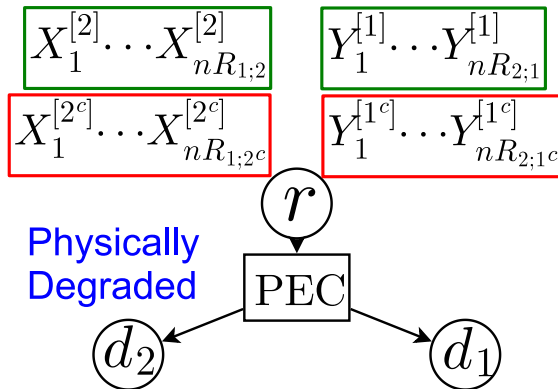
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- Observation 3.1: To send  $nR_{1;2^c}$  to  $d_1$ , we at least need  $\frac{nR_{1;2^c}}{p_{r;1}}$  symbol use, which can be achieved by using an MDS code that encodes  $nR_{1;2^c}$ .



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- Observation 3.1: To send  $nR_{1;2c}$  to  $d_1$ , we at least need  $\frac{nR_{1;2c}}{p_{r;1}}$  symbol use, which can be achieved by using an MDS code that encodes  $nR_{1;2c}$ .
- Observation 3.2\*: Then among the  $nR_{1;2c}$  basis vectors received by  $d_1$ , they will contribute at least  $\frac{p_{r;2}}{p_{r;1}} nR_{1;2c}$  interfering basis vectors at  $d_2$ .



# Cap. 2-user PEC BS w. SI

The cap. outer bound:

$$d_1 \text{'s perspective: } nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}$$

$$d_2 \text{'s perspective: } nR_{2;1} + nR_{2;1^c} + \frac{p_{r;2}}{p_{r;1}} nR_{1;2^c} \leq np_{r;2}.$$





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The achievability: A 2-stepped coding scheme.



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The achievability: A 2-stepped coding scheme.

- Step 1: Randomly mix the packets from  $nR_{1;2}$ ,  $nR_{2;1^c}$ , and  $nR_{2;1}$  and send

$$\max \left( \frac{nR_{1;2} + nR_{2;1^c}}{p_{r;1}}, \frac{nR_{2;1^c} + nR_{2;1}}{p_{r;2}} \right) \text{ such randomly gen. pkts.}$$



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- Step 2: Randomly mix the packets from  $nR_{1;2^c}$  and send

$$\frac{nR_{1;2^c}}{p_{r;1}} \text{ such randomly gen. pkts.}$$



# Combine it with $s_i \rightarrow r$ coding

$$d_1 \text{'s perspective: } nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}$$

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Given that  $R_1 = R_{1;2} + R_{1;2^c}$ , maximizing  $R_2$  is equivalent to allocating the smallest  $R_1$  to  $R_{1;2^c}$ .



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- By  $s_1$  performing random linear NC, we max. the overhearing

$$nR_{1;2^c} = (nR_1 - np_{s_1;2})^+$$

- By  $s_2$  performing random linear NC, we max. the overhearing

$$nR_{2;1^c} = (nR_2 - np_{s_2;1})^+$$



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$$d_1 \text{'s perspective: } nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}$$

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- By  $s_2$  performing random linear NC, we max. the overhearing

$$nR_{2;1^c} = (nR_2 - np_{s_2;1})^+$$

- Therefore:  $R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$

$$R_2 \leq p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - p_{s_1;2})^+$$



# The Capacity Regions (Cont'd)

$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

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# The Capacity Regions (Cont'd)

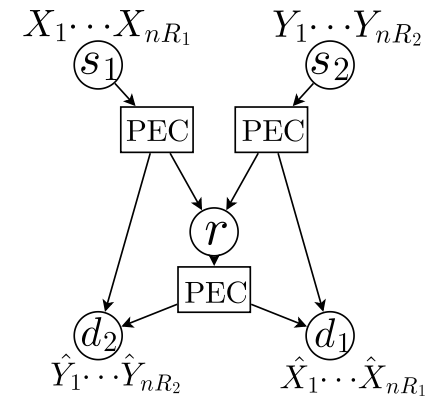
$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

$$R_2 \leq p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - p_{s_1;2})^+$$

● Final Results:

$$R_1 \leq \min \left( p_{s_1;r}, p_{r;1} - (R_2 - p_{s_2;1})^+ \right)$$

$$R_2 \leq \min \left( p_{s_2;r}, p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - p_{s_1;2})^+ \right)$$





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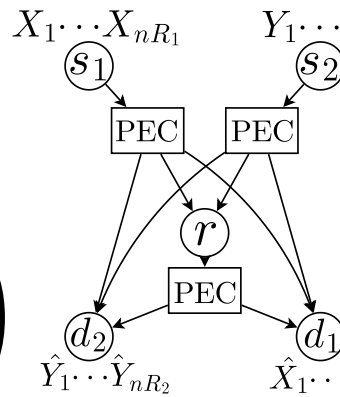
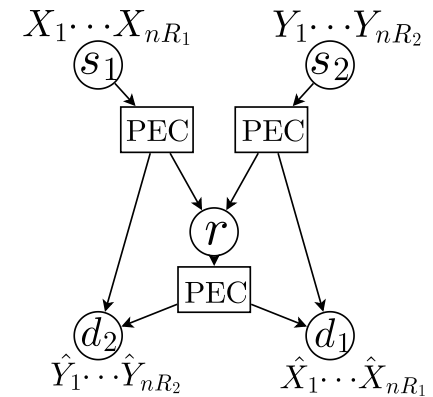
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- With opp. routing (jump over 2 hops):

$$R_1 \leq \min (p_{s_1;1Ur}, p_{s_1;1} + p_{r;1} - (R_2 - p_{s_2;1U2})^+)$$

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# The Capacity Regions (Cont'd)

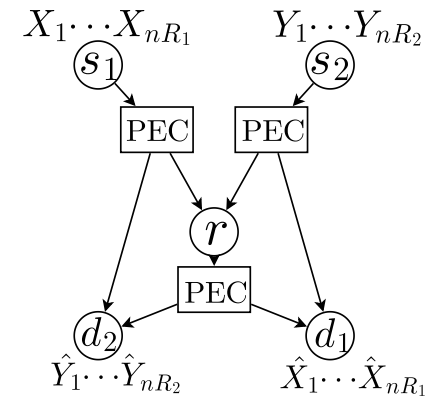
$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

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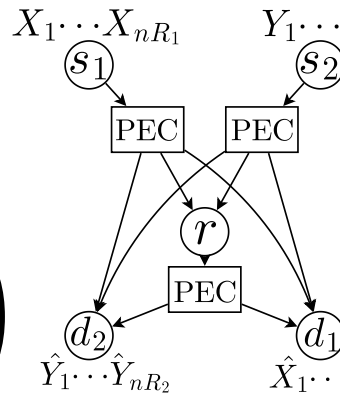
$$R_2 \leq \min \left( p_{s_2;r}, p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - p_{s_1;2})^+ \right)$$



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- Each round of  $3n$  packets  $\Rightarrow$  variable scheduling  $t_{s_1}, t_{s_2}, t_r$ .



# The Capacity Regions (Cont'd)

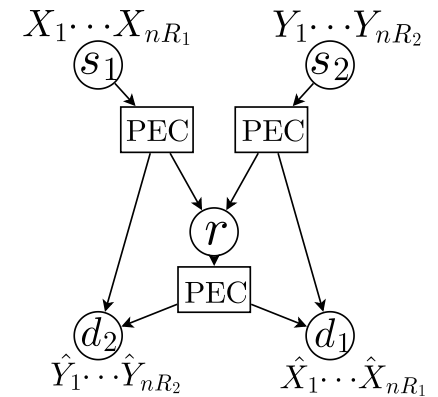
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● Final Results:

$$R_1 \leq \min \left( t_{s_1} p_{s_1;r}, t_r p_{r;1} - (R_2 - t_{s_2} p_{s_2;1})^+ \right)$$

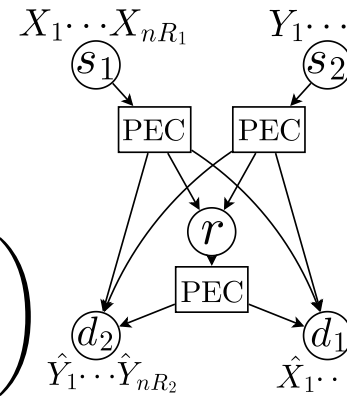
$$R_2 \leq \min \left( t_{s_2} p_{s_2;r}, t_r p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - t_{s_1} p_{s_1;2})^+ \right)$$



● With opp. routing (jump over 2 hops):

$$R_1 \leq \min \left( t_{s_1} p_{s_1;1Ur}, t_{s_1} p_{s_1;1} + t_r p_{r;1} - (R_2 - t_{s_2} p_{s_2;1U2})^+ \right)$$

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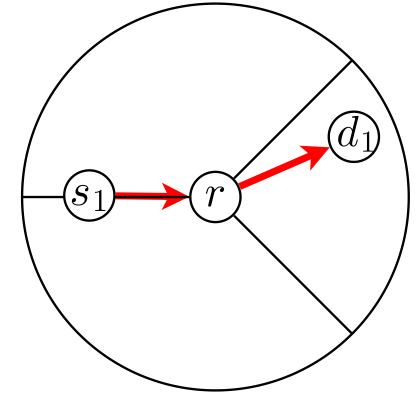
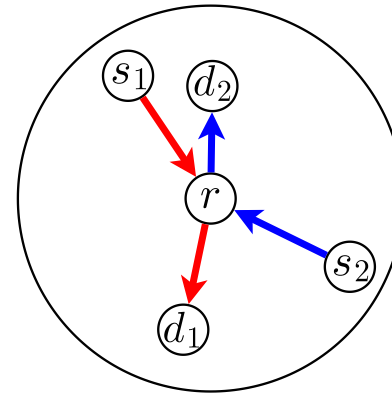
# Results

- Our linear formulation contains three orthogonal components.
  - Intersession NC (INC): Whether to use the new cap. region.
  - Opp. Routing (OpR): Randomly vs. pre-planned 2-hop tx.
  - Cross-layer (CL): Fixed schedule  $1/3, 1/3, 1/3$  vs.  $t_{s_1}, t_{s_2}, t_r$ .
- Mixed & match: (INC, OpR,  $\times$ CL), (INC,  $\times$ OpR, CL), ...
  - The baseline scheme: **Preplanned multipath routing with fixed schedules** ( $\times$ INC,  $\times$ OpR,  $\times$ CL).
- The capacity region has been generalized to  $N = 3$  sessions, and empirically tight upper and lower bounds for  $N > 3$ .

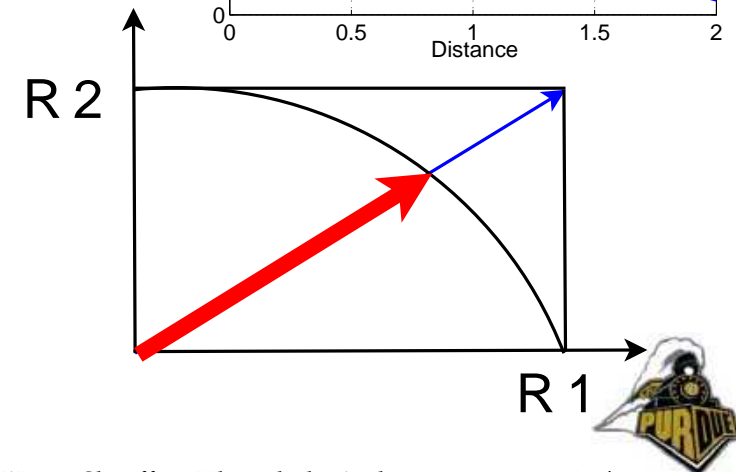
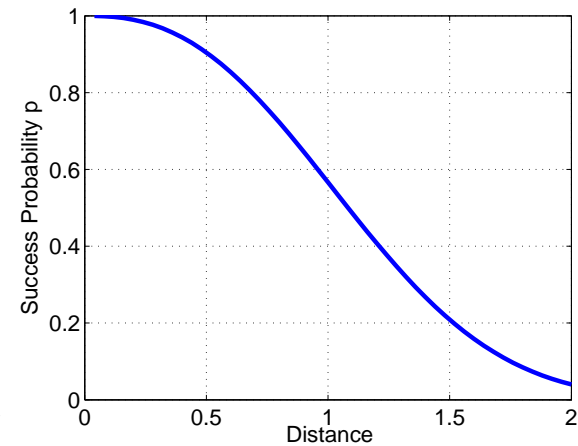


# Simulation Settings

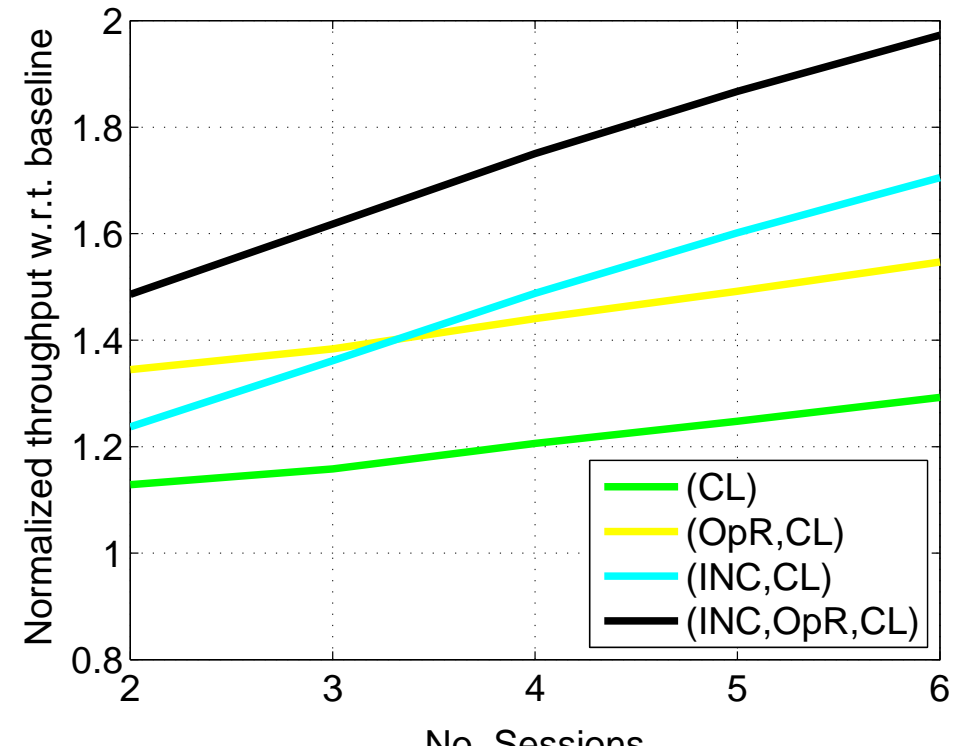
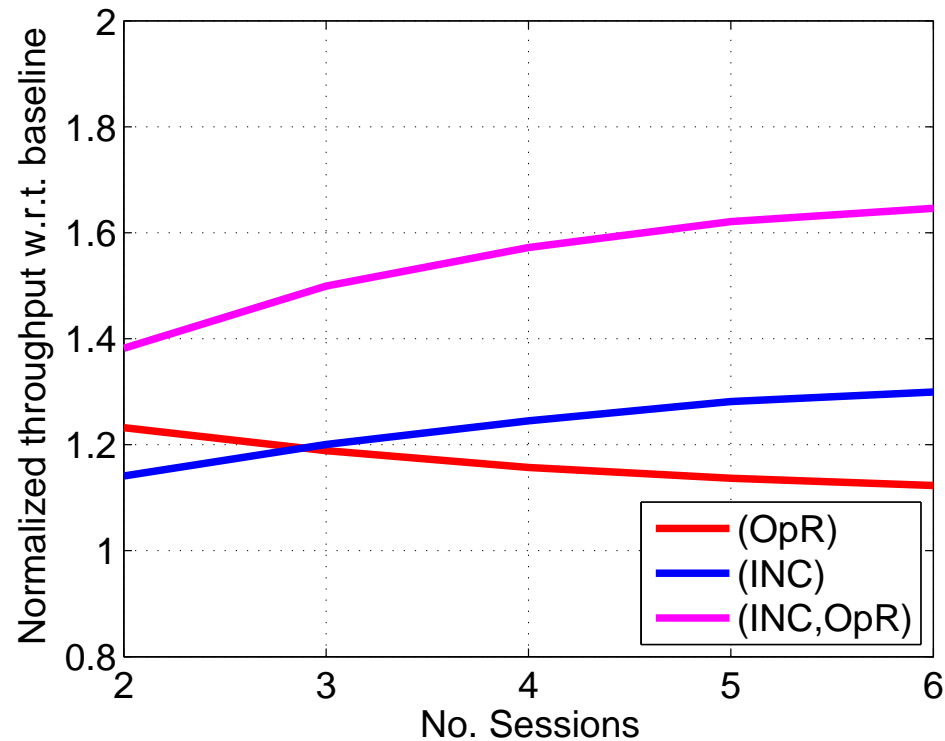
- The 2-hop random networks.



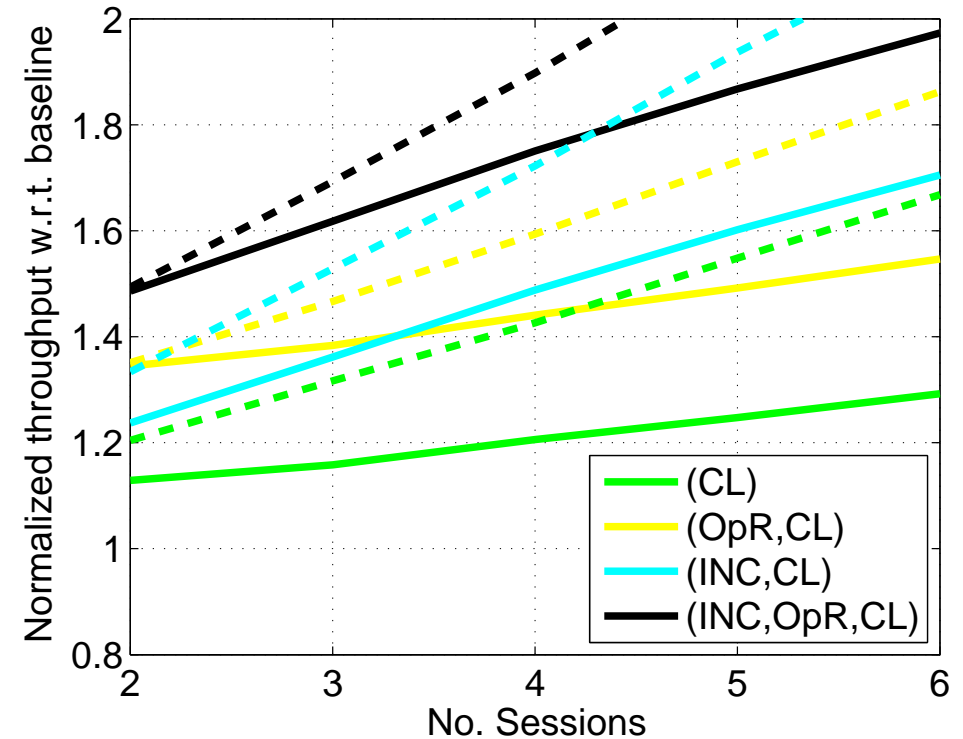
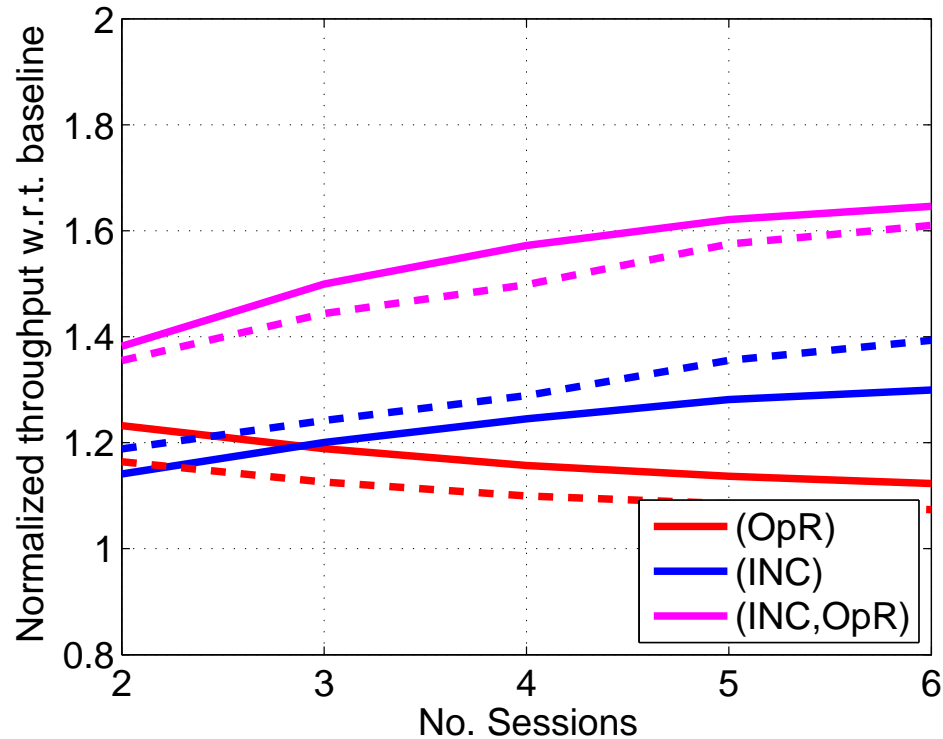
- The success probability versus distance (By Rayleigh fading channels w.  $\alpha = 2.5$ ):
- Objective function: Achieving the percentage of the unicast capacities.
- Baseline scheme: ( $\times$ INC,  $\times$ OpR,  $\times$ CL)
- 2000 random topologies



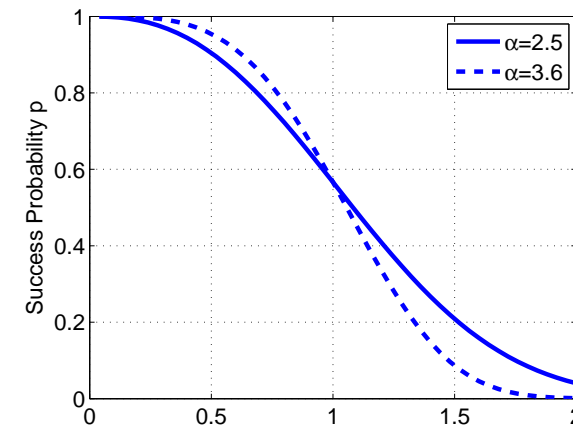
# The throughput improvement w.r.t. $N$



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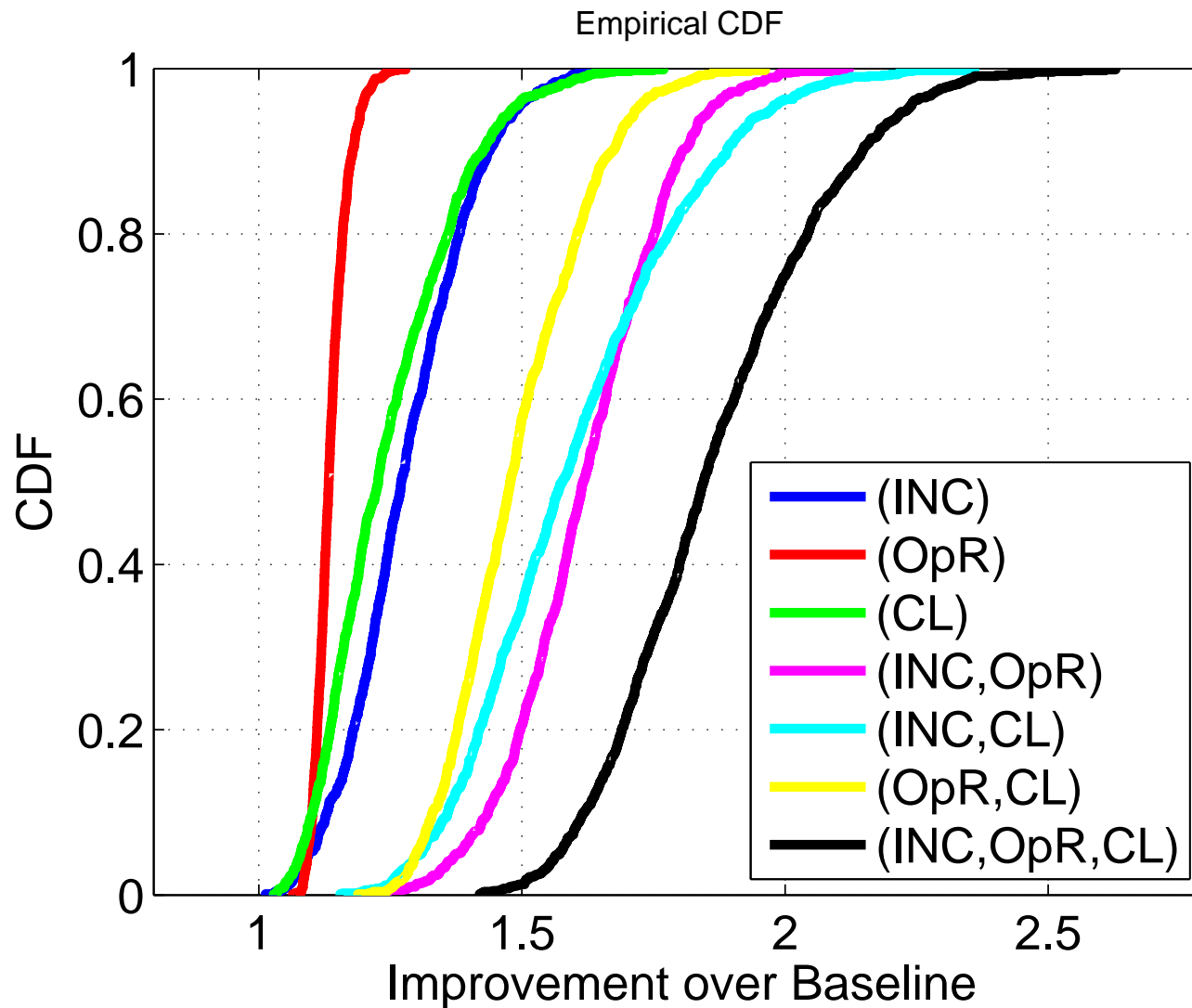


For different channel conditions:  $\alpha = 3.6$ .



# The throughput improvement w.r.t. $N$

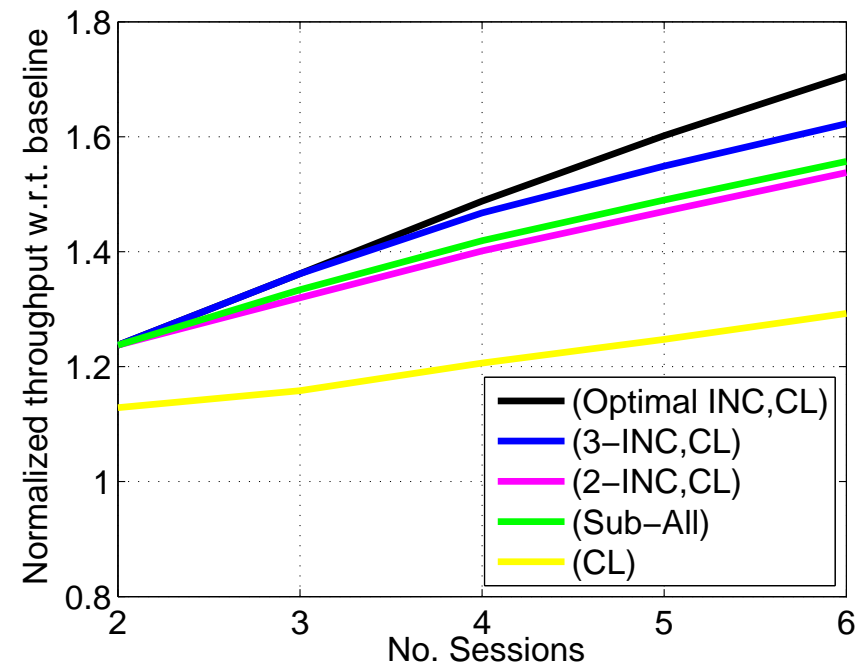
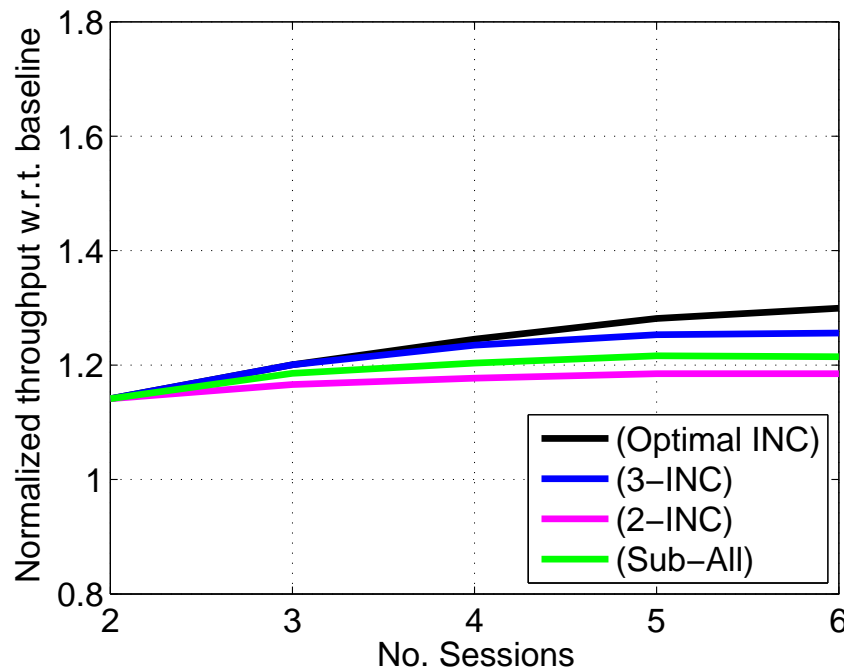
The cdf of the percentage gain when  $N = 5$ .





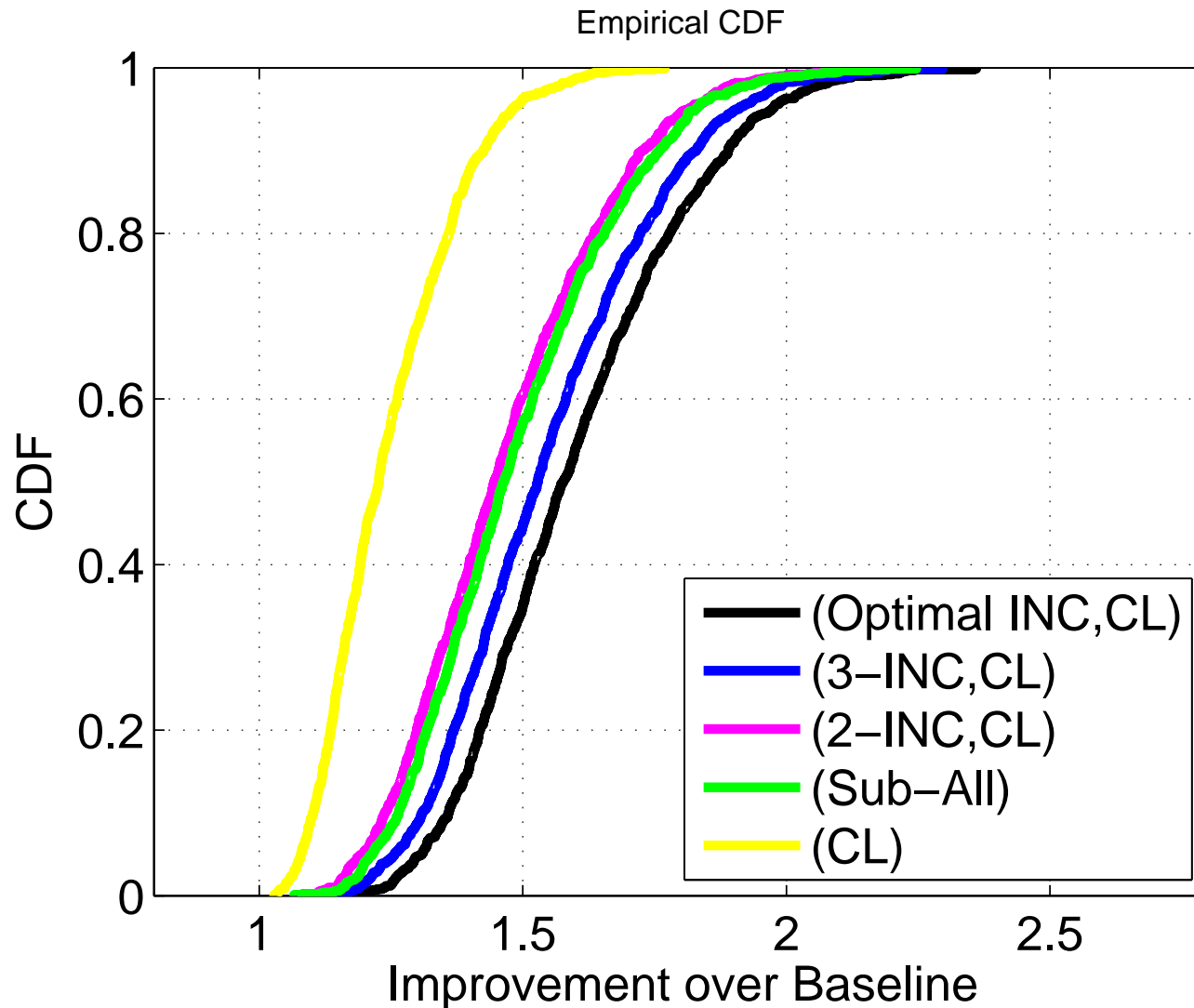
# The Practical Schemes

- Optimal INC scheme is complicated (Optimal). All w.  $\times$ OpR.
- Optimal INC on Limited # of coded sessions + optimal time multiplexing: 2-session INC (2-INC), and 3-session INC (3-INC)
- Suboptimal INC on all coded sessions + optimal time multiplexing: (Sub-All). The importance of good INC schemes.



# The Practical Schemes (cont'd)

The cdf of the percentage gain when  $N = 5$ .



# Conclusion

- Multi-hop-based vs. 2-hop-based Intersession Network Coding
  - Wireless-to-wireline conversion: 15-20% throughput improvement over non-coded solution.
  - The settings may not be practical.
- Practical 2-hop relay network capacity.
  - PEC, round-based tx, and infrequent reception report feedback.
  - Examine the performance gain of three orthogonal techniques: Intersession NC, Opp. Routing, & Cross layer.
  - An optimal scheme has 100-120% net improvement over **feedback-free** multi-path routing
  - Cross-layer is powerful, but is a global optimization. Both OpR and INC in this paper are local single-hop computation.
  - NC is a promising technique.

